BASICS OF ANALOG COMPUTERS

T. D. TRUITT and A. E. ROGERS

basics of

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PREFACE

Living in a dynamic world we see around us continually changing conditions. All natural and man-made devices are subjected to the unending action of forces — electrical, mechanical, and chemical. The many forms the forces take, the many apparently different kinds of energy, cause these devices to appear to behave in an infinite variety of ways. There appears superficially to be no relation whatsoever between their different behaviors, particularly between those of physical systems that to the human senses are quite distinct. The mechanical vibration of an automobile on a bumpy road appears unrelated to the oscillation of a voltage in an electrical circuit or to the fluctuation of magnetic field strength in a coil. On closer inspection, however, one can often discern similarities between the variations of certain characteristics of these seemingly distinct devices and systems. There is an analogy between them that relates the otherwise distinct devices in such a way that the behavior of two, or possibly many, can be determined by experimenting with one. This correspondence in behavior is used most effectively in an analog computer where building blocks are provided to construct conveniently-controlled models which behave similarly to engineering systems of interest. The vibration of the automobile suspension, the strength of the magnetic field, the frequency of sound from the loudspeaker diaphragm, the temperature of the missile nose-cone, each can be investigated through study of the voltage behavior within the electronic circuits of the analog computer.

A book describing the principles and operation of analog computers has long been needed for the instruction and interest of scientifically minded readers. Endeavoring to fill this need, the authors have written this book for the college student desiring to increase his understanding of the basic principles of engineering computation, the engineer and technician concerned with computer applications in their fields, and others alert to ever-expanding technological developments.

Analog computing in this book is presented graphically, and introduced and dealt with in its simplest terms. The most useful computing devices, those most easy to use in the analog model-building process, are discussed, and the essential mathematical tools are presented. Volume 1 includes descriptions of many kinds of analog computers and devices. Volume 2 gives detailed attention to the computer that is most flexible and easy to use — the d-c electronic differential analyzer. Volume 3 presents some of the programming techniques and interesting applications common to the field of analog computation. By the liberal use of art work, and simple descriptions, an attempt has been made to present this fascinating but demanding subject in such a way that little or no previous knowledge is required to understand it.

In the preparation of the manuscript and artwork for publication the thoroughness and energetic help of Miss Cynthia Stephens is acknowledged with appreciation.

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T. D. Truitt
A. E. Rogers

Princeton, N. J.
December 1960

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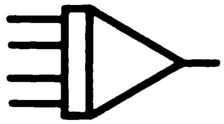
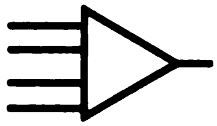
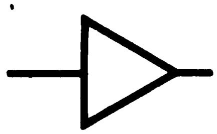
SYMBOLS



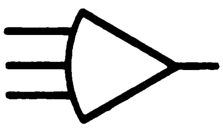
Integrating amplifier



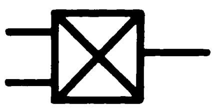
Summing amplifier



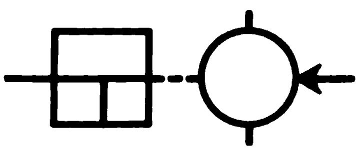
Inverting amplifier



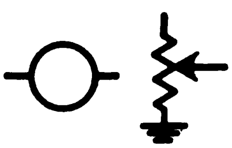
High-gain amplifier



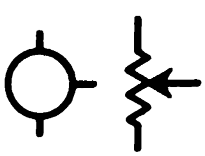
Electronic multiplier



Servomultiplier



Hand-set potentiometer, grounded



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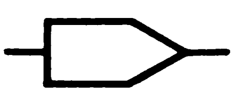
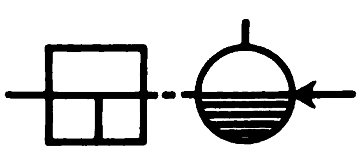
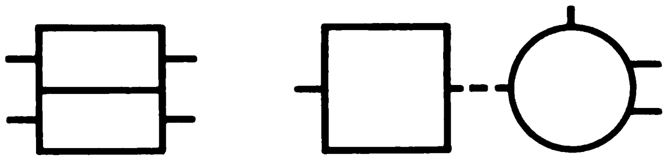


Diode function generator



Pot-padder function generator (servo-driven)

SYMBOLS



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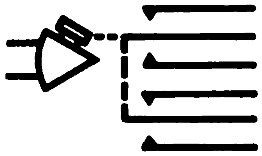
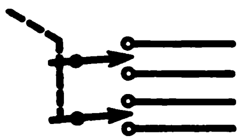
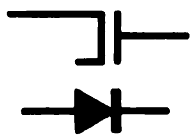


Relay amplifier, relay comparator



Function switch, manually operated



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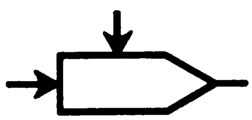

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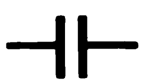
Resistor



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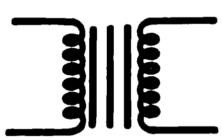
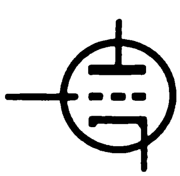
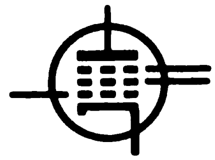


Transformer

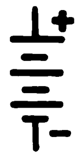


Triode vacuum tube

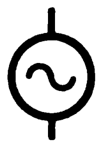
SYMBOLS



Pentode vacuum tube



Battery



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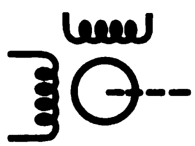

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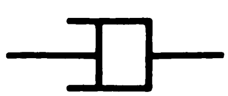


A dashpot, symbol for a
mechanical damping force

Spring

Abbreviations*

ac or a-c	alternating current; referring to variables which alternate sinusoidally from plus to minus values
afc	automatic frequency control
agc	automatic gain control
cps	cycles per second
dc or d-c	direct current; referring to variables which do not alternate in signs periodically
DDA	digital differential analyzer
dfg	diode function generator
dvm	digital voltmeter
hga	high-gain amplifier
IC	initial conditions
kc	kilocycles per second
mc	megacycles per second

* See also NOTATION, p. 3-86.

Volume 1

ANALOG COMPUTING PRINCIPLES AND TECHNIQUES

Chapter 1

INTRODUCTION TO ANALOGS

Experimental Science

From the early dawn of scientific thought, researchers have relied heavily upon empiricism and inductive logic. Only by years of constant observation, trial and error experimentation with physical objects, living organisms, and often models of larger systems, could investigators learn the detailed behavior of their subjects, from which to induce the general order of things. Observation of nature, and experimentation, gradually built up a body of scientific knowledge upon which general principles and physical laws could be based.

Centuries of such fact finding and theory building have brought about the evolution of three groups of scientists (Fig. 1-1). Always there is the

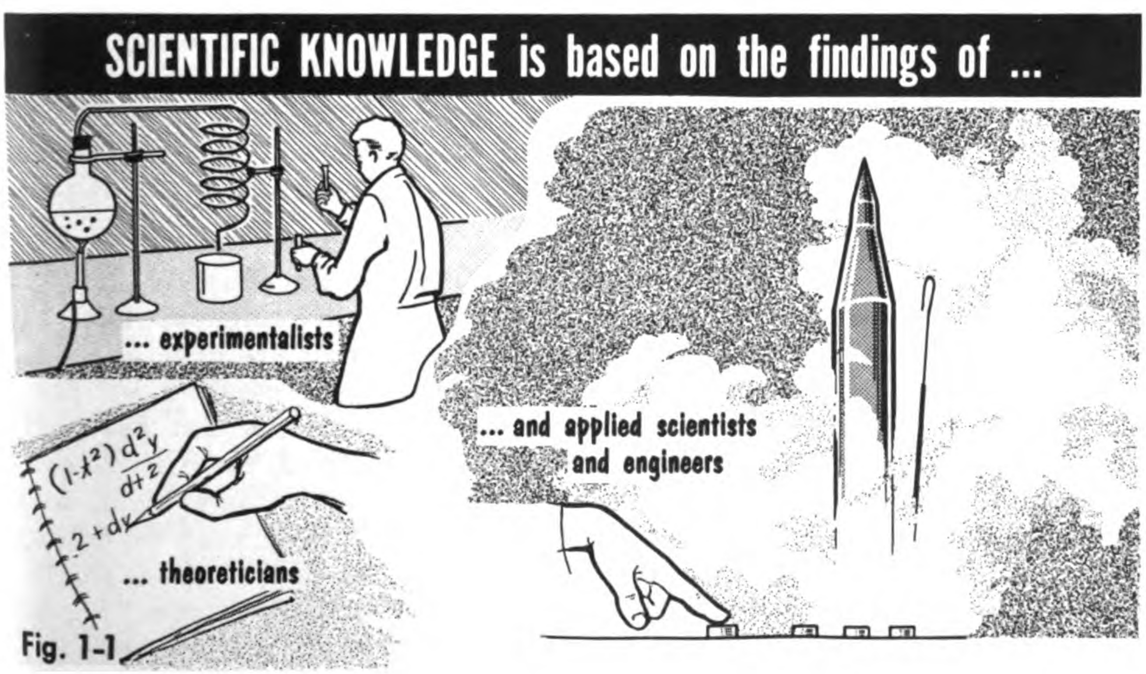


Fig. 1-1

researcher, the investigator, the *experimenter*, ever in search of new facts to support or refute old theories, or to indicate a path to new theories. Then there is the *theoretician* deducing new theory from established principles, predicting results of untried and seemingly impossible experiments. Finally, the applied *scientist* and engineer. No purists, their quest of knowledge and truth is secondary to the desire to *do* something with the acquired knowledge of science; they must find new ways to put ideas to work for man — to raise the standard of living or to wage war.

Each of these scientists, regardless of his point of view or objective, must ultimately resort to the experiment, the test, or the trial run. For the hypothesis is vulnerable lacking a proof; the proposition worthless in the face of a counterexample. As for the engineer, his employers know there is many a failure twixt the thought and the product. A breadboard circuit, a working model, a pilot plant, or a trial run, are engineering experiments that have saved millions of dollars.

Experiments of today, however, have advanced far beyond the apple dropping or pH testing level. Each piece of research is a very careful probing of perhaps, the behavior of a simple but elusive subatomic particle, of a complex organic structure, or of a complicated system of interdependent physical devices and subsystems. Thus in all branches of science we find men who are intimately concerned with the dynamic behavior of physical (sometimes hypothetical) systems subject to particular environments, and to environmental disturbances.

Models

The ways in which a scientist goes about studying his subject are myriad. In some cases the simplest experiment may throw light on the most profound and universal theory. The Michelson-Morley experiment (1880) that disproved through the use of rotating mirrors, the existence of an undetectable ether substance surrounding the earth which was purported to act as a propagation medium for light waves, is the classic example. On the other hand, to establish a relatively insignificant fact may require multimillion dollar atom smashers.

It is helpful for the purposes of this book to classify the subjects of scientific investigation into three groups: (a) those that are phenomenological and require the usual empirical methods; (b) those that are developmental and direct experimentation, but which may be well handled by scale models; and (c) those research and development problems that have sufficient structure to be either mathematically well defined, or to have well-established patterns of behavior. As vague as the latter may seem, it is precisely this group (c) that forms the basis for what we shall be calling *analog simulation*. We will see that if a system can be described mathematically, experiments are not restricted directly to that system; it may be possible to find another system of different substance but described by the same equation. If so, depending on the nature of the two systems,

great economies of effort may be afforded by experimenting with the second system. Consider for example, the possibility of investigating the motion of celestial bodies with a second system which is an electronic circuit.

Even when a subject system is not mathematically describable, if a second system can be found which behaves in an analogous fashion, experimentation with the latter to exactly duplicate the behavior of the primary system may well lead to greater understanding of both.

The investigator is always faced with the design of an experiment. If he cannot work directly with his subject, or if the experiment is too expensive or dangerous, he must seek a suitable model. Either a scale model, a mathematical model, or an *analog model* may help him perform the desired experiment. One purpose of this book is to introduce the vast store of *analog models* available for the investigation of scientific problems.

WHAT IS AN ANALOG?

Analog

The word "analog" (or "analogue") has been used and misused. It has one meaning to some people, and a variety of uses to others. Webster speaks of a thing which maintains a "relation of likeness with another, consisting in the resemblance not of the things themselves, but of two or more attributes, or effects". Let us then accept this general definition of devices which may differ in substance or structure but agree in dynamic behavior, as being dynamic analogs or just analogs. To avoid including everything from bicycles to toasters, we will be somewhat more specific and divide the field into three groups: (1) scale models; (2) automata; and (3) what we have called *analog models* (Fig. 1-2). Each of these is discussed in the succeeding pages.

Before proceeding further, let us identify the real subject of this book, the *analog computer*. The adjectives "digital" and "analog" are in common use today in reference to electronic computers. Specifically, they refer to the general purpose digital and the general purpose d-c electronic analog computers. The latter is dealt with in Volumes 2 and 3 of this book. It is important to recognize that while *analog computer* refers most commonly to this one specific type of analog computer, it can just as well refer to certain mechanical and hydraulic devices, to general purpose a-c electronic computers, and to a variety of special purpose computers. All of these have one characteristic in common — that the components of each computer or device are assembled to permit the computer to perform as a model, or in a manner analogous to some other physical system. Thus all analog computers fall in the third category. We will see that these analog computers are flexible tools that give the scientist a means of studying, in his laboratory, a system that would be too expensive, too large, or impossible to build.

What is an Analog?

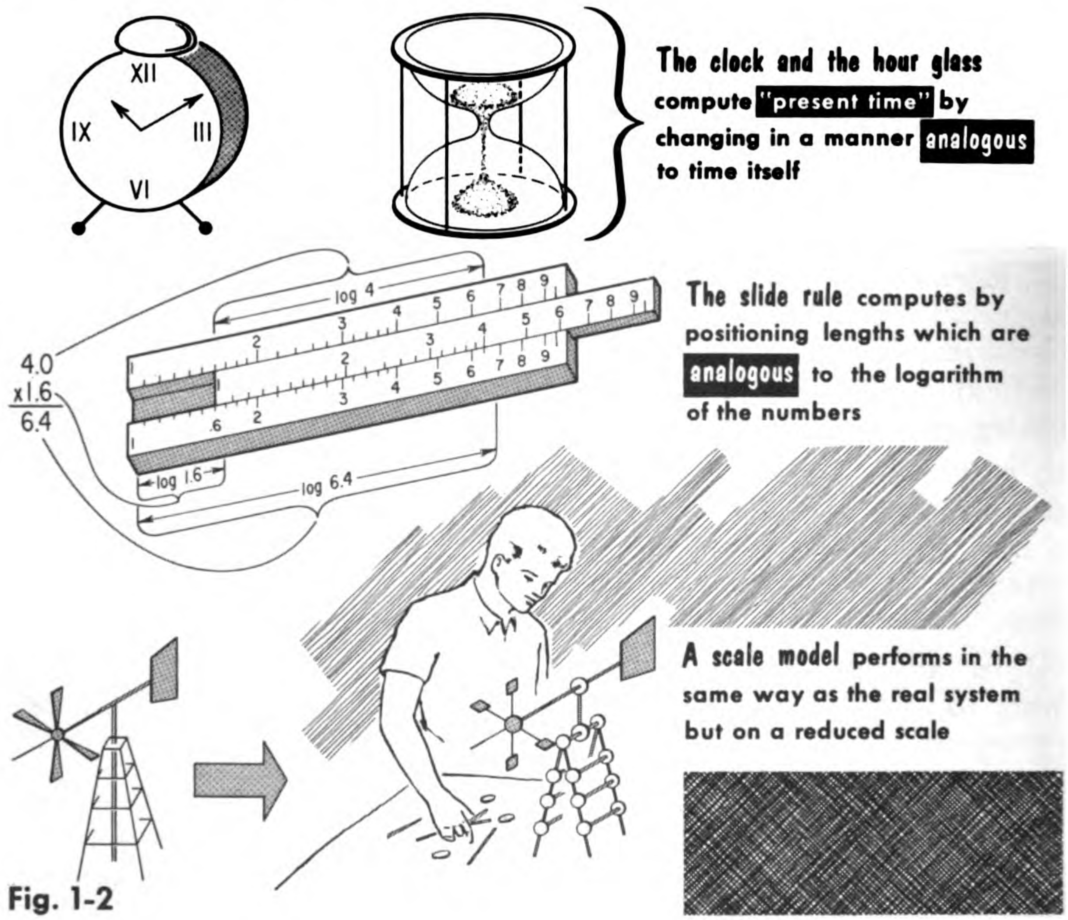


Fig. 1-2

Group 1: Scale Models

Scale models assume the same substance but not the same structure as the primary system. Small models are built every day in laboratories in the process of experimental design. Large scale models are often constructed as semifinal tests of semicomplete designs: pilot plants, wind tunnel models, and ship models are common examples (Fig. 1-3). Often there is no choice but to construct scale models; construction of the primary system may be prohibitive, and analog models may be ruled out because of insufficient understanding of certain phenomena. In fact, it is often necessary to obtain by experiments with a scale model in a wind tunnel, data of coefficients of lift and drag for an airfoil, *before* an analog computer study can be made.

Group 2: Automata

All man-made devices purporting to "behave" in a fashion analogous to human behavior may be classed as automata. To the extent that the

analogy is valid they may be considered also as analogs. Many fascinating studies have been made of the problems of mechanizing human responses, decision-making processes, and other simple behavior patterns. In some cases working models have been constructed, such as the electronic "mouse" which finds its way through a maze, and a battery-operated "turtle" which, while its sole objective is to seek and find a light, is able to circumvent most obstacles, and scoot back to its garage for recharging when batteries are low. No known automaton has been able to enslave its creator! How-

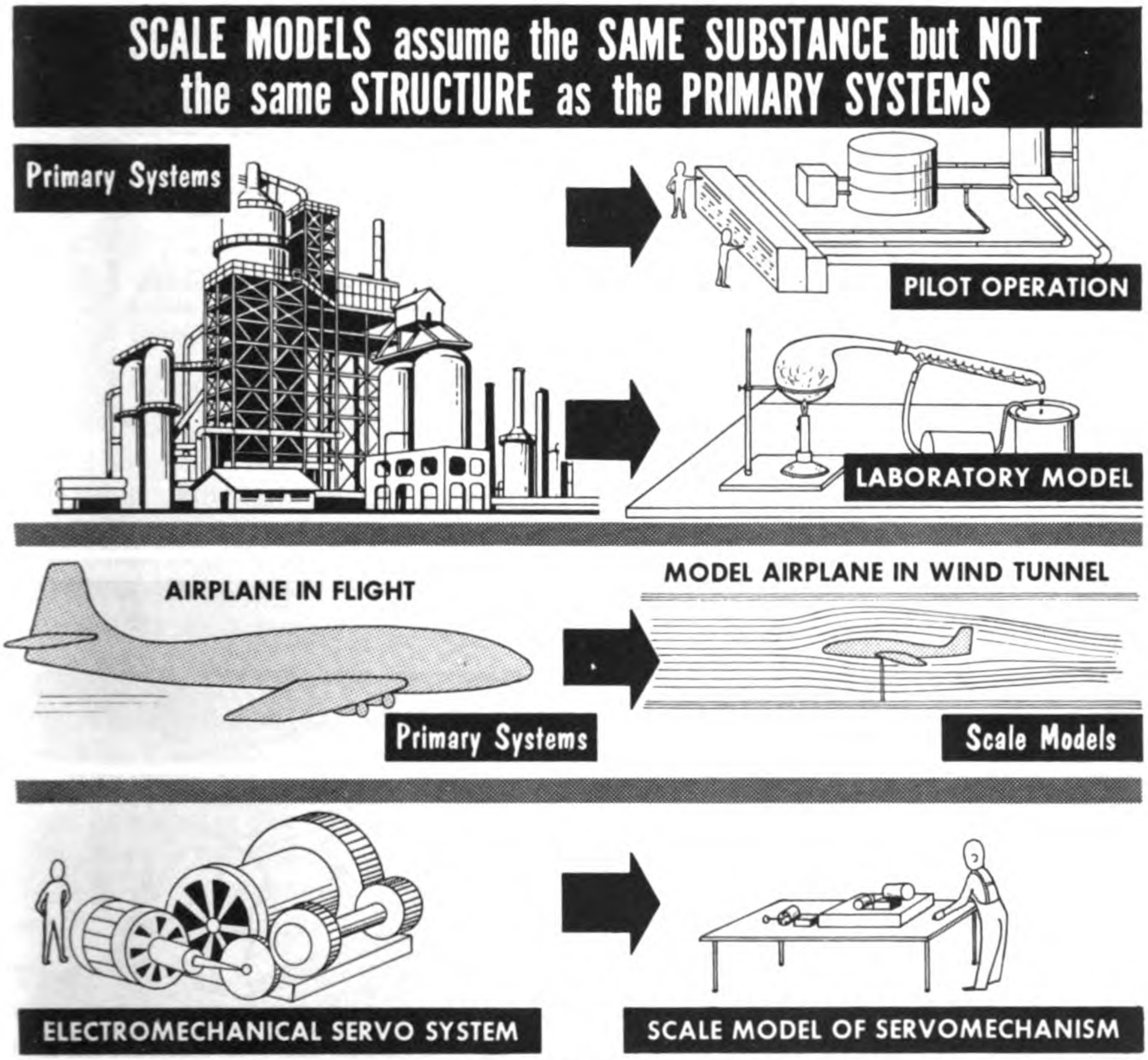


Fig. 1-3

ever, one must admit that many equally fascinating, if not spine-chilling, automata, have been developed by science fiction writers.

On a more prosaic plane, we include in group 2 that class of systems known as *automatic control systems* and *regulators*, in as much as these systems perform simple and routine tasks which might otherwise be done by a human being.

The thermostat in your house (Fig. 1-4A) is a regulator that continually observes the room temperature and turns your furnace on or off whenever the temperature deviates beyond preset tolerance limits. Since the thermostat, in effect, on reaching an uncomfortable state as the temperature goes

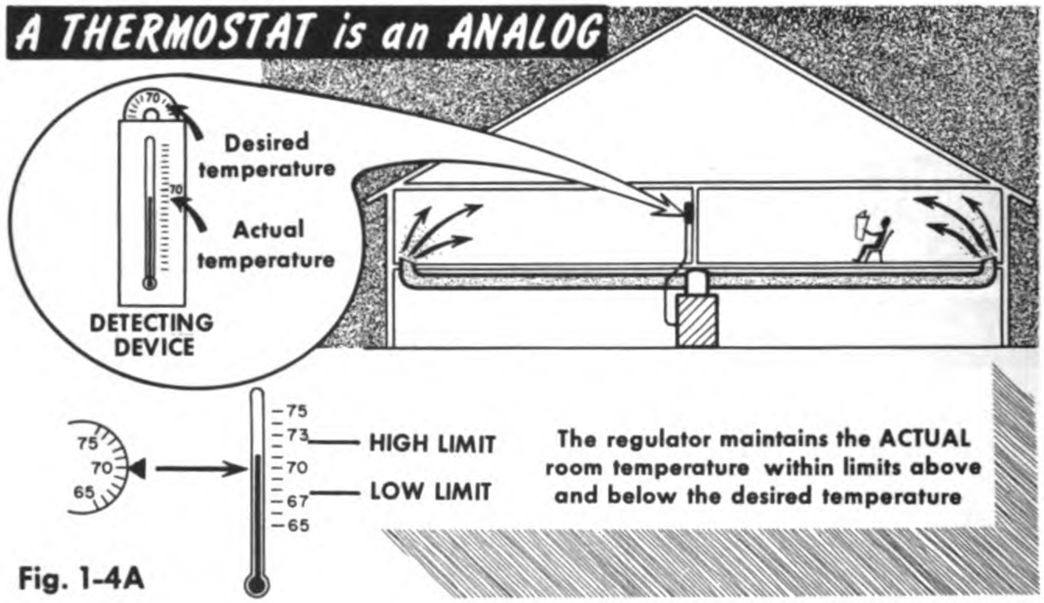


Fig. 1-4A

beyond the limits, promptly does something about it, this regulator behaves in a manner analogous to human behavior.

The autopilot in an aircraft (Fig. 1-4B) is a control system that continu-

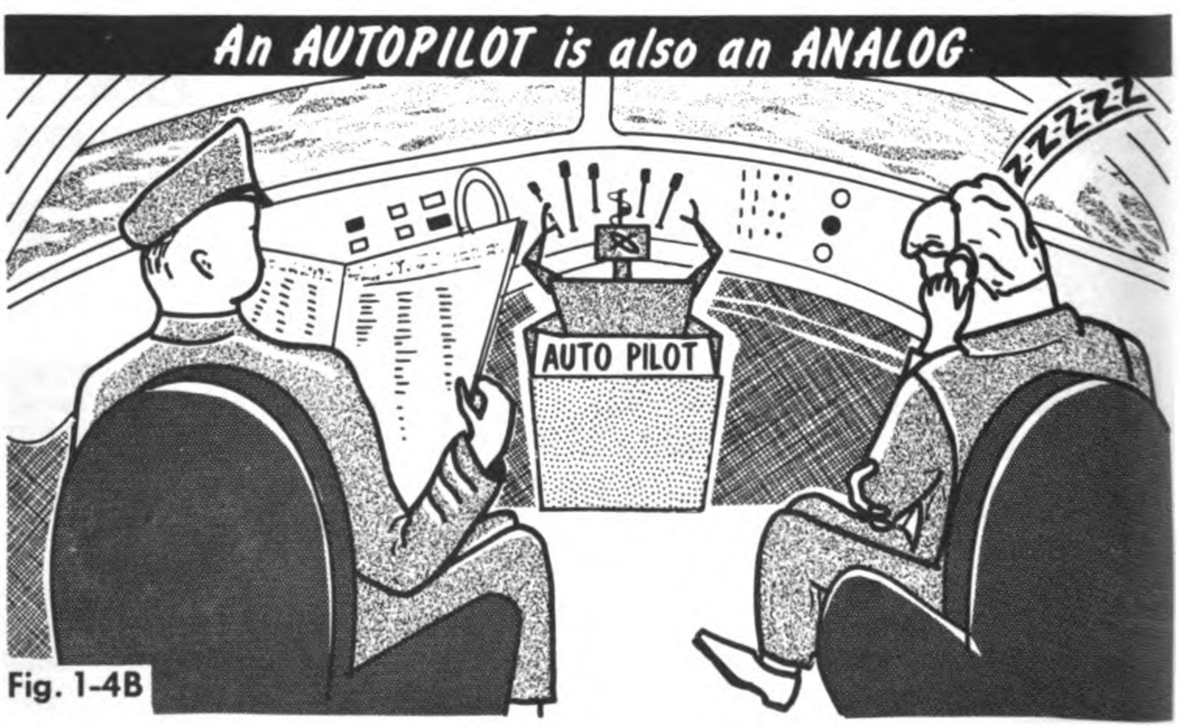


Fig. 1-4B

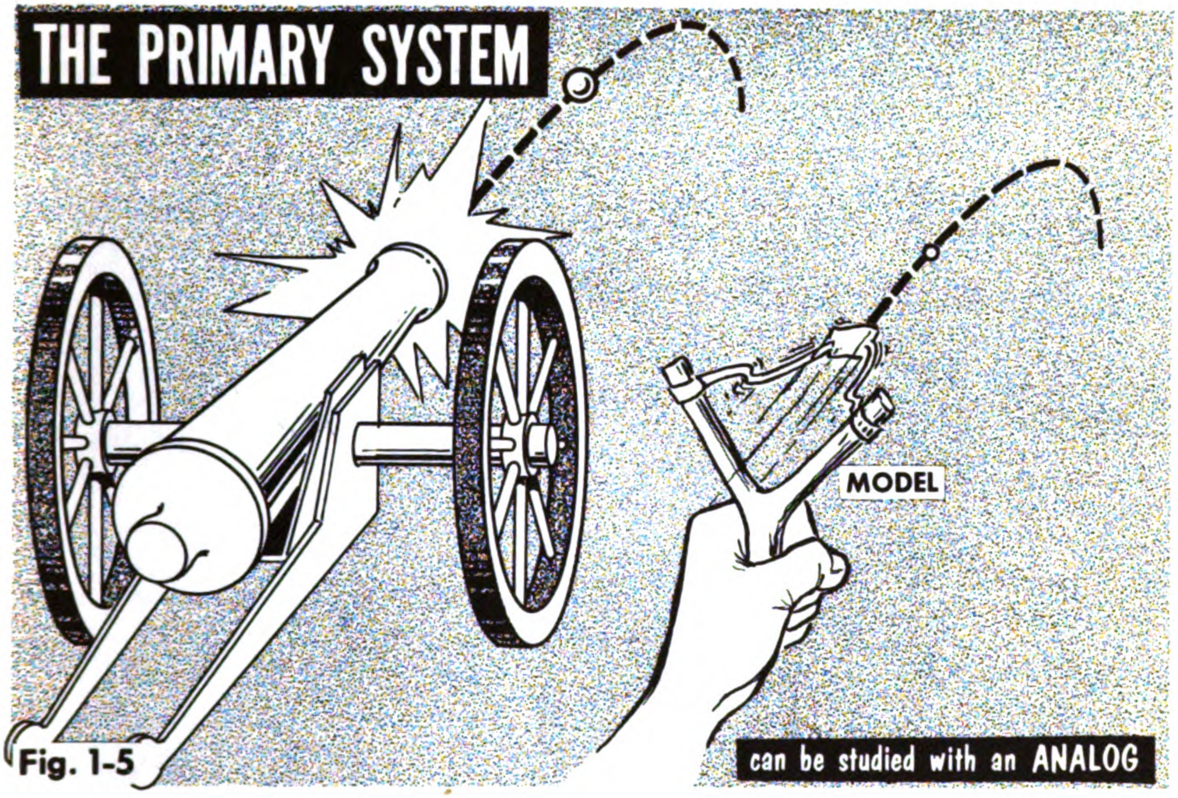


Fig. 1-5

ally observes the altitude and heading of the airplane, compares them with the desired (preset) altitude and heading and reduces the errors to zero as rapidly as possible. Thus it not only acts as the pilot, but may even do a better job.

Group 3: Analog Models

Analog models are devices that behave in a fashion analogous to others simply because they obey the same or similar fundamental laws of nature. The use of these devices has the same advantages as for group 1, namely, that large or expensive or explosive primary systems can be simulated and studied on a smaller, cheaper, safer scale (Fig. 1-5).

These devices however, are not models in the sense of scale models. They bear no resemblance to the primary system except that some characteristic of the one, changes in a manner analogous to the variations of some characteristic of the other. For example, a voltage in the analog (Fig. 1-6) changes in a manner analogous to the variations of the velocity of a missile.

The word model *is* often used when one speaks of an *electric model* of a mechanical (or other) system. In fact, we shall use the terms *electric model* and *computer model* quite frequently in this book when referring to Group 3 analogs. The reader is cautioned against confusing these terms with the simple models of Group 1.

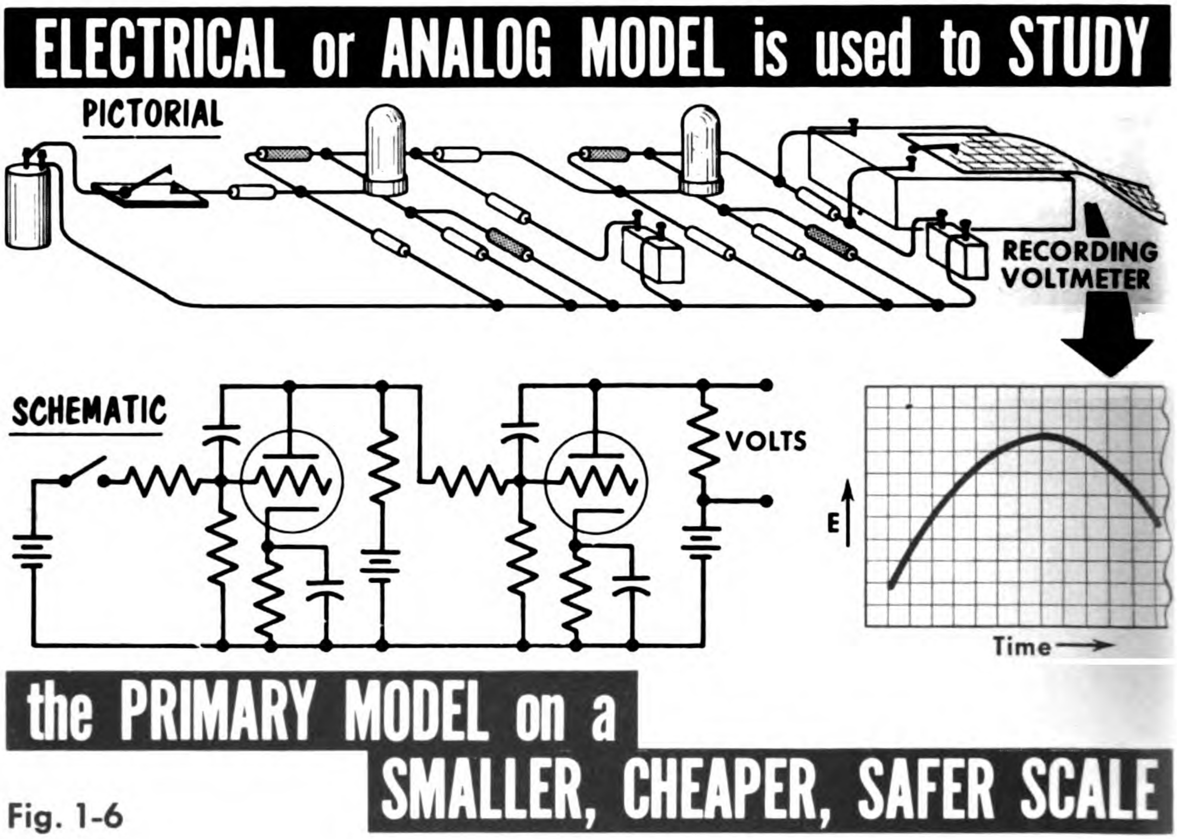


Fig. 1-6

ANALOGS AND PHYSICAL LAWS

Group 3 Analogs vs. Scale Models

It is analog devices from the *third* group that provide the basis for analog computers, for with these the engineer is no longer concerned with building a model airplane to a super-precise scale, nor with the building of a model chemical reactor with its associated problems of handling heat, fluids, acids, etc. *Now* an electronic device can simulate the aircraft characteristics, and a complicated mechanical gear-and-shaft device might represent the chemical reactor. The only requirements are that the engineer know the fundamental laws which govern the behavior of his primary systems, and that he find analog devices that obey similar laws. Properly assembled these analog devices will reproduce the behavior of his primary system, in volts instead of miles/hour, or feet of altitude; or for a mechanical analog device, in rpm or degrees of shaft rotation, instead of calories of heat, or degrees Fahrenheit.

Provided he can describe the behavior of his primary system with mathematics, the engineer with a desk-size (or larger) analog computer (Fig. 1-7) can often obtain the answers he needs without ever leaving his air-conditioned office. With an analog computer, rather than a scale model such as a laboratory or pilot-plant reactor, he may avoid the mess and fuss

of handling acids, implementing delicate instrumentations, and constructing equipment to function at high temperatures, high pressure or high vacuum. Furthermore, when he has completed his analog-computer investigation he is not left with a lot of special purpose equipment of little value to

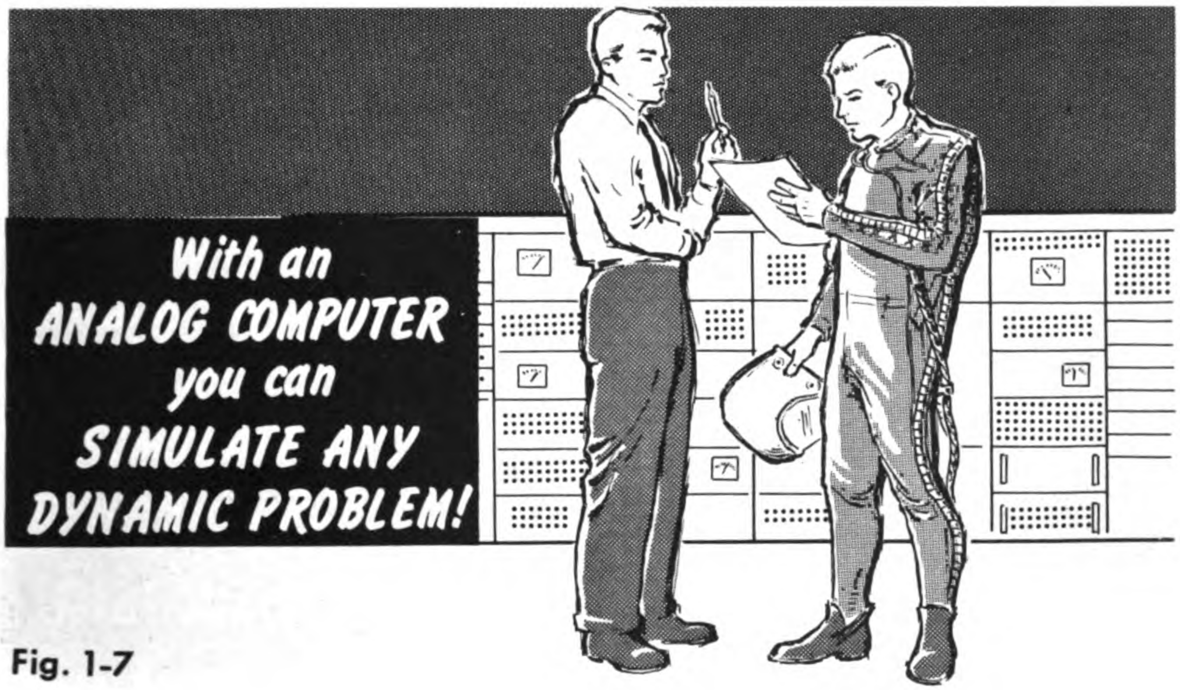


Fig. 1-7

anyone. In fact, he has only to disassemble his computer model and he is ready to investigate bigger and better systems immediately.

Force Laws

If all this sounds a bit farfetched, let us look at some of the basic physical laws. Consider the three different force laws (for two isolated bodies) illustrated in Fig. 1-8, A through C.

Each of these two-body force laws has the same form. In general, the magnetic, electrostatic, and gravitational fields have similar properties, and appropriate bodies moving in these fields are subject to like forces and behave similarly.

Newton's Second Law

Of all Sir Isaac Newton's contributions to man's knowledge of the universe his formulation of the classical laws of mechanics are the most well known. Every school boy learns that Newton's second law of motion states that the net force on a body is equal to its mass times its acceleration (Fig. 1-9A).

The acceleration of a body is the rate of change of its velocity, while the velocity is the rate of change of position (Fig. 1-9B).

Fig. 1-8A

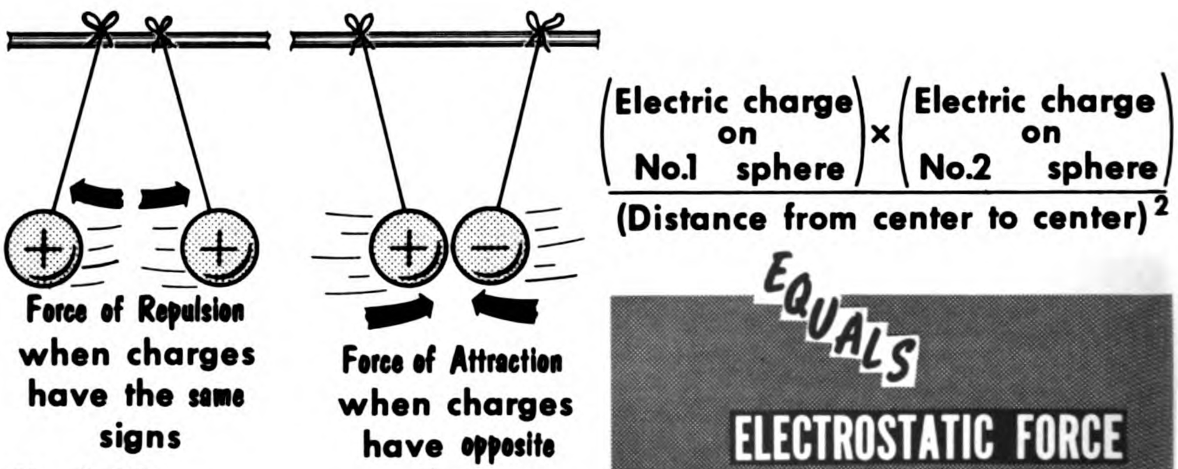


Fig. 1-8B

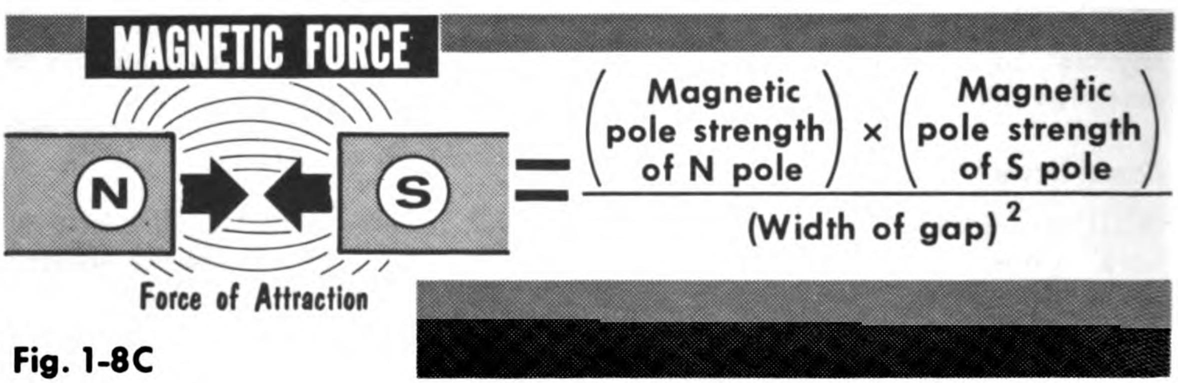


Fig. 1-8C

When Newton's law is applied in the mass and spring system shown in Fig. 1-10A, it leads to the relationship:

Mass × acceleration = gravitational force

+ {a spring force *downward* when spring is compressed, or *upward* when spring is extended:

+ {a force which slows the motion down due to air friction, or due to the dissipation of energy by moving the surrounding air.

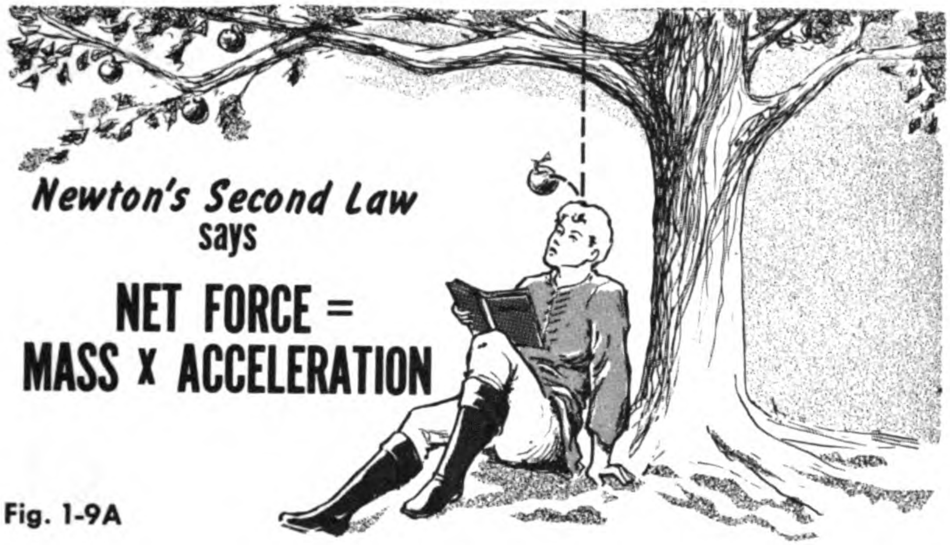


Fig. 1-9A

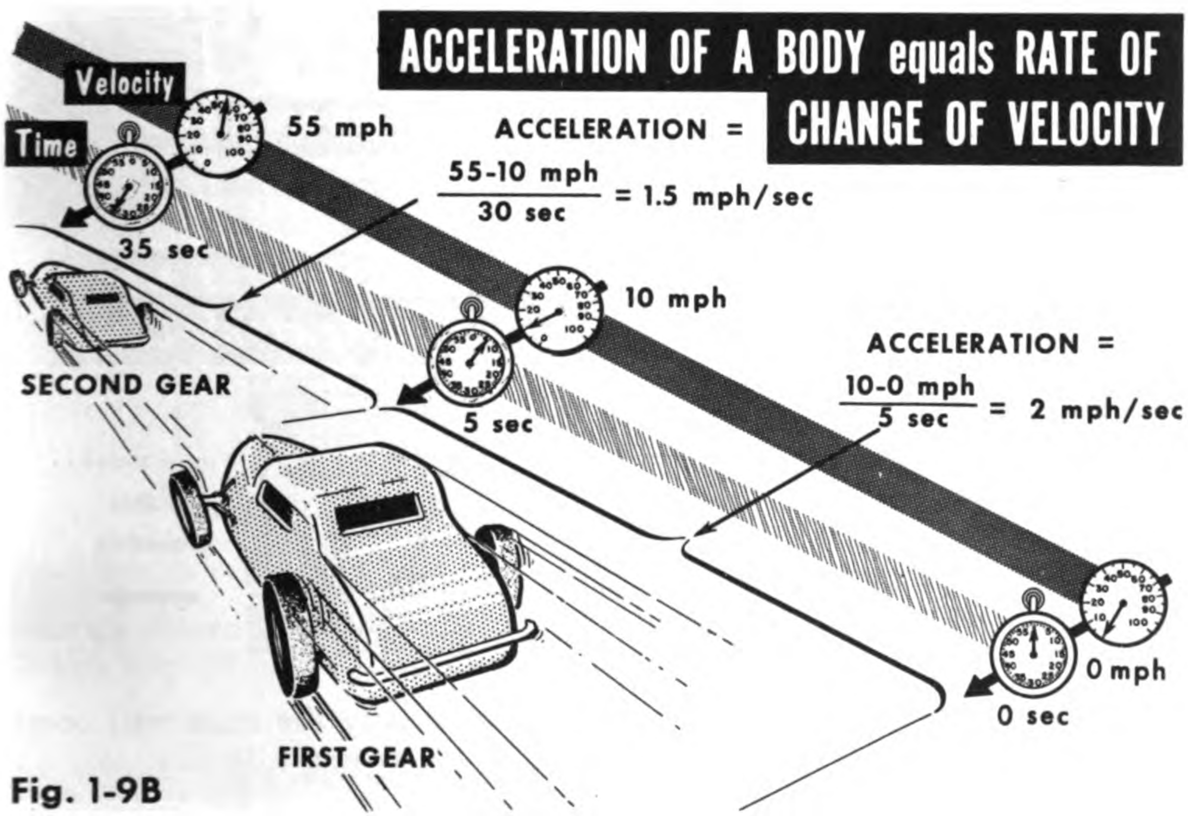


Fig. 1-9B

If a pen were attached to the mass so that it could write on a long strip of paper moving past the pen, and then the mass was disturbed, the pen would trace a curve which would represent the variation of the position of the mass with time, and we would discover that the position of the mass varies up and down in a periodic manner (described later, we will see, as a sinusoidal function). In fact, due to the air resistance, we can see that the oscillations are decreasing in amplitude (Fig. 1-10B). The system is said

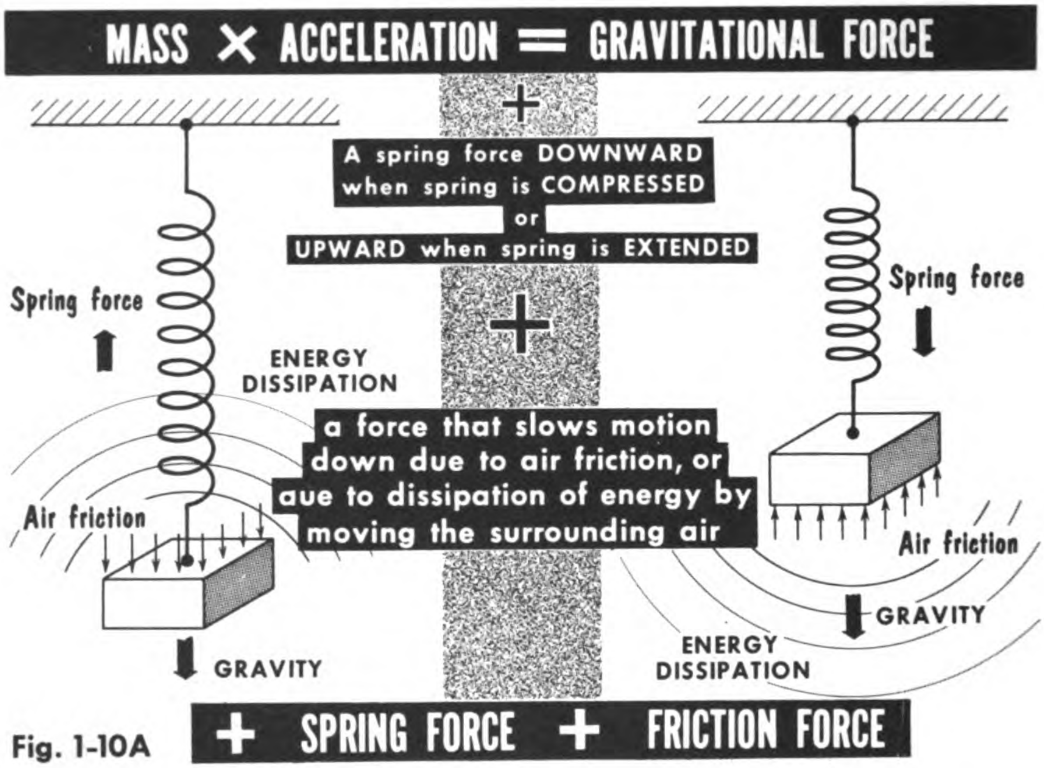


Fig. 1-10A

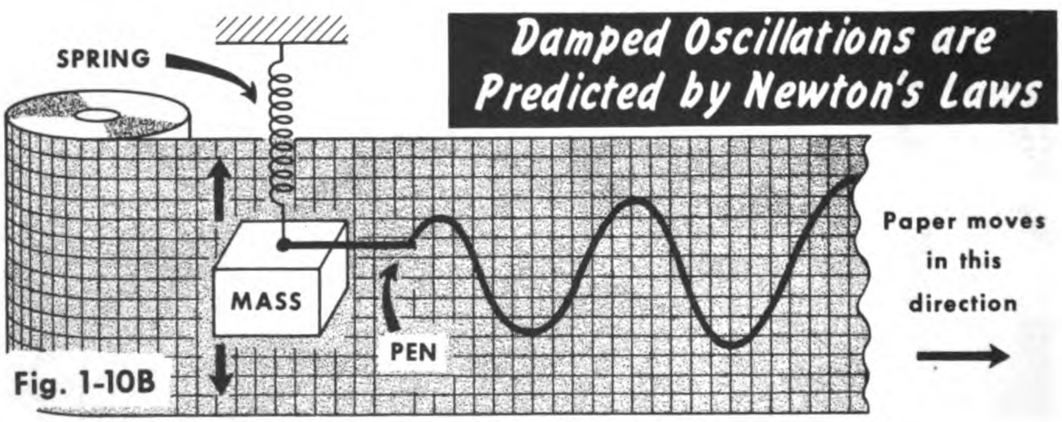


Fig. 1-10B

to exhibit a *damped* oscillatory behavior. Eventually the mass will come to rest. *This behavior is precisely predicted by Newton's laws.*

Kirchhoff's Voltage Law

To find an electric analog of the mechanical system just described, we consider the electrical equivalent of Newton's law. One way of stating Newton's law is to say that the vector sum of all forces on a body (including the inertial force: mass × acceleration) is always zero. An electric circuit law that appears to be very similar, is Kirchhoff's voltage law, which states that the algebraic sum of all voltage drops around a closed circuit is always zero. When applied to a circuit containing a coil, this law states that the sum of the electric-potential forces (batteries, voltage

sources) that act upon the electric charge in a coil (inductor) is proportional to the rate of change of the flow of the charge (that is, the rate of change of the current).

Potential forces (voltages) = L × rate of change of current

where

L = constant of proportionality, called inductance (measured in henries)

and,

Current = rate of change of charge at a given point

Remember from p. 9 that

$F = M$ × rate of change of velocity

where

Velocity = rate of change of position

In looking for the electric analog we might first choose to consider the

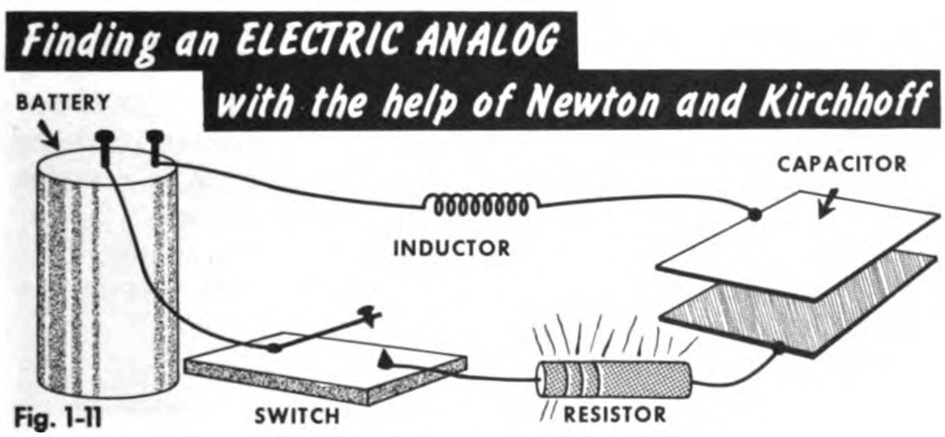


Fig. 1-11

circuit shown in Fig. 1-11. Doing so, we would find that Kirchhoff's law when applied to the circuit would say:

L × rate of change of current = battery voltage

$+ \left\{\right.$ a *positive* potential force when the *top plate* of the capacitor is charged positively, or a *negative* potential force when the *bottom plate* of the capacitor is charged positively;

$+ \left\{\right.$ a potential force due to the voltage across the resistor when current flows through it; this force reduces the flow of charge. The resistor dissipates electric energy by converting it into heat energy. This is a "damping" force.

Compare this with the expression on p. 10.

Now let us set this system in motion (by closing the switch) and with a recording meter of appropriate type, record on a strip of paper the time variation of the charge on the capacitor (Fig. 1-12).

As might be expected, the result is perfectly analogous to the result on p. 10. Thus, it is clear that the two systems could be used interchangeably in analyzing the dynamic behavior of systems described by the same

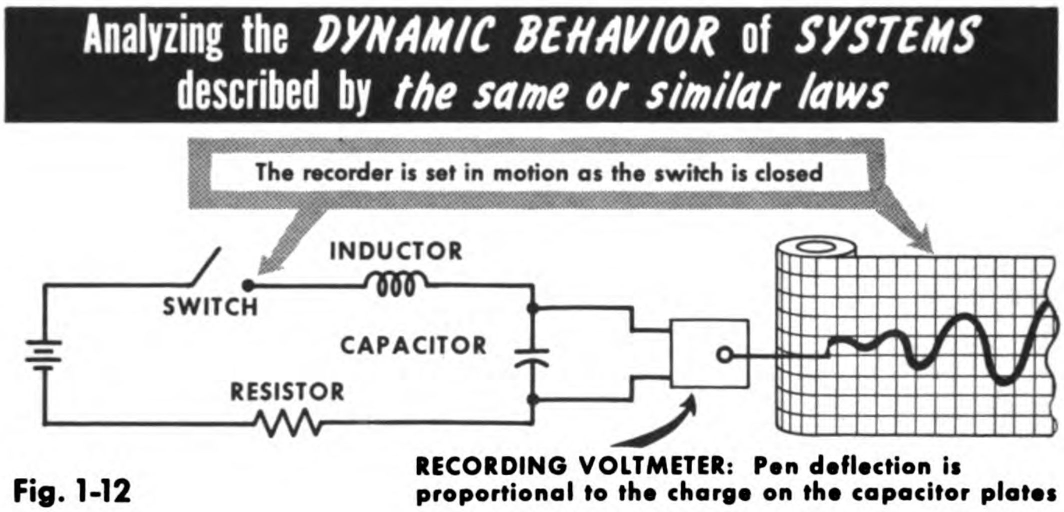


Fig. 1-12

or similar laws. By "dynamic behavior" we mean the time-function transient response of the device or system to an impulsive disturbance. To use the *electric* circuit to study the *mechanical* system it remains only to establish the correspondence between circuit parameters (L, capacitance and resistance) and the mechanical parameters (mass, spring and damping constants of proportionality). We will see later that this is easily done by comparison of the laws for the two systems.

Static Analogs and Potential Fields

The principle of studying a system with which experimentation is difficult, by observation of a completely different system that obeys physical laws of the same form as the laws for the first system, is so important that it will be reiterated many times in this book. It is fundamental to analog computation. So let us consider two more examples.

In the first example, a d-c electrolytic device is used to simulate a high-frequency a-c device. In the second, a thin rubber membrane becomes the analog of a d-c electrolytic system.

Consider the tuning capacitor of a small radio [Fig. 1-13 (A)]. With the plates partly enmeshed, a design engineer may wish to know the electric field pattern between the plates. He would be unable to measure this field because the field strength would be too small for any measuring device, and the presence of the meter would disturb the field. However, he could construct the following device.

STUDYING the ELECTRIC FIELD where FIELD STRENGTH is TOO SMALL for a MEASURING DEVICE

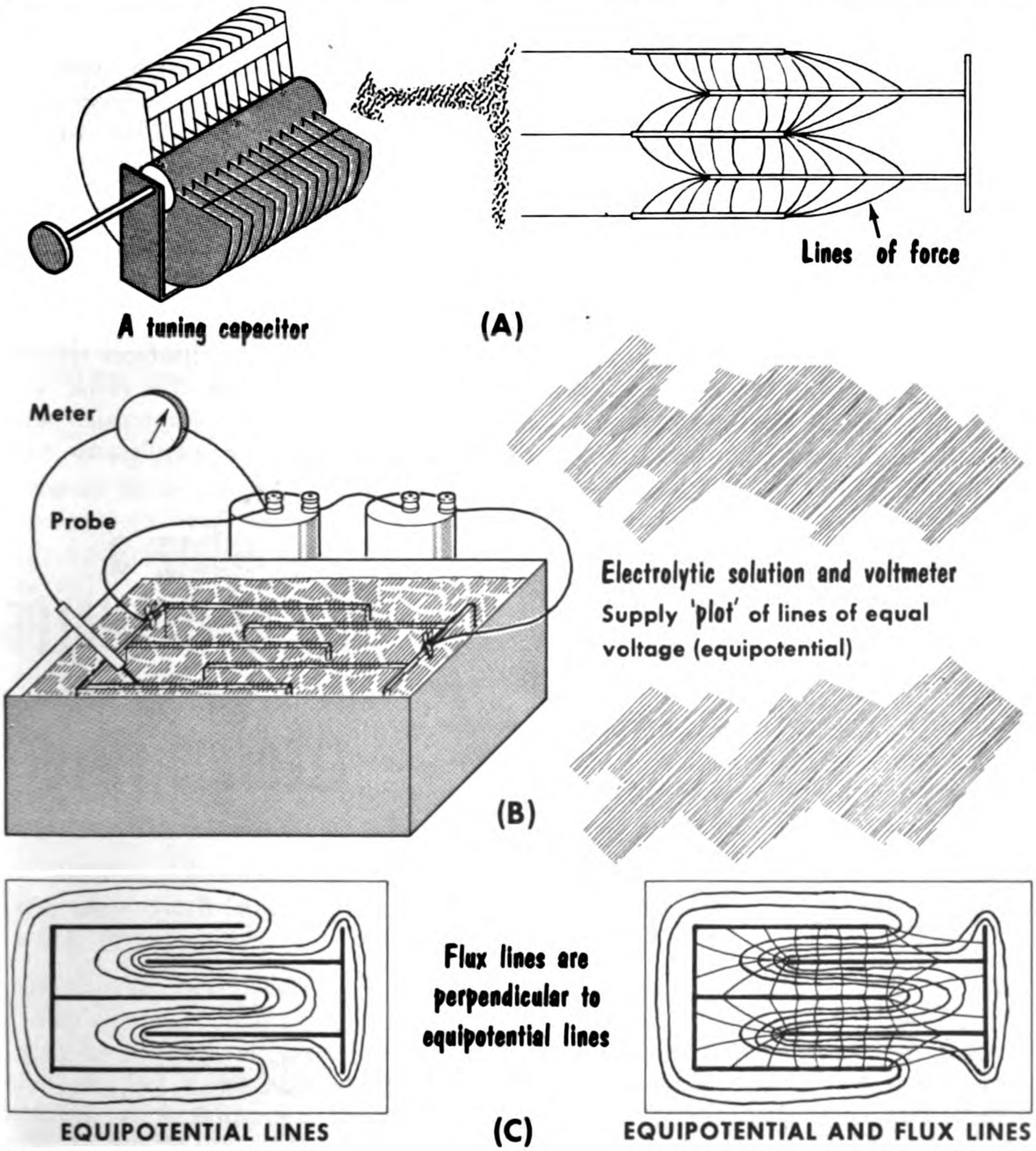


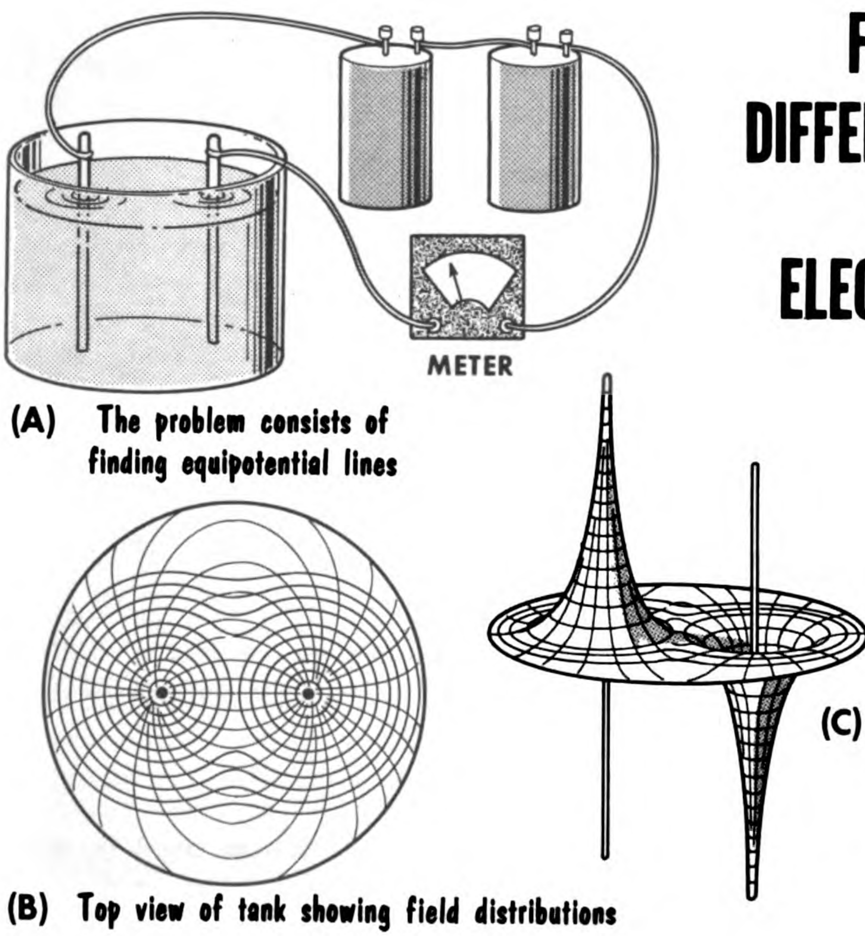
Fig. 1-13

The tank, Fig. 1-13 (B), contains an electrolytic solution as used in electroplating tanks and in wet-cell storage batteries. A d-c electric field exists between the plates. This field has the same pattern and is analogous to the a-c field pattern in the capacitor. It is necessary, of course, that the

tuning capacitor plates and the tank plates be the same shape (though not the same size). The field pattern in the tank can be conveniently measured by moving a probe around in the liquid while recording the reading of a voltmeter connected between the probe and ground. Since an electric current will flow in the electrolytic solution it is possible to make the measurement with an ordinary voltmeter. Actually, what one chooses to measure and plot on paper are the lines of equal voltage, equipotentials [Fig. 1-13 (C)]. These yield a pattern resembling a contour map with lines of equal elevation, or a weather map with lines of equal barometric pressure. The electric field lines (electric flux) are drawn perpendicular to all equipotential lines, as shown in Figs. 1-13 (B) and 1-13 (C).

Now for illustration of a different analog, let us suppose we have two bars in a metal tank with an electrolytic solution [Fig. 1-14 (A)], and we do not wish to use a probe to measure the equipotential lines.

We could take a thin, highly elastic membrane, held in a rigid frame, and use it in such a way as to stimulate the potential distribution in the electrolytic tank. If two rods, spaced as the electrodes in the tank, are used to stretch the membrane upward for the positive electrode, and downward an equal amount for the negative electrode, the membrane will



Finding a DIFFERENT ANALOG for an ELECTRIC FIELD

(A) The problem consists of finding equipotential lines

(B) Top view of tank showing field distributions

(C) The HEIGHT at any point is COMPLETELY ANALOGOUS to the STRENGTH of the ELECTRIC POTENTIAL in the tank at the CORRESPONDING POINT

. 1-14

assume a position such that its height at any point is completely analogous to the strength of the electric potential in the tank at the corresponding point [Fig. 1-14 (B) and (C)]. Note that the membrane is like a three-dimensional contour map.

Membranes and electrolytic-tank analogs are often used today by engineers to solve complex, potential problems.

PROBLEM SOLVING WITH ANALOGS

Solving Problems: An Example

We have seen that analogous behavior exists in certain physical systems that would otherwise appear completely dissimilar. Furthermore, we have seen two examples of how an analog might assist someone investigating potential distributions. Before discussing computers and computing devices, it seems desirable to re-emphasize what is meant by "solving" problems by analog techniques.

In general, the problems one wishes to solve may occur in the design of a new turbine, missile, refrigerator, or automatic fish scale. The designer wishes to know how big, small, heavy, or how fast to make some component, so that the overall system will perform correctly and stay within the specifications. It is usually necessary to do this without building the actual component. Consequently, to remove the guesswork from the design, a model may be constructed. It may be a scale model, an electric model built in the laboratory, or a simulation on a general purpose analog computer. In any of these cases analog techniques are employed.

On the other hand, not all problems are engineering problems. Some may be purely mathematical, others may require calculation of profits, production scheduling, or manpower requirements. Consider a simple example that we all faced at some time in our early schooling:

• *Problem:* John can dig a certain ditch in 48 hours, and Robert can dig the same ditch in 40 hours; how long will it take if both John and Robert work together?

Of course, bright little Euclid in eighth grade quickly works out that John's rate of digging is 1/48 ditches per hour, that Robert's rate of digging is 1/40 ditches per hour, and that therefore their combined rate is $1/48 + 1/40 = 88/1920 = 11/240$ ditches per hour. Consequently, it will take $240/11 = 21.8$ hours of steady work from both John and Robert to complete the work (Fig. 1-15).

Suppose, however, that while we are not so mathematically inclined as young Euclid, we are very clever in the use of analogs. We might find that we could represent the work of John and Robert as a flow of water down two pipes from a large tank of water, representing work to be done (Fig. 1-16). We will make the flow of water in each pipe in gallons per hour proportional to the work rate of each boy. The valve on each pip

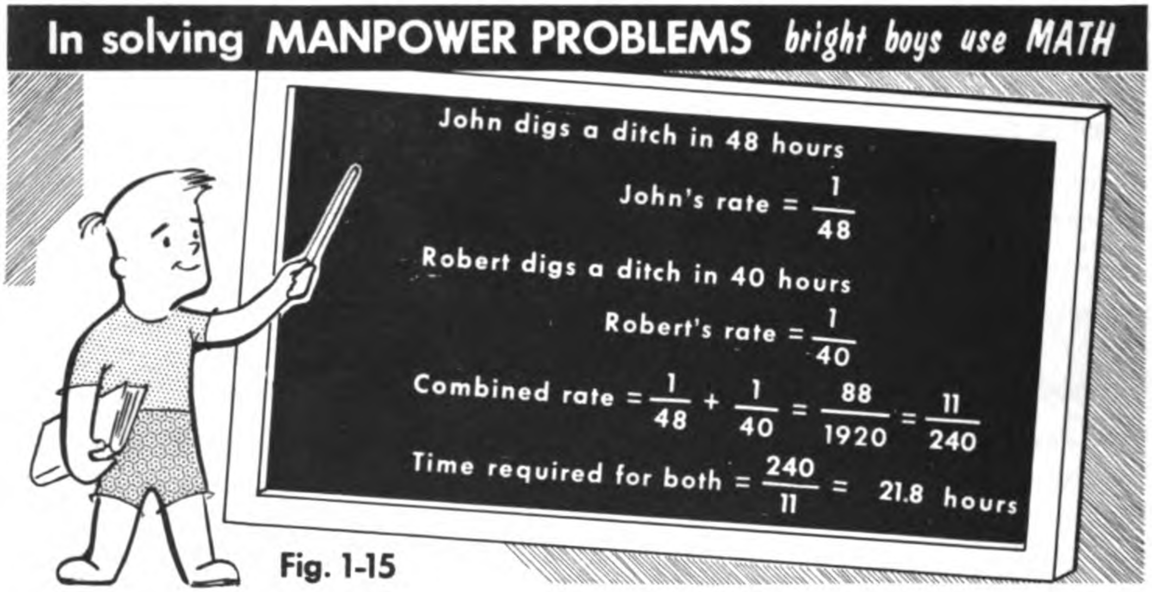


Fig. 1-15

can be calibrated in shovelfuls per hour, if desired. Now if the upper tank is large enough, the pressure head will not vary appreciably during the operation of the device, and the calibrated valve will satisfactorily gauge the proper flow of water. When the lower tank is filled with a number of gallons proportional to the volume of earth to be removed, the problem simulation is over: "The ditch has been dug". If John's valve were the only one open, the solution would require a certain number of minutes corresponding to 48 hours of actual work. Likewise, with only Robert's

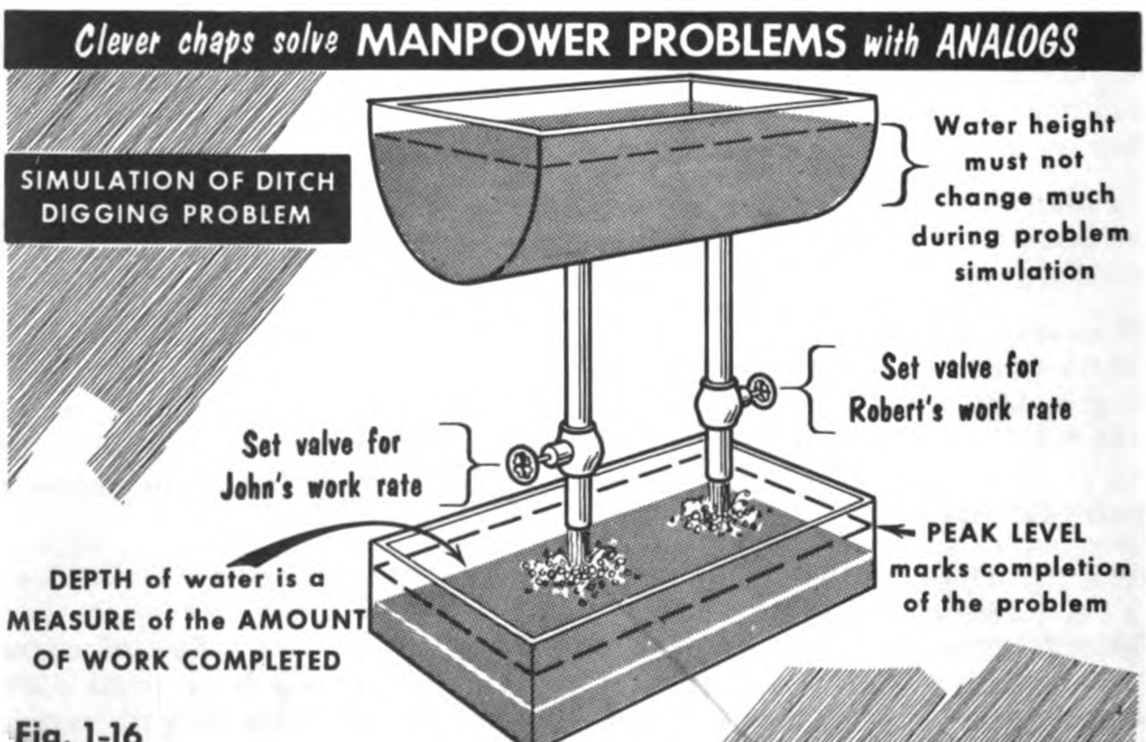


Fig. 1-16

valve open, the minutes required to fill the lower tank correspond to 40 hours of Robert's labor. With both valves open, the time to fill the tank will provide the answer to the problem.

A More Difficult Example

All the foregoing is very fine, and as with the hare and the tortoise, young Euclid has so far left us well behind in arriving at an answer. But let us

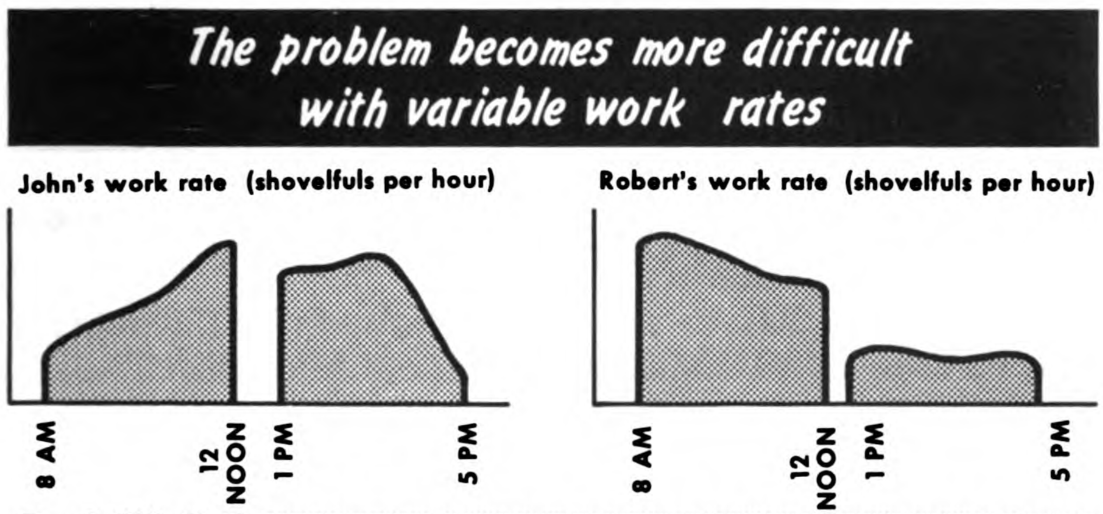


Fig. 1-17A

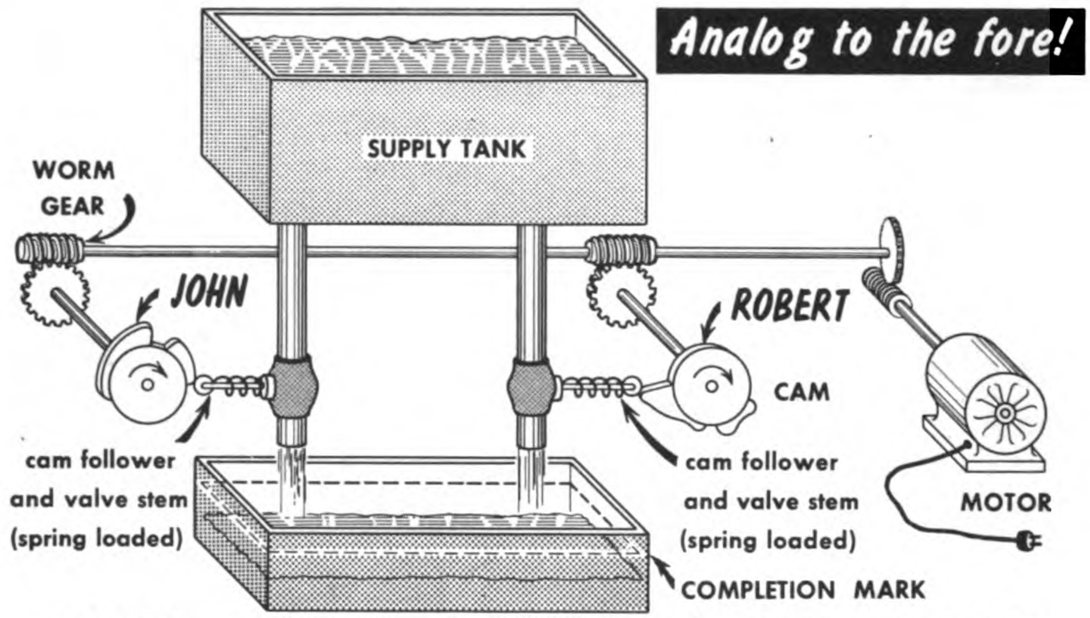
now make the problem more interesting. Suppose John is a slow starter and finds the shovel very heavy in the early morning. He reaches a peak in his work rate just before noon and he takes an hour off for lunch. Refreshed, he works hard in the early afternoon but finds the hot sun slows him down. John's work rate pattern is shown for a typical day (Fig. 1-

Fig. 1-17B

17A). Now Robert, on the other hand, you can see is a fast starter, but he apparently gets sleepy after his half-hour lunch period.

Now how long does it take to dig the ditch?

Unfortunately, little Euclid is hard put to it, to solve this one in his head, (Fig. 1-17B) and the man with the analog device may beat him. For since



CAMS are CUT to John's and Robert's WORK RATE

Fig. 1-17C

we are so clever with analog methods we quickly see that by cutting two cams, patterned from the curves illustrated, we can easily vary the water flow rate in a manner analogous to the variation of work rates. The cams are turned slowly by a small motor and gear boxes, and the spring-loaded

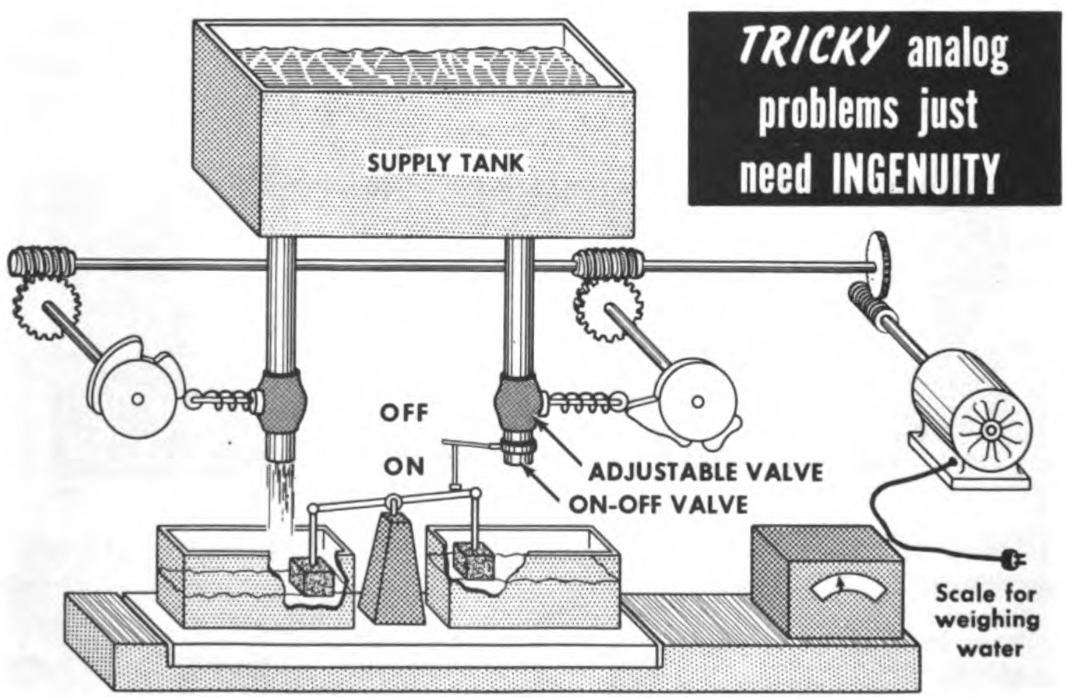


Fig. 1-17D **FLOATS are added to cause ROBERT'S valve to close whenever the water level in the right hand container exceeds that in the left**

cam followers vary the two valve positions (Fig. 1-17C). The rest of the problem is the same.

Just to be tricky, let us assume that Robert, though a faster worker, does not like to do more than his share, and therefore slows down, or even stops, whenever he gets ahead of John. He then waits for John to catch up.

To simulate this, we accumulate the water from the two pipes separately, and with two floats cause Robert's valve to close whenever the water level in the right-hand container exceeds that in the left (Fig. 1-17D). The time to complete the ditch digging can now be determined by weighing the two cans of water.

Although this example borders on the ridiculous, it clearly illustrates the great strength of the analog-simulation approach to solving problems.

Analog Devices for Computing

There are many devices used for computing. Some are so simple they may not be generally thought of as computing anything, while some are quite

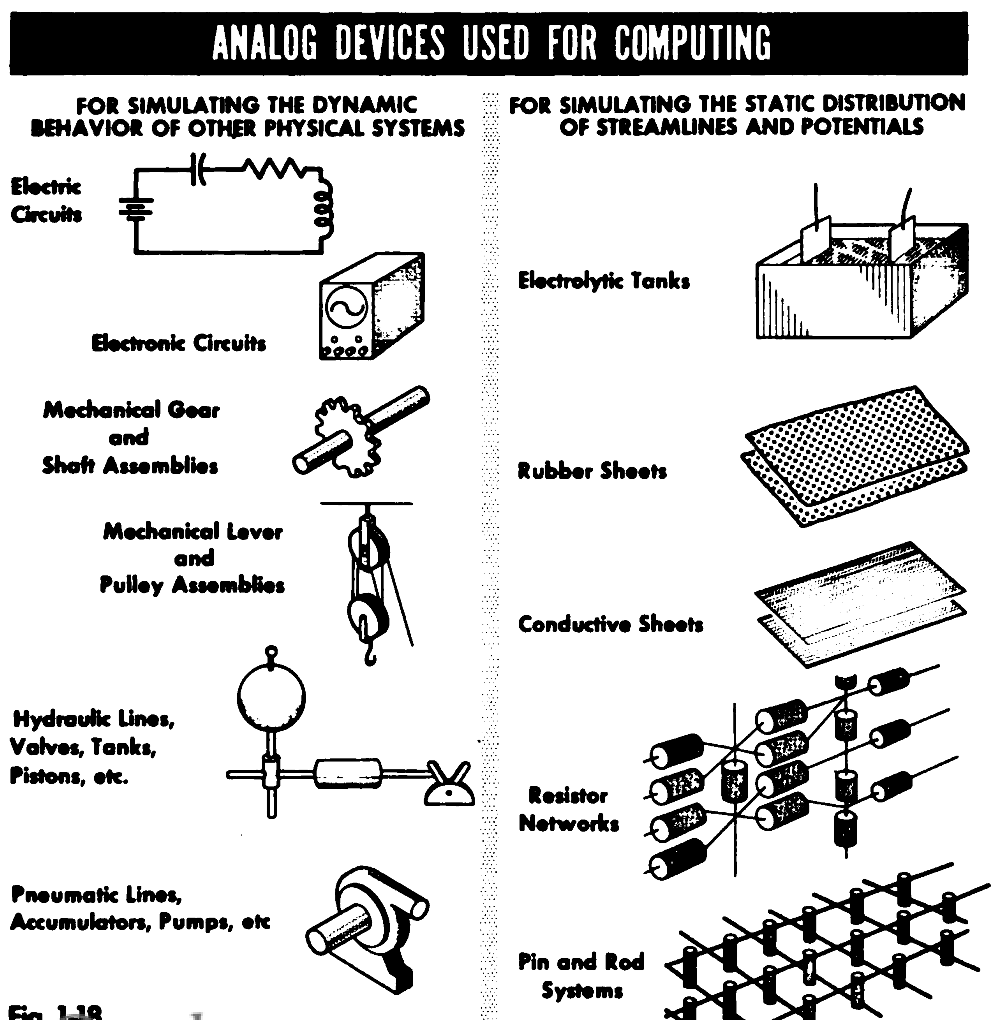


Fig. 1-18

complex and difficult to understand. Before we are through we shall have described in some detail examples of devices from all the major categories of such devices.

Computer devices may be classified in several ways. A convenient grouping according to substance is shown here to provide a preview of the areas to be discussed. Note that a major division is shown between devices for static and dynamic simulations. This is a very important division between physical problems: problems that are *time varying* and those that are *independent of time*. The two problems are quite distinct, as are the computing devices used to solve them (Fig. 1-18).

QUESTIONS

1. What is the difference between a scale model and an analog?
2. What are Newton's first and second laws?
3. What are Kirchhoff's voltage and current laws?
4. Discuss how two of the four laws required in Questions 2 and 3 describe analogous situations in electrical and mechanical systems?
5. What kinds of primary systems can be simulated with "static analogs"?
6. What kinds of primary systems can be simulated with "dynamic analogs"?
7. Discuss the relative merits of analog simulations and "pencil and paper" calculations.
8. Why is model building a useful technique of analysis?
9. What advantages and disadvantages does analog model building appear to have over scale model building?

Chapter 2

WHY ANALOG?

ANALOG CHARACTERISTICS

"Continuousness"

The characteristic most commonly associated with analog devices is the continuous nature of their behavior. That is, the time variations of their several physical features are *smooth* changes. Between any two positions of a physical variable there are an infinite number of intermediate positions. Unfortunately, some people use the adjectives "analog" and "continuous" interchangeably. This usage is not correct, for although the two terms are loosely related, their association lies only in the fact that the majority of primary systems which we want to simulate by analog means are continuous devices.

In contrast to systems with continuous behavior there are systems whose variables can assume a certain number of discrete states or levels. These systems change from one state to another in an abrupt or discontinuous manner. Examples are: a relay, an adding machine, a telephone switching system, punched-card accounting machines, and digital computers. Such systems are called discrete-state or digital systems. It would be possible to simulate a discrete-state system by analog means by building a model from discrete state devices, and these then would be called *analog devices.* However, the point to be made is that without too great a stretch of the imagination we can still say that the model simulates the primary system in a *continuous* manner, for the behavior of each is the same for *every instant* in time. Thus the analog characteristic of "continuousness" might be best expressed as; maintaining correct correspondence between analog and primary system *for every instant of time.* If a discrete state device is used to simulate a continuous system, then correct correspondence of the two will occur only at intervals. For the simulation to be useful the difference in discrete states must be small so that the intervals between points of correspondence are also small. For example, a continuous curve can be approximated by a discontinuous curve (Fig. 2-1).

Although a continuous device may assume an infinite number of positions between any two given positions, in any practical analog device it is not always certain *which* position is assumed. This is due to the ever-present random variables, mechanical vibration and electrical noise. Conversely, the condition of a discrete-state device is always known and its resolution is often adjustable to provide better and better approximations to the con-

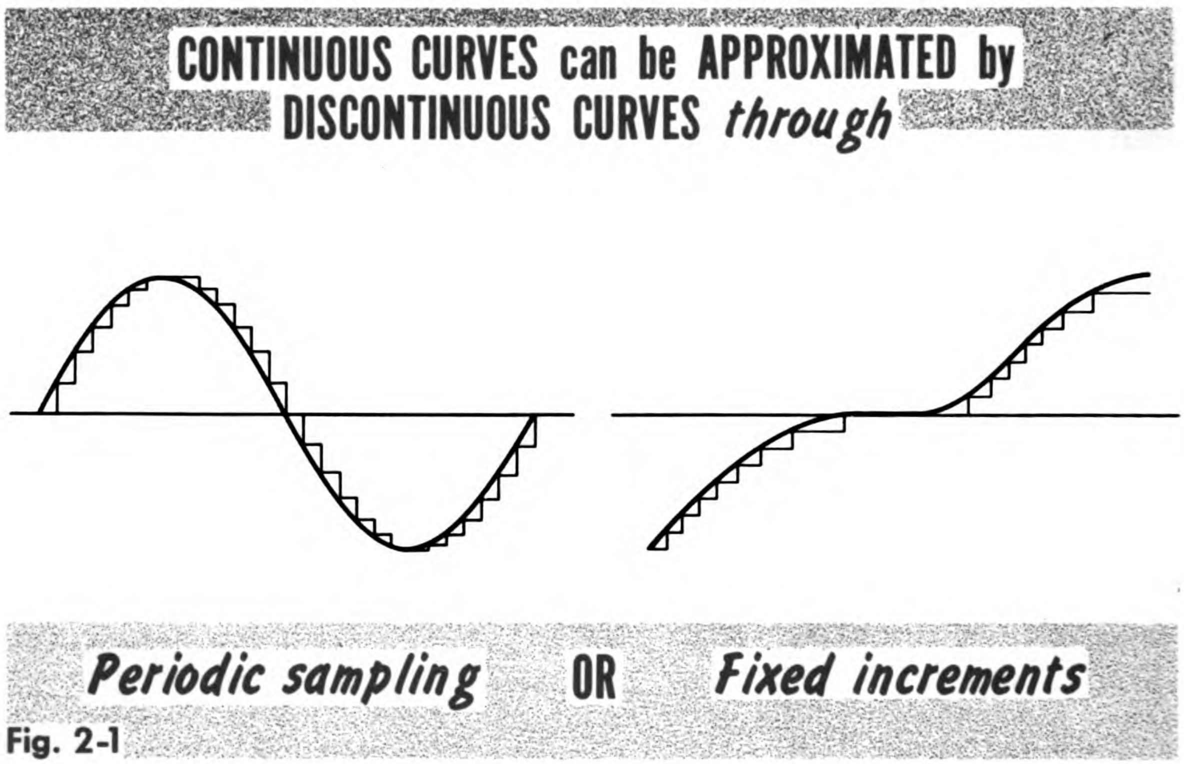


Fig. 2-1

tinuous variable (at some cost in speed or equipment). Thus the choice between continuousness and discrete devices is not so simple and depends upon the particular applications. Common examples of discrete and continuous variables are shown in Fig. 2-2.

One-to-One Correspondence

A very important characteristic of analog devices and computers is that for every action of the primary system there is a corresponding action of its analog, and every variable has its counterpart in the analog. Thus we speak of there existing, a *one-to-one correspondence* between the primary system and its analog [Fig. 2-3, (A), (B), and (C)]. This characteristic is fundamental to all model building and analog computation and simulation. It is of the greatest benefit to the computer operator for it permits him to observe the performance of each component or each variable of his primary system through its corresponding component in the analog. In fact, many variables which are often virtually impossible to measure in the primary system are actually computed in the analog and are available to operator.

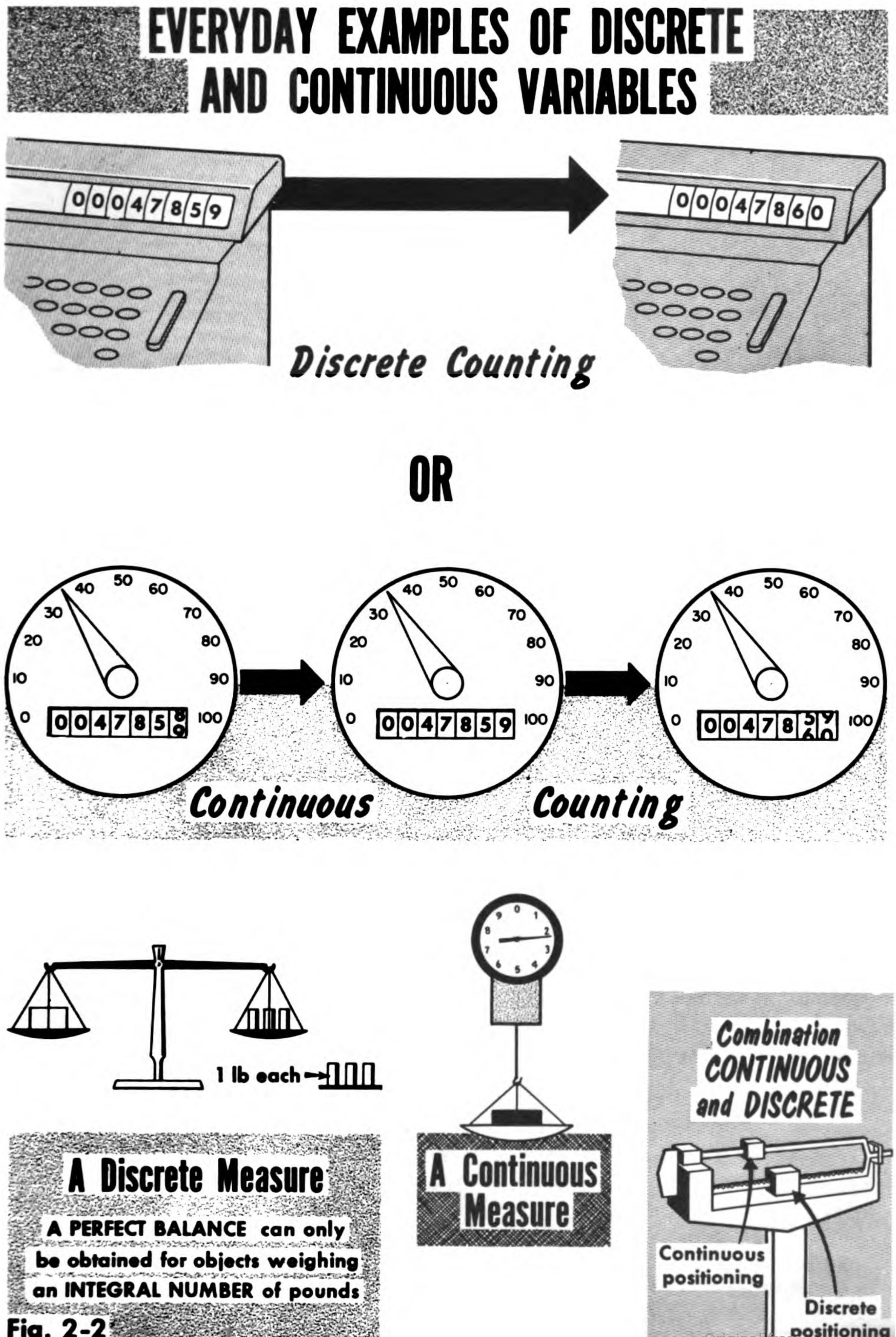

EVERYDAY EXAMPLES OF DISCRETE AND CONTINUOUS VARIABLES
00047859
00047860
Discrete Counting
OR
Continuous Counting
A Discrete Measure
A PERFECT BALANCE can only be obtained for objects weighing an INTEGRAL NUMBER of pounds
1 lb each
A Continuous Measure
Combination CONTINUOUS and DISCRETE
Continuous positioning
Discrete positioning

Fig. 2-2

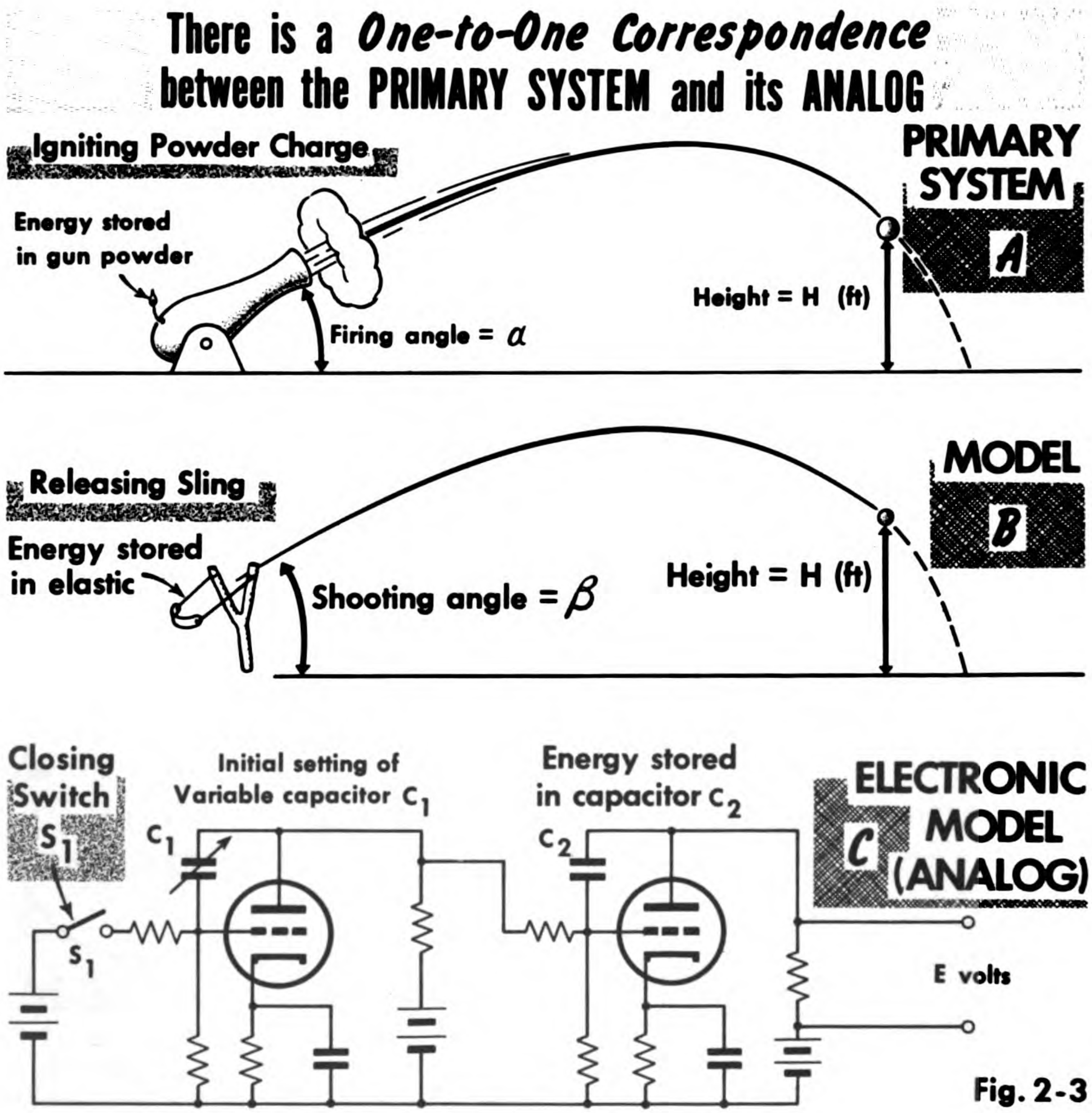


Fig. 2-3

PRIMARY SYSTEM [Fig. 2-3 (A)]	MODEL [Fig. 2-3 (B)]	ELECTRONIC MODEL [Fig. 2-3 (C)]
Igniting powder charge	Releasing sling	Closing switch S_1
Firing angle α	Shooting angle β	Initial setting of the variable capacitor C_1
Height, H, in feet	Height, H, in feet	Voltage, E, in volts
Energy stored in gun powder	Energy stored in elastic	Electric energy stored in capacitor C_1

Example: Electronic Simulation of Exterior Ballistics

In the analysis of a simple exterior ballistics problem as illustrated on the previous page, only the angle α and the muzzle velocity are needed to compute the trajectory (Fig. 2-4). The muzzle velocity depends upon the

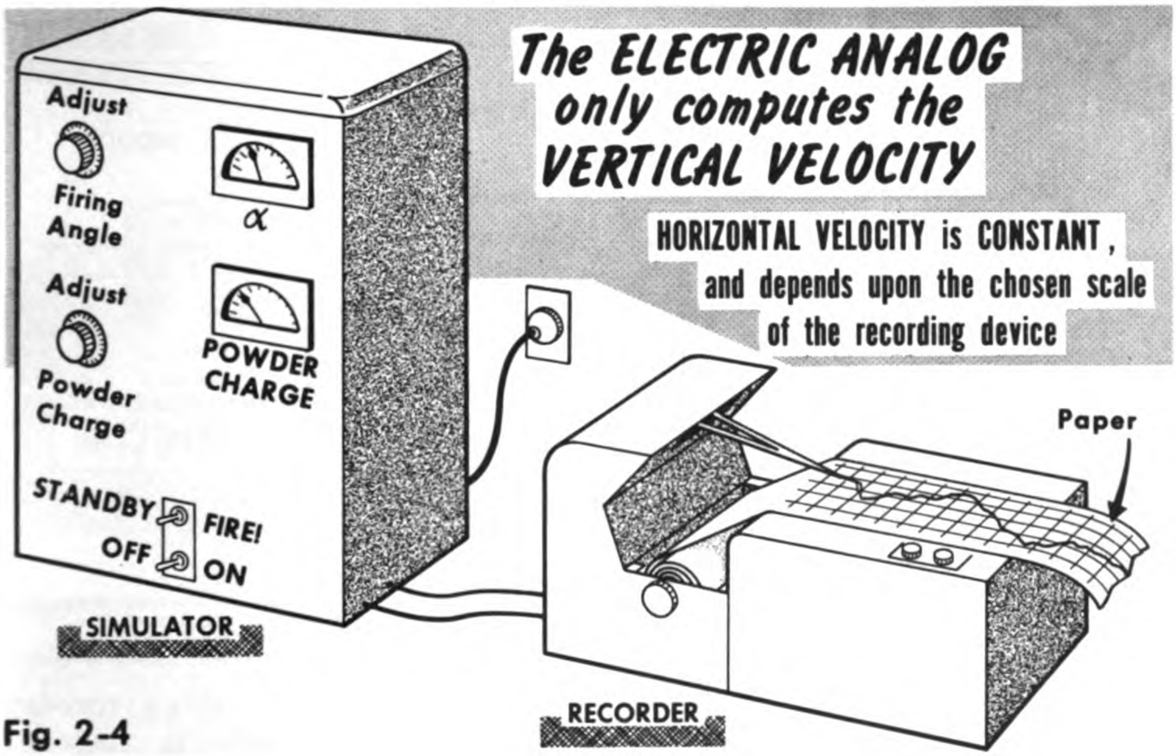


Fig. 2-4

powder charge, and the projectile and barrel characteristics. In the electronic model only the setting of C_1 and the initial charge on C_1 (or the voltage measured across C_1 before the switch is closed) are required to complete the set-up of the model and to compute the trajectory.

Actually in this case the electronic analog only computes the vertical velocity, since the horizontal velocity is known to be constant (neglecting air resistance), and is taken into account by choosing the appropriate horizontal scale on the recording device.

Correspondence of Simple Electrical and Mechanical Systems

One of the simplest and most basic mechanical systems and its electrical analog are shown below. We have seen that these systems *are* analogs, for their dynamic performance is governed by physical laws whose mathematical expressions are the same.

Note the one-to-one correspondence between the elements and variables of the two systems (Fig. 2-5).

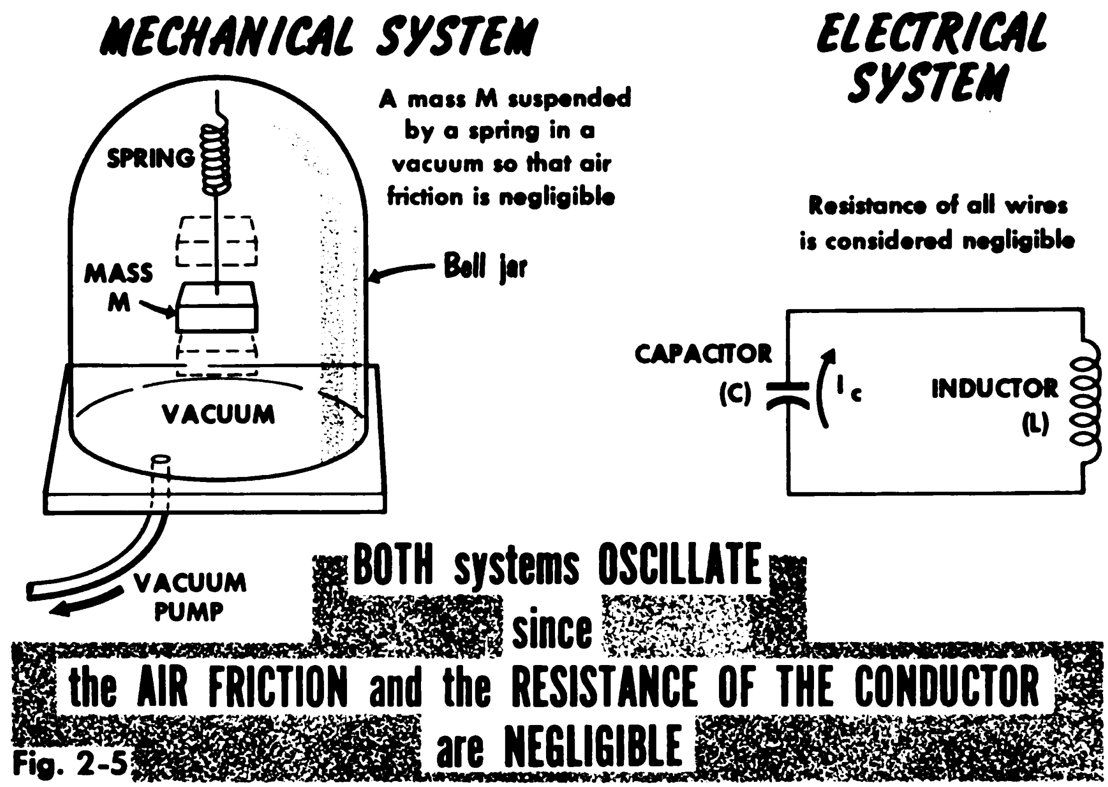


Fig. 2-5

MECHANICAL SYSTEM	ELECTRICAL SYSTEM
1. When the mass is moving there is *kinetic* energy stored in it (inertia).	1. When a current flows through the inductor there is *magnetic* energy stored in it.
2. When the spring is compressed or extended there is *potential* energy stored in it.	2. When a voltage exists across the capacitor, there is electric energy stored in it.
3. Once the system is set in motion the mass oscillates up and down. As it reverses direction the velocity is instantaneously zero and hence the inertia or kinetic energy is zero. But the energy must have gone somewhere, and we find that it is stored in the compressed (or extended) spring, in the form of potential energy. The energy imparted to the system is exchanged back and forth between mass and spring; between kinetic and potential forms.	3. Once the electrical system is "excited", current will oscillate between clockwise and counterclockwise flow. When the current reverses direction it is instantaneously zero, and the energy stored in the magnetic field of the inductor is also zero. At this instant all the energy is stored in the capacitor as electric energy. The initial excitation energy will be exchanged back and forth between inductor and capacitor, between magnetic and electric forms.

Both systems will oscillate indefinitely, since it is assumed that neither air friction nor wire resistance exist. The presence of these quantities would introduce a dissipation of energy and hence would damp out the oscillations.

Electrical and Mechanical Systems With Damping

Consider now a more realistic case, where both wire resistance and mechanical friction are present to some small degree. Also let us provide a driving force for each of the two systems (Fig. 2-6) as follows:

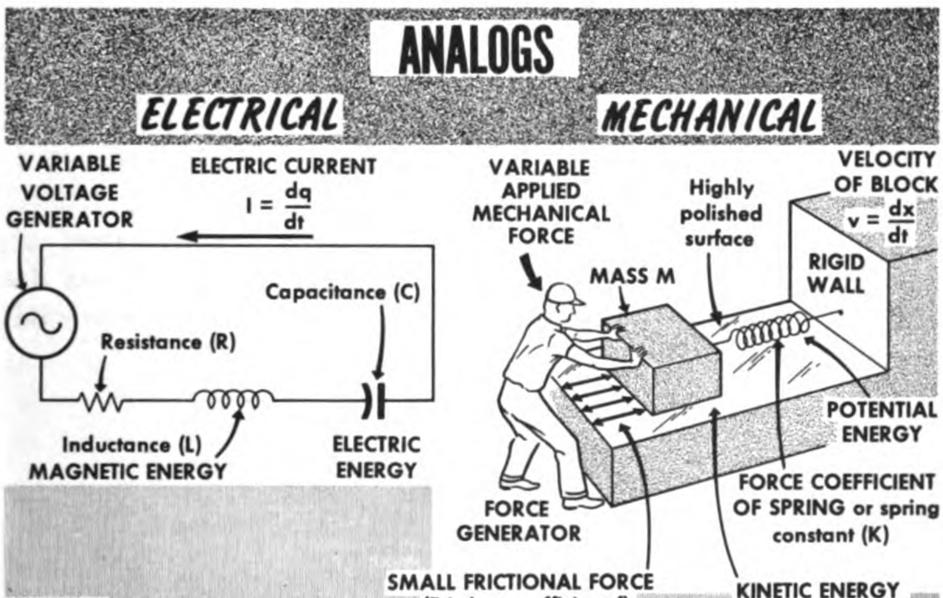


Fig. 2-6

ANALOGS

⟵⟶

Mechanical	Electrical
Force "generator"	Voltage generator
Coefficient of friction, f	Resistance, R
Mass of the block, M	Inductance, L
Force coefficient of spring, or spring constant, K	Capacitance, C
Kinetic energy	Magnetic energy
Potential energy	Electric energy
The velocity of the block, $v = dx/dt$	The electric current, $I = dq/dt$

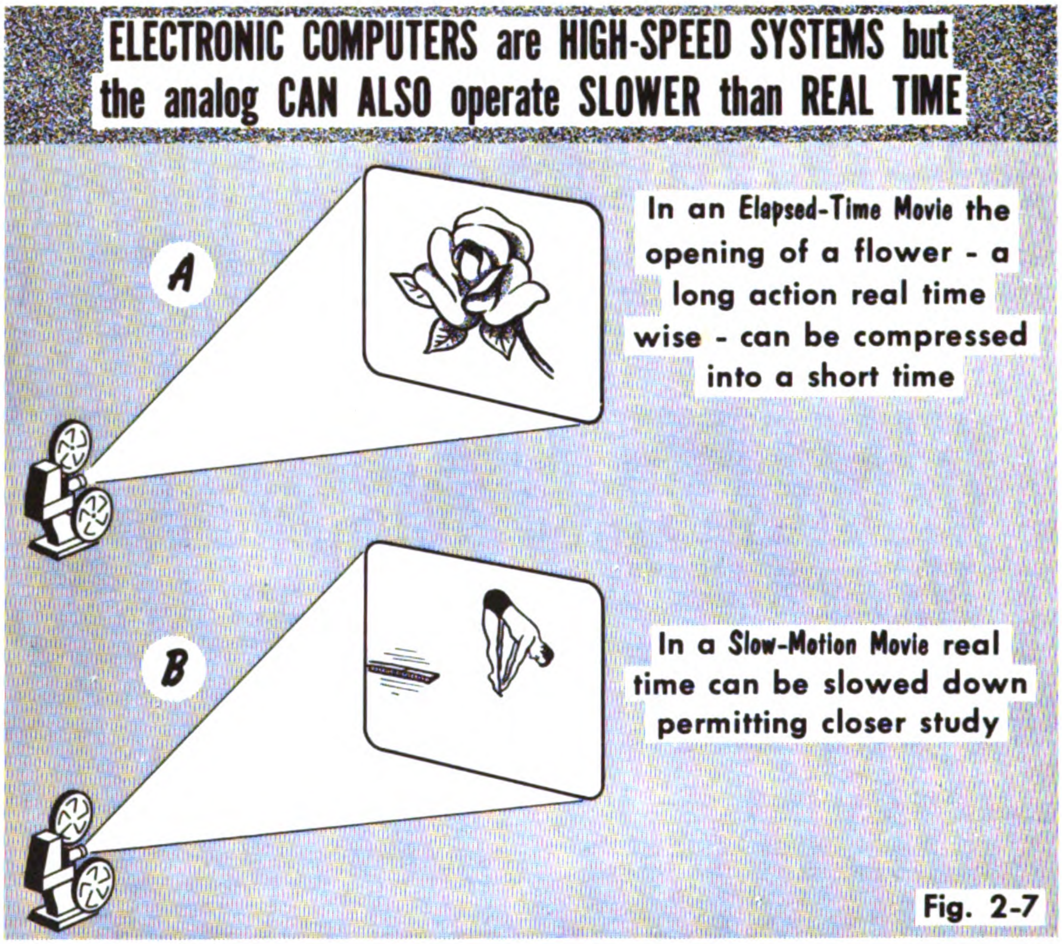


Fig. 2-7

The friction and resistance will damp out any free oscillations by continually converting a small amount of the circulating energy into heat. If the voltage and force generators are connected to their respective systems, however, motion and current will be sustained according to the amount of energy furnished by the generators.

Computing Speed

All modern electronic computers are high-speed systems. Most mechanical computers are relatively slow-speed systems due to the limitations on speed imposed by stresses on moving parts. When comparing the speed of different kinds of computers, one must be careful to define what he means by speed. For example, digital computers are extremely fast machines for processing bits of data, that is, extracting numbers from a storage medium, adding numbers, and storing numbers in a storage medium, all in a matter of microseconds (millionths of a second). On the other hand the time required to solve complex mathematical problems may be quite long—minutes, or hours. When compared on this basis, but analog computers and devices are even faster, for they acquire and add information instantaneously and continually. Although a large number of components may be required for the solution of complex mathematical problems, the time of

solution is usually of the order of seconds to minutes. Accurate comparisons of this nature are really not possible. There are more useful ways of describing computing speed. For the purpose of the present discussion of analog devices *we shall always compare the behavior of the analog device with the behavior of the primary system*. If they are *in step*, then we say the analog "operates in real time". If the analog *responds faster than the primary system*, then it operates "faster than real time". Finally, the analog can operate "slower than real time".

A. If the analog device is a computer and it is operating in real time then

one second of computer time = one second of real time

B. When computing at a rate *five* times real time [Fig. 2-7 (A)]

one second of computer time = five seconds of real time

That is, the analog computer performs in one second an operation that requires five seconds by the primary system.

C. When computing at a rate one tenth of real time [Fig. 2-7 (B)]

ten seconds of computer time = one second of real time

That is, in order to see better the performance of some rapid process, the computer takes ten seconds to do what occurs in one second in the primary system.

Time Scaling

The flexibility afforded by the ability to compute at a rate faster or slower than real time is an important characteristic of analog systems. It is known

TIME SCALING PERMITS COMPUTING at a rate FASTER or SLOWER than REAL TIME

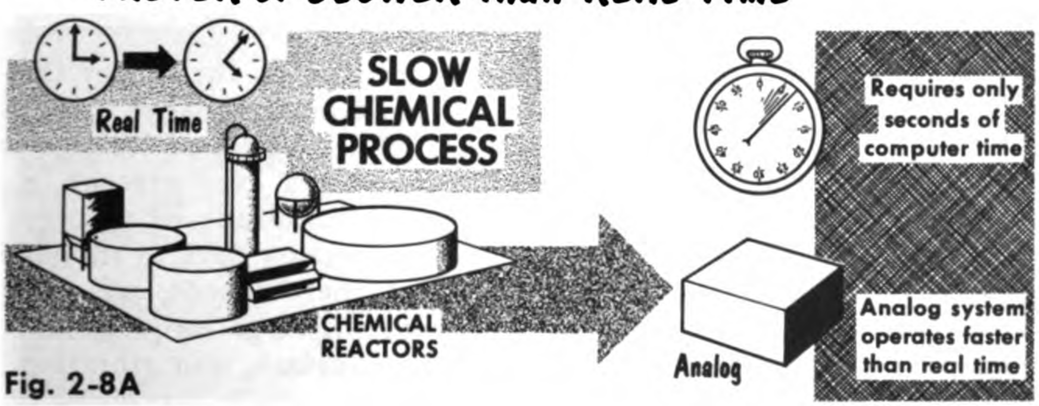


Fig. 2-8A

as *time scaling*, since the measure of the independent variable, time, can be adjusted to suit the immediate need.

It is possible by proper time scaling to compress a slow chemical process taking hours, into an analog representation taking only minutes (Fig. 2-8A). Conversely, a fast process can be slowed down in the analog system

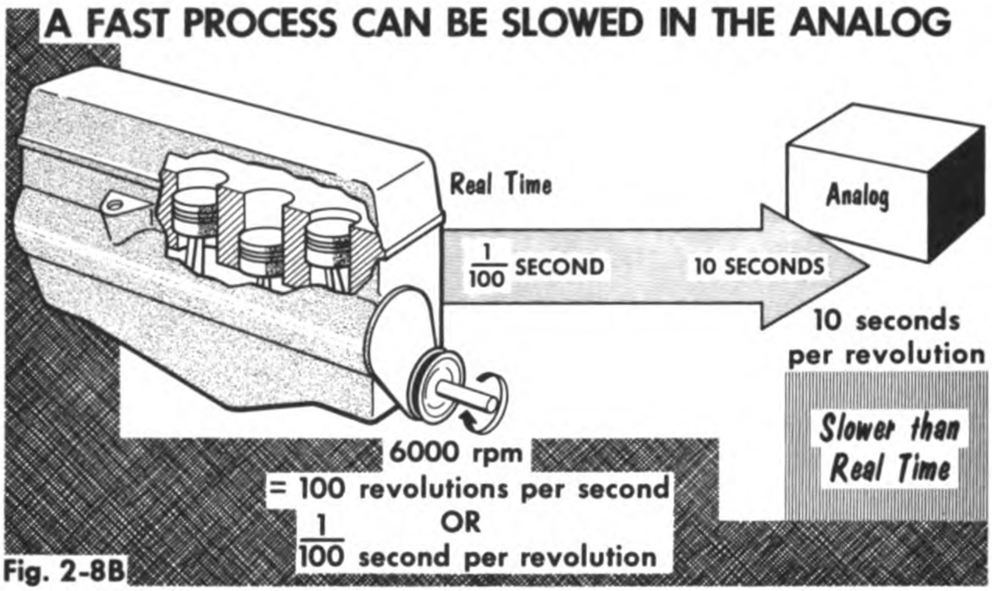


Fig. 2-8B

so that the details of operation can be closely observed and measured (Fig. 2-8B).

Component Limitations

In building analog devices it is impossible to obtain perfect components, that is, bearings without friction, or gears without backlash; capacitors without leakage, inductors without resistance, hydraulic components without leakage (Fig. 2-9). On the other hand, when high-quality components are employed in well-designed devices the effects of component imperfections are often negligible. In such cases the assumption of ideal component characteristics is justified.

The imperfections do, however, impose limitations on the use of the components. Thus, most capacitors might be used to hold a charge for a matter of minutes but not for hours, for the accumulated effect of leakage would not then be negligible. The small "noise" voltages ever present in electronic amplifiers place a lower limit upon the amplitude of voltages that may be distinguished as signals. In mechanical analog devices translational and rotational speeds, and rate of change of these speeds, are restricted by the inertia of heavy components because of the large torques and forces required to accelerate them, and by friction, backlash, and vibration.

Accuracy Limitations

In addition to time scaling limitations due to component imperfections, analog computing faces a basic accuracy limitation related to the precision

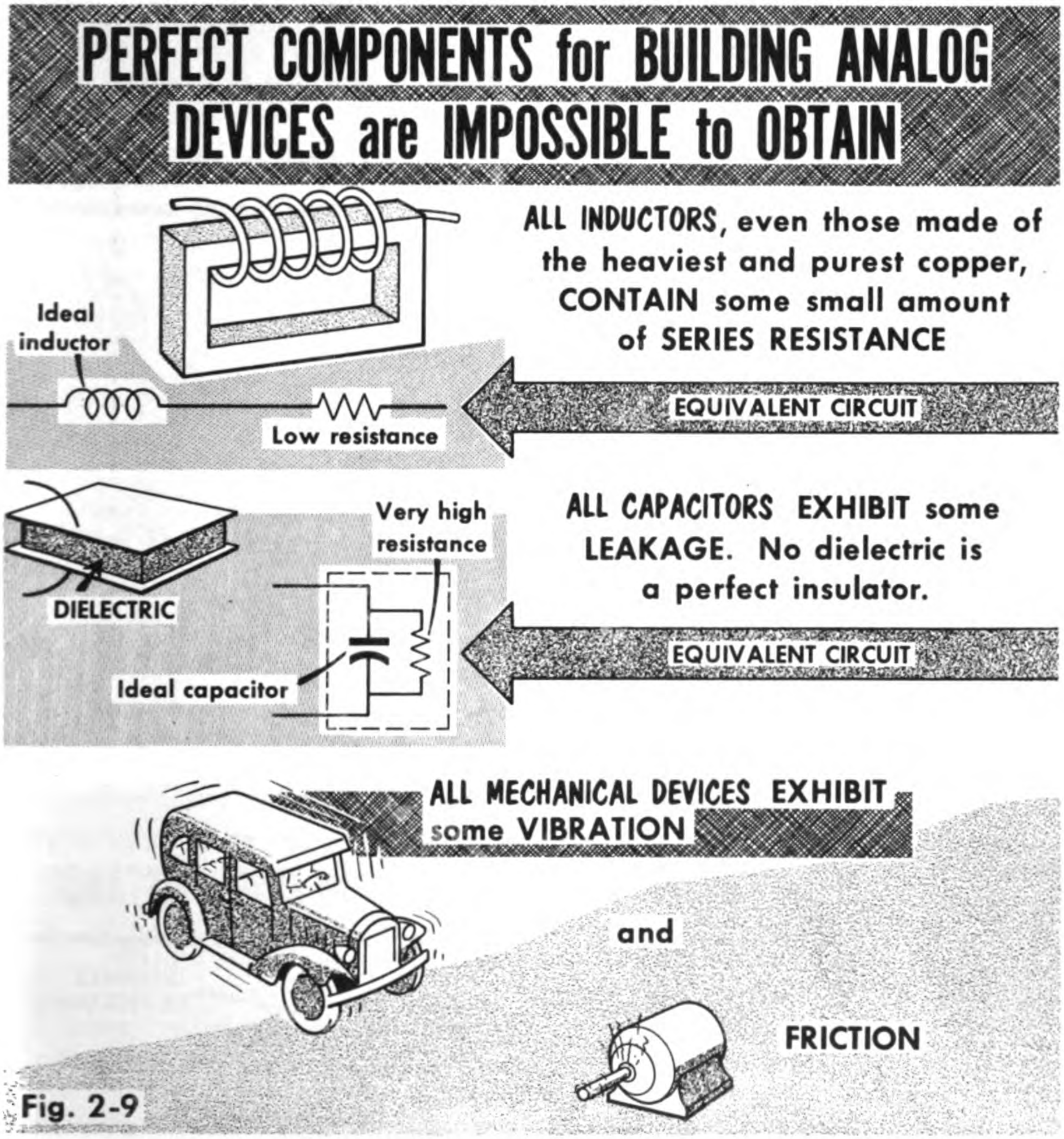


Fig. 2-9

with which it is possible to measure and record the commonly used analog variables of electric voltage and current, and the mechanical position of shafts or gears.

- How precisely can you read a voltmeter? (Fig. 2-10A-B).

Common meter . . .	1 part in 100, or 1%
Expensive meter . .	1 part in 1000, or 0.1%

- How precisely can you measure a distance? (Fig. 2-10C-D).

With an ordinary scale or a ruler	1 part in 100, or 1%
With expensive laboratory equipment . .	1 part in 5000, or 0.02%

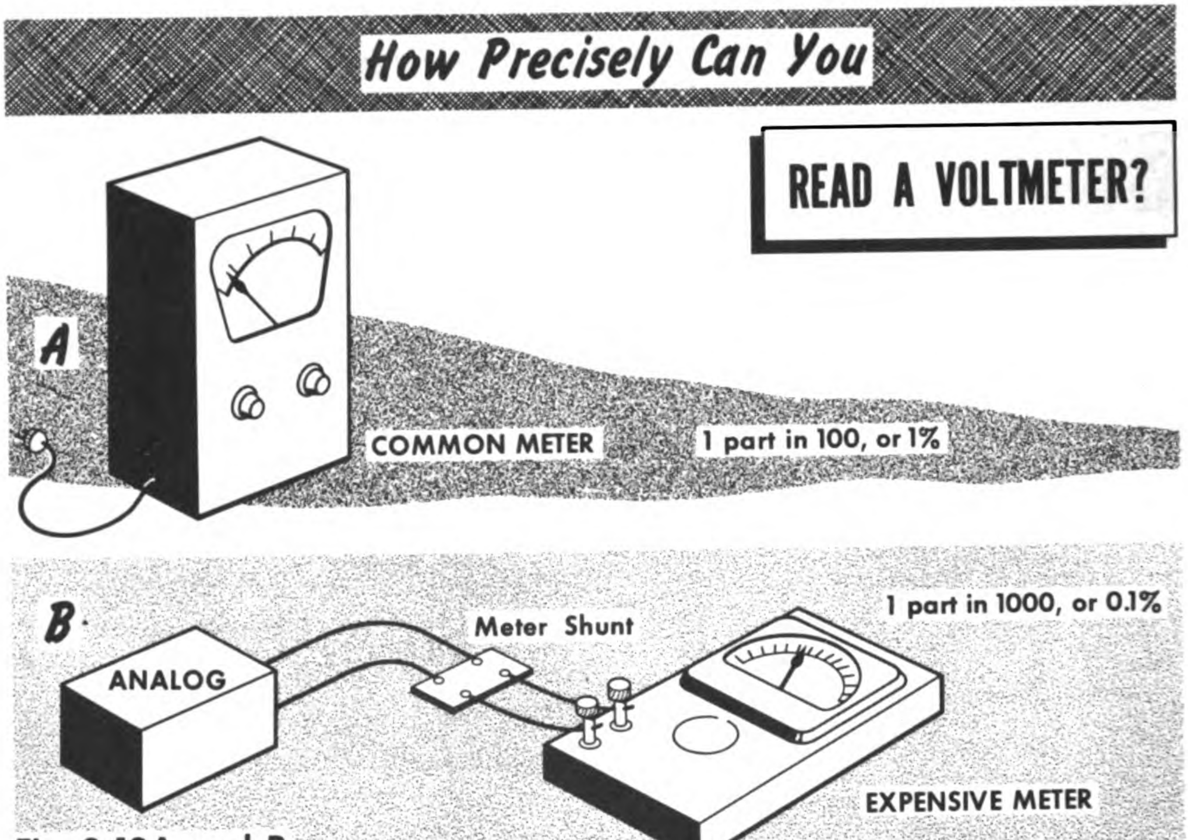


Fig. 2-10A and B

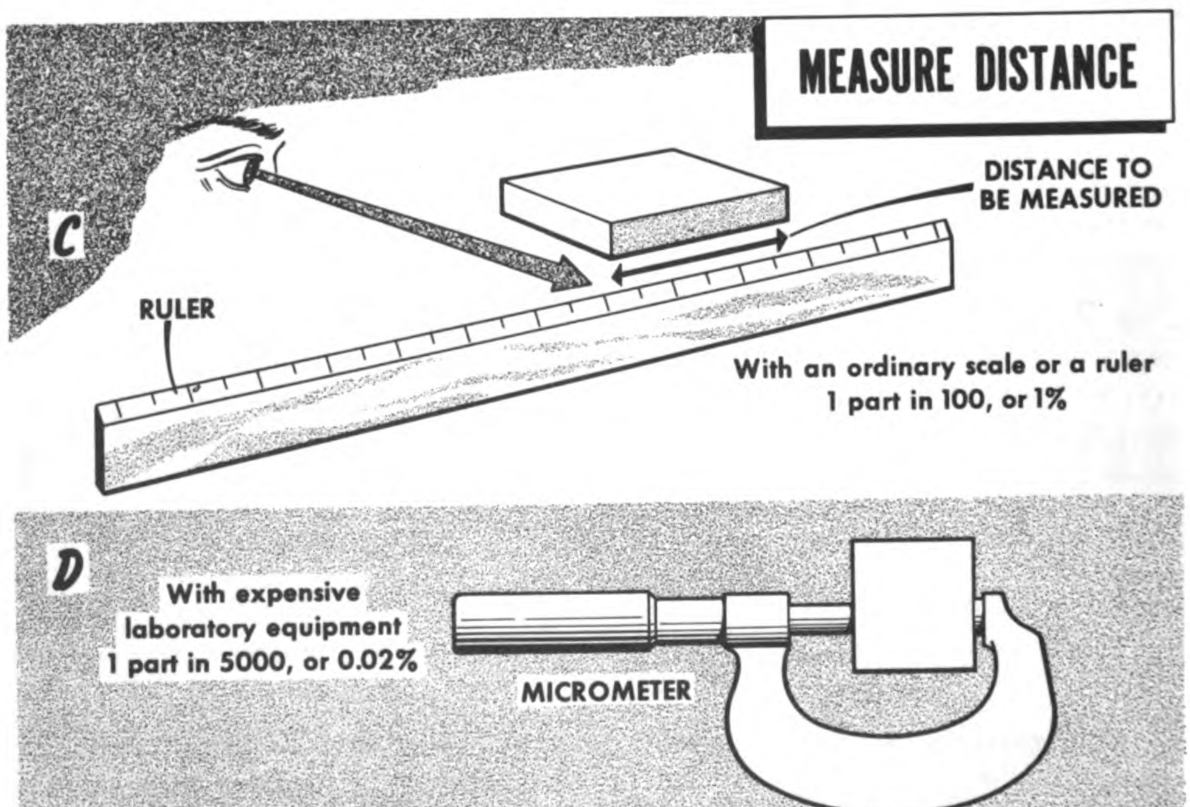


Fig. 2-10C and D

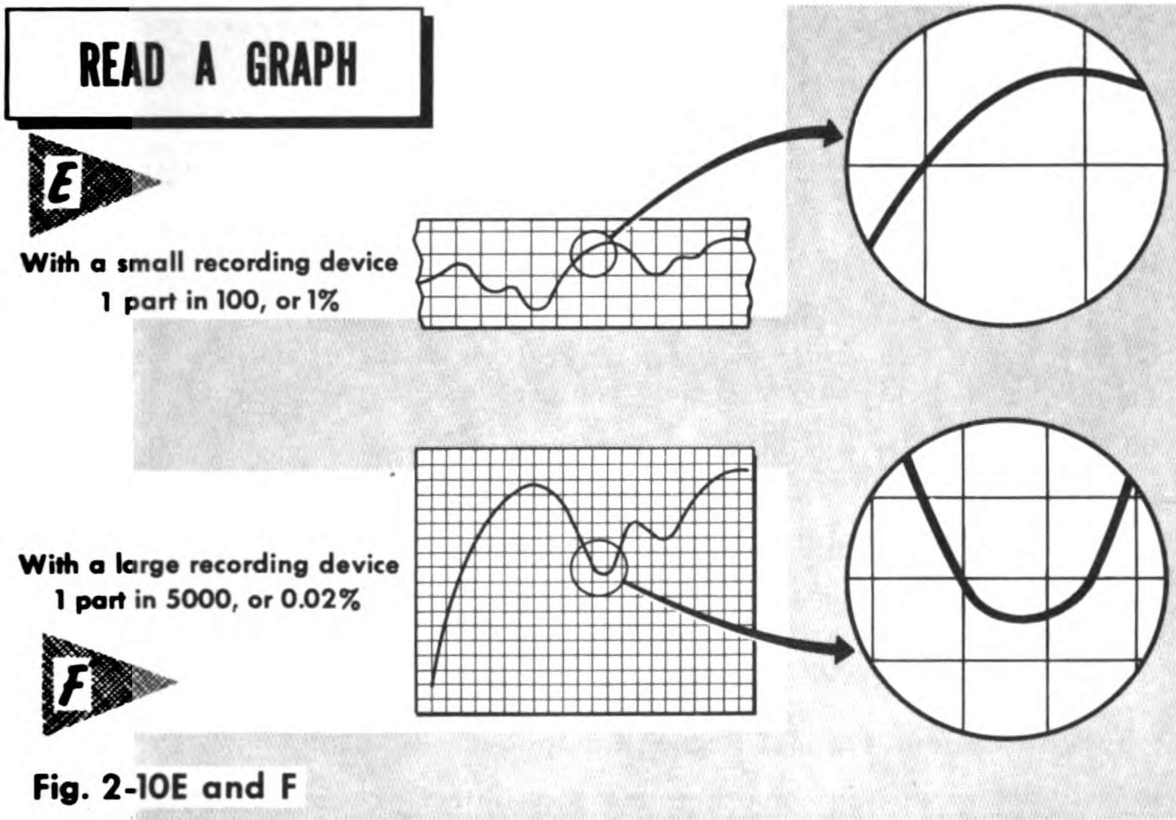


Fig. 2-10E and F

- How accurately can you compare the position of a line with the grid of a piece of graph paper? (Fig. 2-10E-F).

With a small recording device . .	1 part in 100, or 1%
With a large recording device . .	1 part in 5000, or 0.02%

All these considerations limit the available precision of an analog device. Greater precision can be had at a greater expense, but this is possible only up to a limit, for beyond a precision of 1 part in 10,000 (0.01%) the cost of increased precision is prohibitive. This limit is of no great concern, for the design engineer who is using an analog device does not wish to pay the price of extra precision much beyond the precision of the device he is designing. In fact, if his design tolerances are ± 1%, and he uses the analog device simply to obtain qualitative information by observing the dynamic behavior of his experimental design, then a 5% precision is probably adequate for the analog device. If he wishes to use the results of the analog simulation directly in his design, a precision of 0.05% to 0.1% would be required. Analog devices and computers are frequently used for both qualitative information and quantitative results.

Usually it is possible to find an analog device with sufficient precision for any engineering design problem. Extreme cases do exist where the component limitations seriously affect the usefulness of the analog simulation. More often, the primary source of errors in a computing system is in the

mathematical approximations and simplifications employed in describing the primary system. Concern with this difficulty lies in the field of mathematical analysis rather than the field of computer usage.

ANALOG DEVICES vs. ANALOG COMPUTERS

Earlier, three categories of analog devices were defined:

1. *Models*: scaled versions of the primary systems.
2. *Regulators and control systems*: systems that behave in a manner analogous to the operator they replace; *automata*.
3. *Analog devices* which obey fundamental laws of nature similar to those of the primary system.

It is now necessary to further subdivide the third category as follows:

a. General purpose analog computers
b. Special purpose analog computers
c. Other analog devices.

Group 3(a): General Purpose Analog Computers

General purpose analog computers are the subject of the remainder of this book. For an analog computer to be *general purpose* it must contain a large number of standard computing components. These components are often called computer *building blocks* (Fig. 2-11). By correctly intercon-

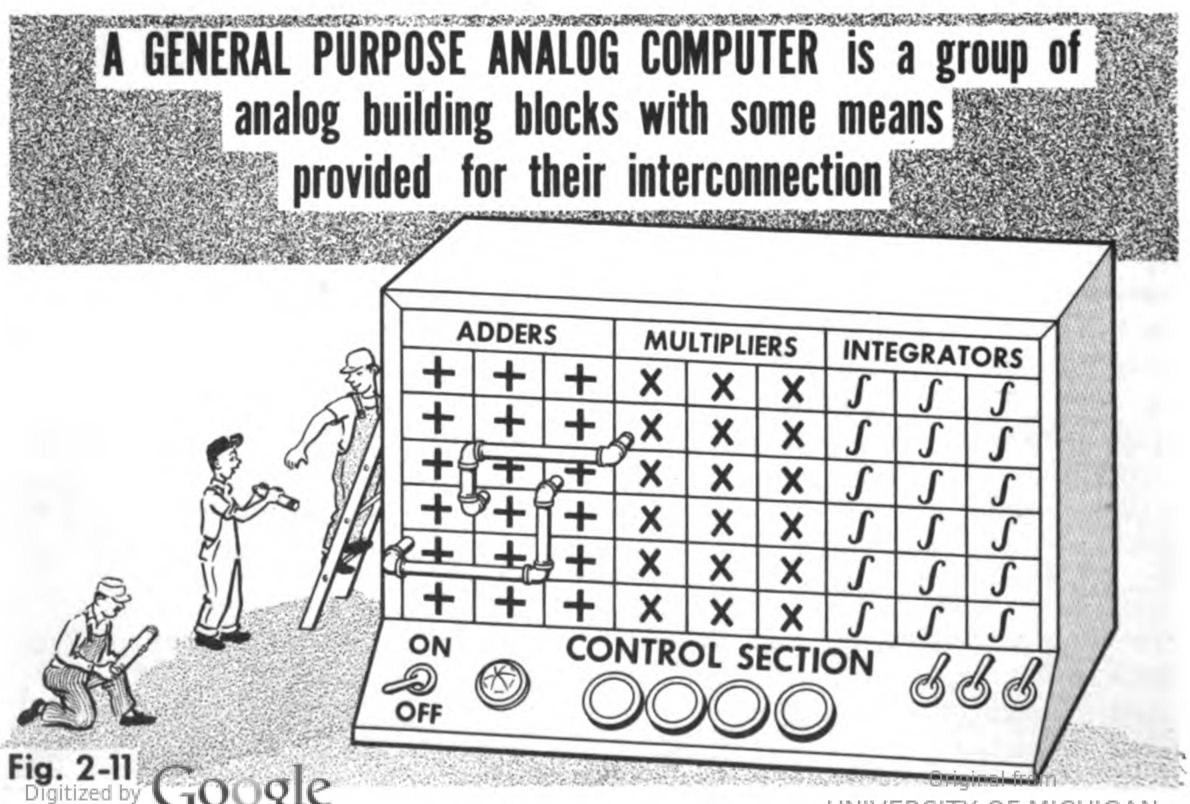


Fig. 2-11

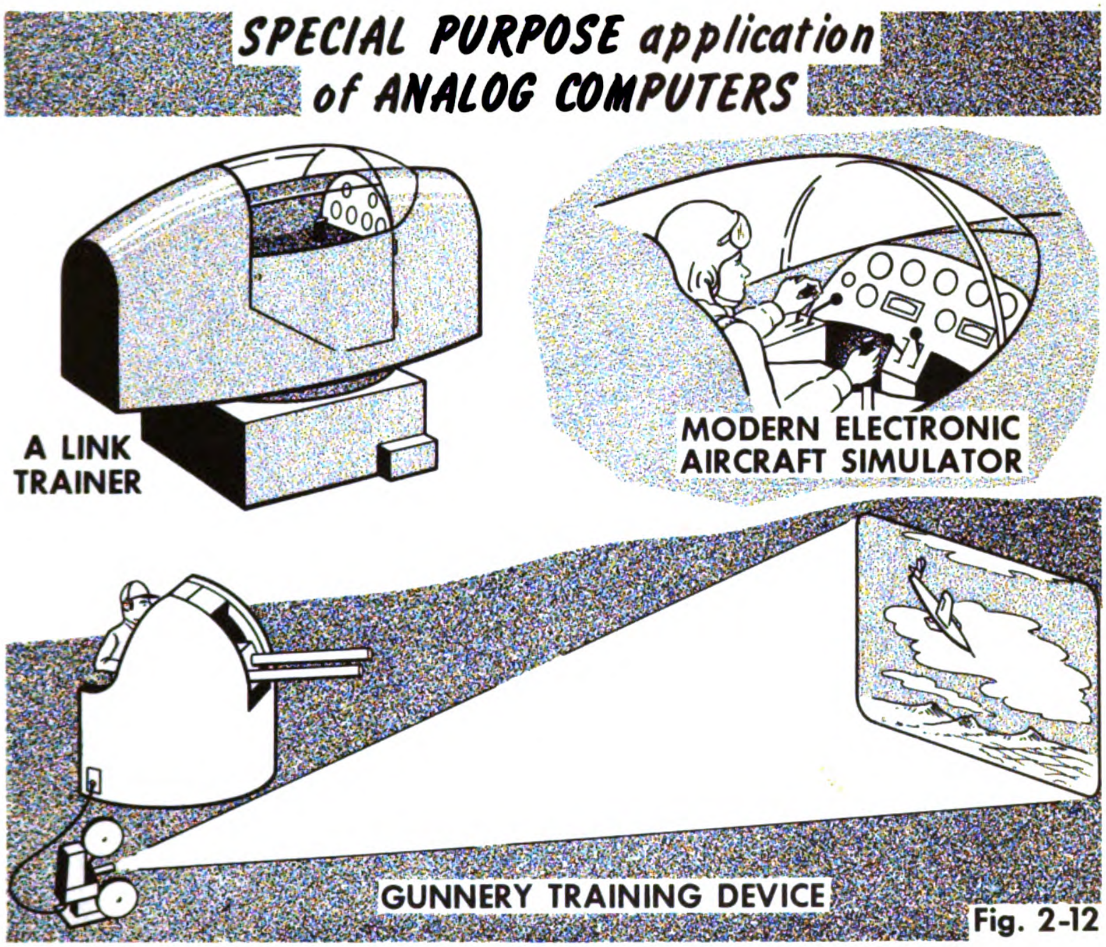


Fig. 2-12

necting these building blocks, the computer operator can build a computer model of any primary system he wishes. The building blocks are themselves analog devices [belonging to Group (3c), for they are not computers by themselves]. They are generally designed to perform a single mathematical operation and are simply described by that operation rather than by the means of achieving it. Thus the building blocks are called *adders*, *multipliers*, *integrators*, etc. Once the primary system is described by mathematical equations it becomes a simple process to connect the building blocks to simulate the behavior of the primary system.

Group (3b): Special Purpose Analog Computers

A large number and variety of special purpose analog computers (Fig. 2-12) have been built. As the name implies, these computers are designed to spend their careers simulating a single primary system. Lest this become boring, provision is generally made for varying certain *characteristics* if the primary system, while the *type* of primary system remains the same.

Special purpose analog computers are in use which simulate nuclear reactors and their control systems, electric power distribution systems, air-

craft control systems and autopilots, and radar systems. Of particular importance is the large number of military training aids which qualify as special purpose analog computers: instrument flight trainers, operational flight trainers (for teaching operating procedures in a particular aircraft),

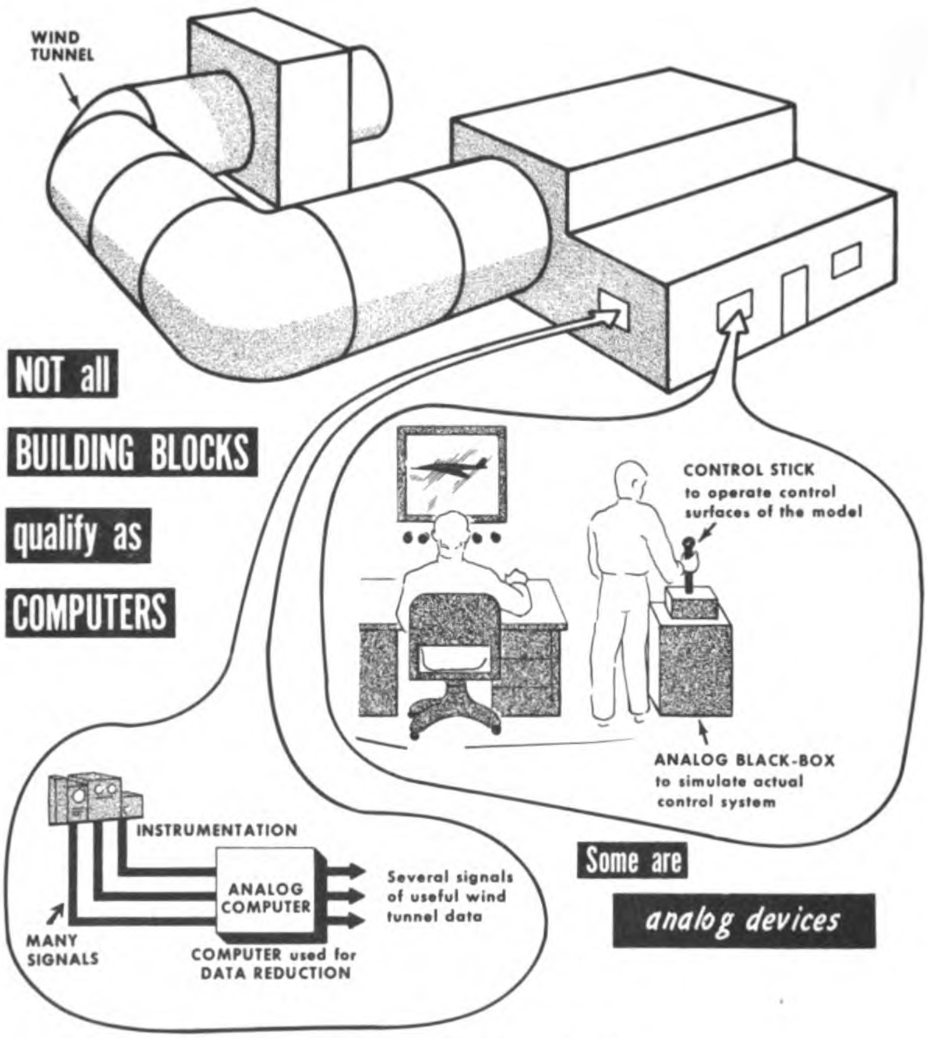


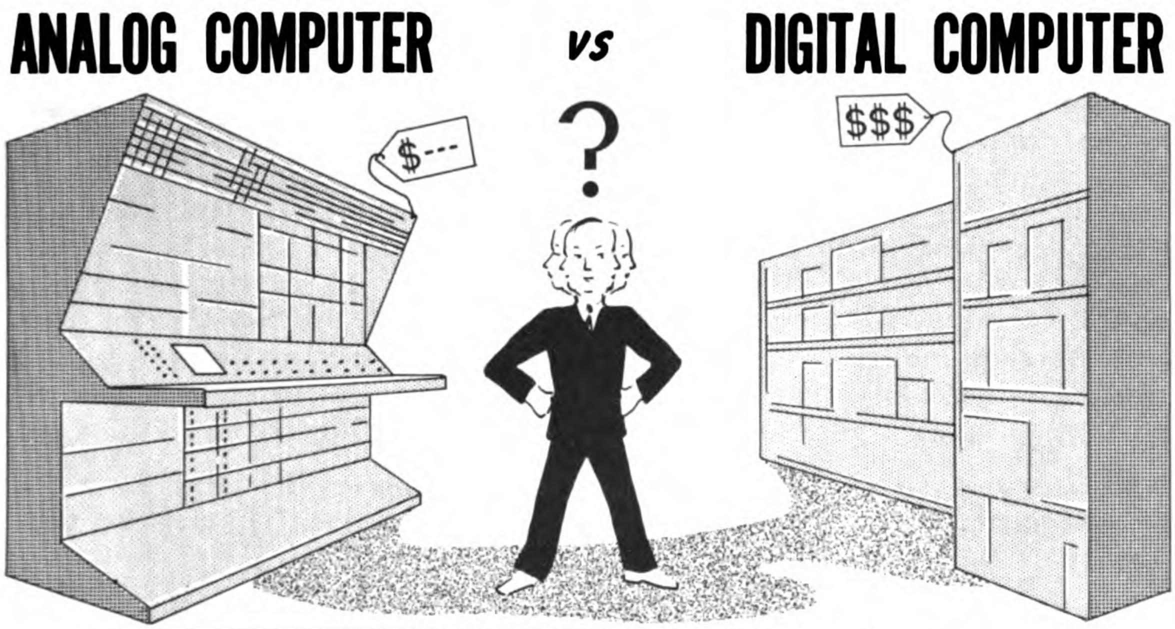
Fig. 2-13 Some are *data processing devices*

submarine trainers, and gunnery trainers. These computers are mechanical, pneumatic, electronic and/or hydraulic. They each simulate one primary system (aircraft, ship, gun turret, etc.) that can assume many situations depending upon the wishes of the trainee and his instructor.

Group (3c): Noncomputer Analog Devices

This group includes:

1. The computer building blocks used in general purpose analog computers and special purpose analog computers which in themselves do not qualify as computers (Fig. 2-13).



PROBLEM Which kind of computer is best? *Analog or Digital?*

FACTORS INVOLVED Price? Speed? Accuracy? Capacity? Ease of Programming?

Fig. 2-14

2. Devices (often hidden in the popular "black-box") used to replace some component in a scale model (such as an electronic analog device to replace the hydraulic system in a model aircraft used in a wind tunnel).
3. Data-processing devices used to evaluate or change continuous data being obtained from a primary system, or from a model in an experiment, or from an operating pilot plant. The data may be in the form of voltages or pressures, etc., and the analog device may be used to add two quantities, average several quantities, or perform other simple mathematical operations before recording the data. A general purpose analog computer or special purpose analog computer might be used, but quite often the operation requires only a single black-box to perform a simple mathematical operation.

ANALOG COMPUTERS vs. DIGITAL COMPUTERS

Although general purpose analog computers have been in use since 1930, the one computer of major importance today, the d-c electronic analog computer, did not come of age until the early 1950's, at which time there was a major expansion in the analog computer field. The aircraft and missile industry provided the chief impetus for the development of this computer, for it was ideally suited to the simulation of aircraft control

systems and the dynamics of flight. At about the same time there was a major expansion of the electronic data processing industry. The earlier development of the ENIAC and other digital computers led to the application of electronic digital techniques to the high-speed processing of large volumes of numeric and alphanumeric data. A natural consequence was the use of digital computers to perform laborious calculations previously performed with hand-operated desk calculators. The enormous success in replacing hand-operated calculators resulted in the use of the digital computer for almost every conceivable type of mathematical problem. As each industry, analog and digital, flexed its muscles for further expansion in the engineering and scientific applications of their computers, a controversy over which was "better" developed (Fig. 2-14).

A critical analysis of the two techniques is not appropriate here, but a few general comparisons will be drawn to give some insight into the complexity of the question.

The General Purpose Digital Computer

The digital computer is only capable of following the simplest instructions requiring arithmetic operations, such as: *add*, *subtract*, *multiply* and *divide*

Fig. 2-15

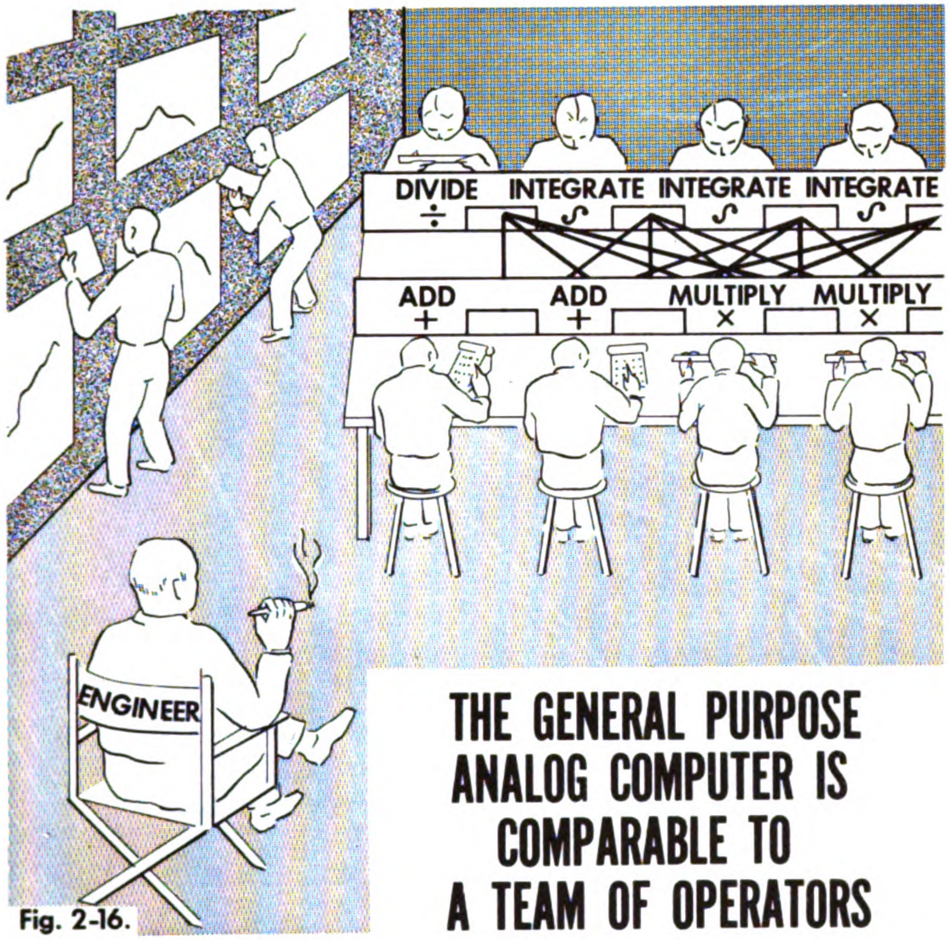


Fig. 2-16.

(Fig. 2-15). Detailed instructions are required for every calculation. The accuracy of the arithmetic operations is limited only by the size of the largest number which can be held in the available registers — as in a desk calculator.

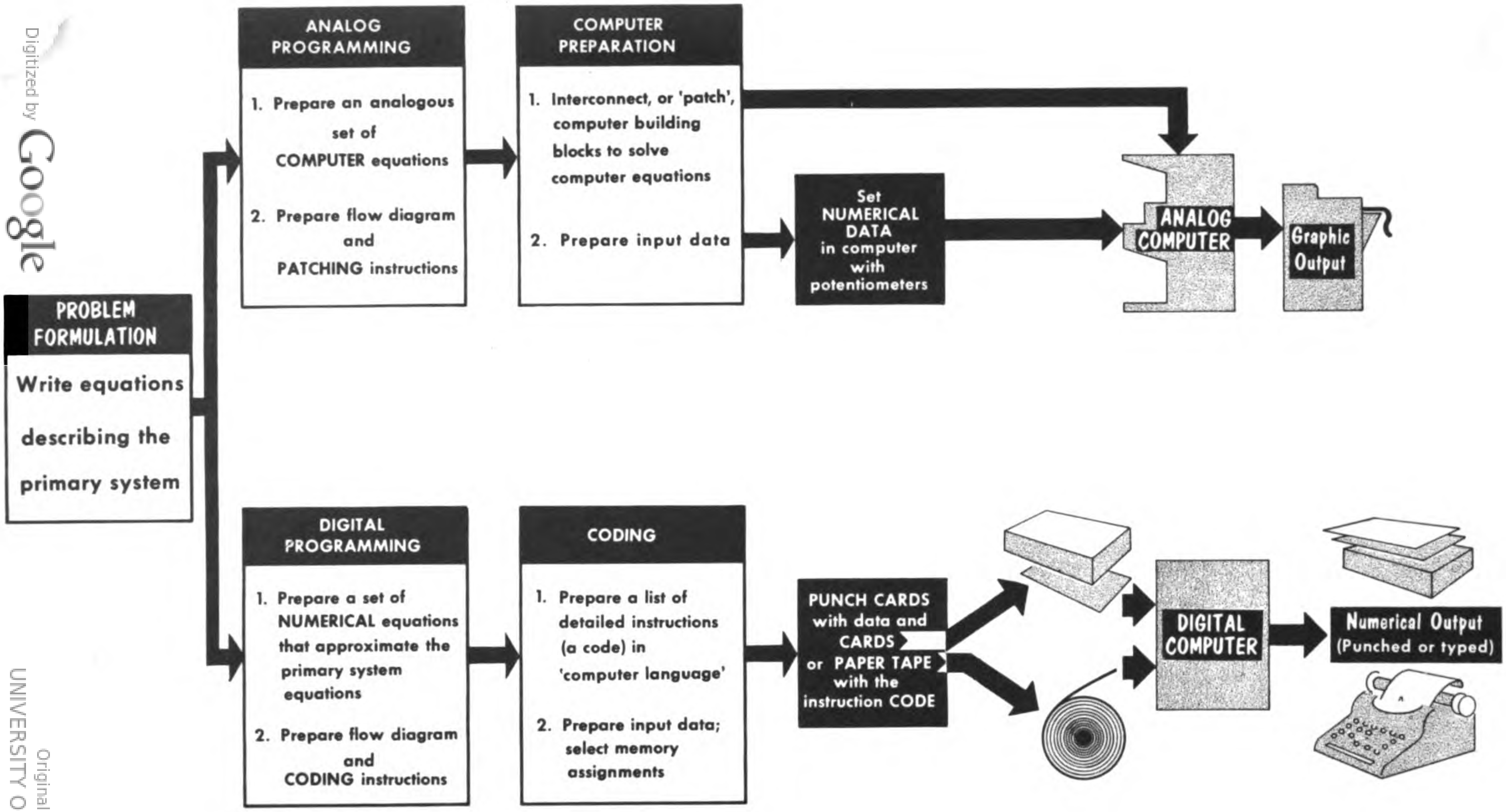
All simple calculations are accomplished *very*, very rapidly. Difficult calculations may require considerable time since only one instruction, or operation, can take place at a time.

The digital computer is ideal for business data processing and reduction of statistical data where a large number of simple calculations are performed and repeated over and over again. Since the hourly cost of operaation of a digital computer is high ($100 to $1000 per hour) many engineering problems, particularly those involving trial and error techniques, are not suited to solution by the use of a digital computer.

The General Purpose Analog Computer

An analog computer (Fig. 2-16) might be described by the following:

1. A team of operators, each with a specific job to do.
2. Instantaneous flow of information between operators.



PREPARATION of a SCIENTIFIC PROBLEM for AUTOMATIC COMPUTATION

Fig. 2-17

3. A continuous, up-to-date, record of all variables within sight of the engineer (no memory required).
4. Extremely fast, since everyone works together; the more complex problems require more operators.
5. The accuracy depends upon the ability and reliability of each operator.
6. Total cost per hour varies from $1 to $100 per hour.

A Comparison of Procedures

A practical illustration of the preparation of scientific problems for automatic computation is shown in Fig. 2-17.

QUESTIONS

1. What is meant by the "continuous nature" of analog devices? Can this include discrete-state devices?
2. Why is the one-to-one correspondence between analog and primary system important?
3. Show how the elements of a one-dimensional spring and mass system correspond to the elements of some electric circuit.
4. Which is faster, a digital computer or an analog computer? What do you mean by speed?
5. What is "real time"? What is computer time?
6. What is time scaling? How does it increase the flexibility of analog simulation?
7. What limits the precision of analog devices?
8. What limits the accuracy of analog computer results?
9. What is the difference between accuracy, precision, and resolution?
10. What are the basic differences between analog and digital computers?

Chapter 3

COMPUTER BUILDING BLOCKS

Building Blocks

A general purpose computer gains its flexibility from the fact that it consists of several groups of standard building blocks which can be interconnected in a variety of ways by the cumputer programmer. The freedom of interconnection makes it possible to solve many kinds of mathematical problems with the same computer. The so-called computer building blocks come with various capabilities. Generally they are designed to perform a single mathematical operation, so that programming the computer becomes a simple matter of choosing the appropriate building block for each of the indicated mathematical operations.

Let us look at the type of building blocks required (Fig. 3-1).

1. Addition of variable x to variable y to form another variable which we may call z; so that $z = x + y$
2. Sign inversion of variable y so that x and *minus* y can be added to get $z = x - y$. Sign inversion is the same as multiplication by -1.

Addition and Subtraction of Several Variables

No subtractor is needed since the combination of sign inverter and adder performs subtraction. A special subtractor could be built, but more freedom is provided with the above combination (Fig. 3-2). For example, the adder may be built to accept more than two input variables, as shown, and if some of the input variables have to be subtracted, sign inverters can be used, as below. Now if special subtractors were used, two subtractors and one adder or two adders and one subtractor would be required, and this would be more expensive than the above method.

Mechanical and Hydraulic Adders

Let us consider some simple examples of the abovementioned blocks. The examples given here are included for illustration only, and are not found

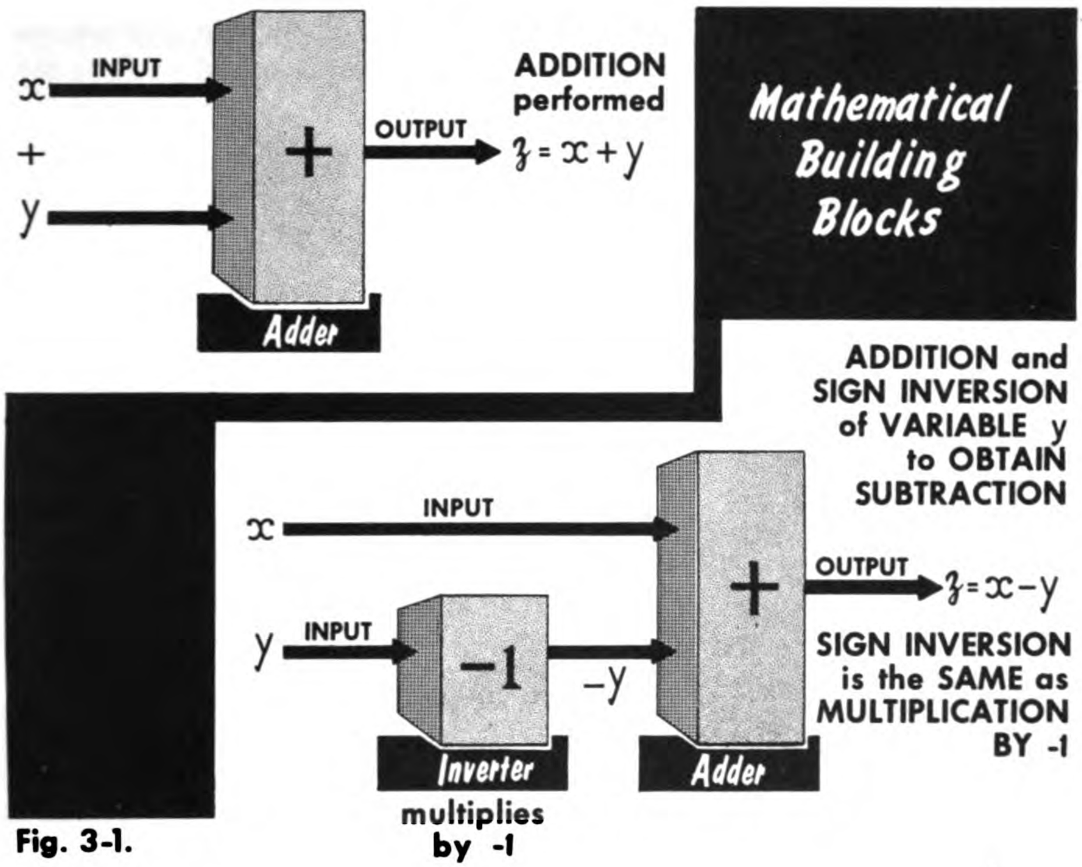


Fig. 3-1.

in any general purpose computer known to the authors. In fact, hydraulic building blocks are probably used *only* in special computing devices.

Suppose our variables x_1, x_2, etc., represent the positions of several sliding rods; then we can build a mechanical adder as shown in Fig. 3-3A.

The distance, z, will always equal the average displacement of the two

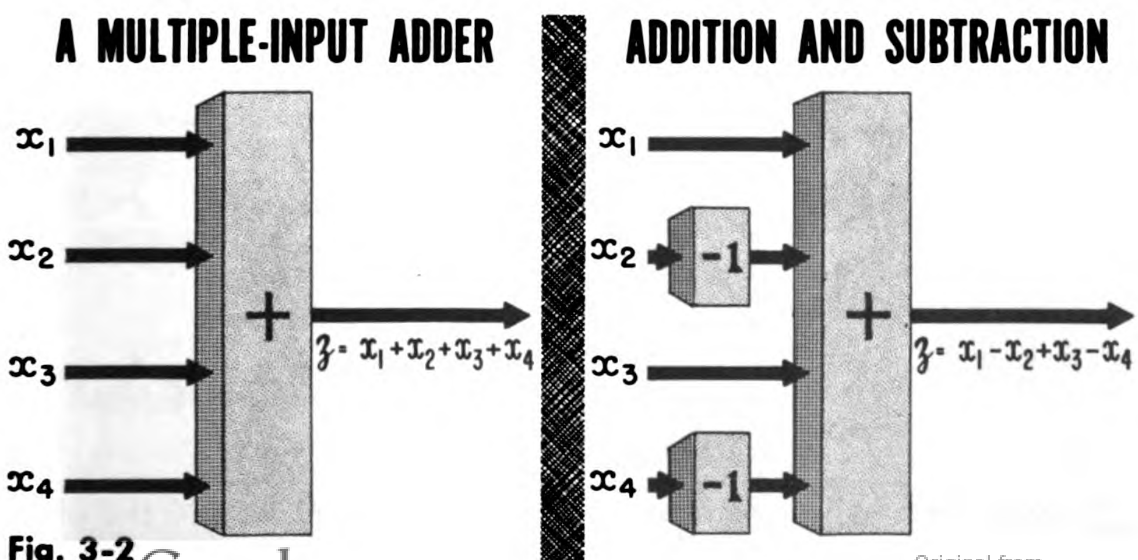


Fig. 3-2

left-hand rods, that is $z = (x_1 + x_2)/2$. Since multiplication by a constant factor of 2 can be easily accomplished by a simple lever, the above system is really an adder.

Now suppose the variables are different pressures from a hydraulic system. That is, we shall consider an hydraulic adder (Fig. 3-3B).

For this system to be in balance, the force exerted by the right-hand piston

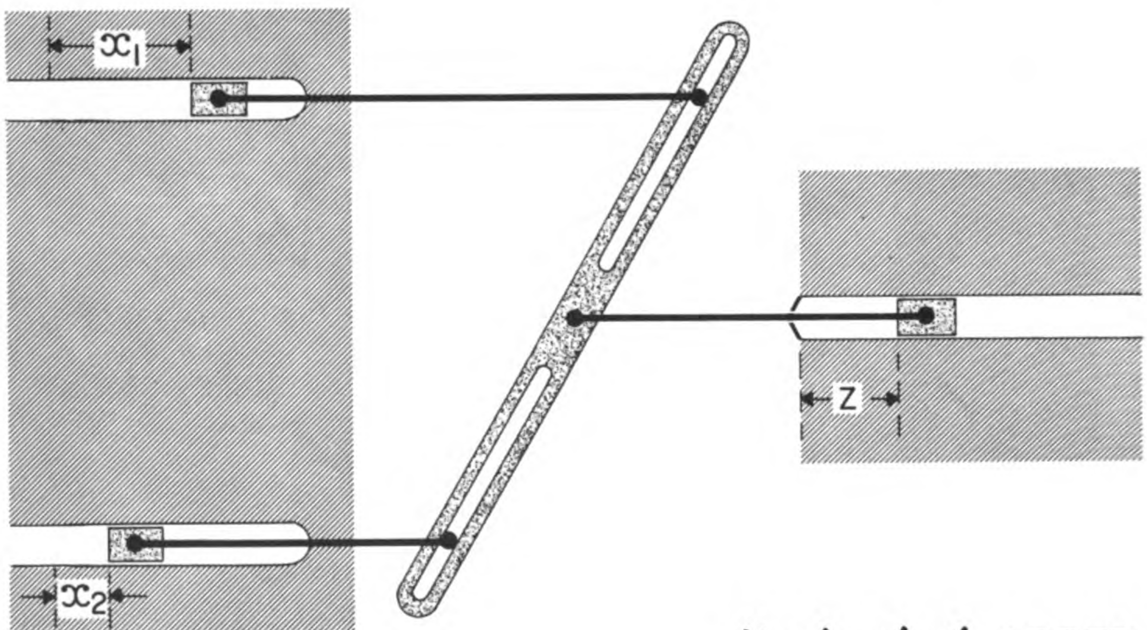


Fig. 3-3A ***Mechanical* ADDER**

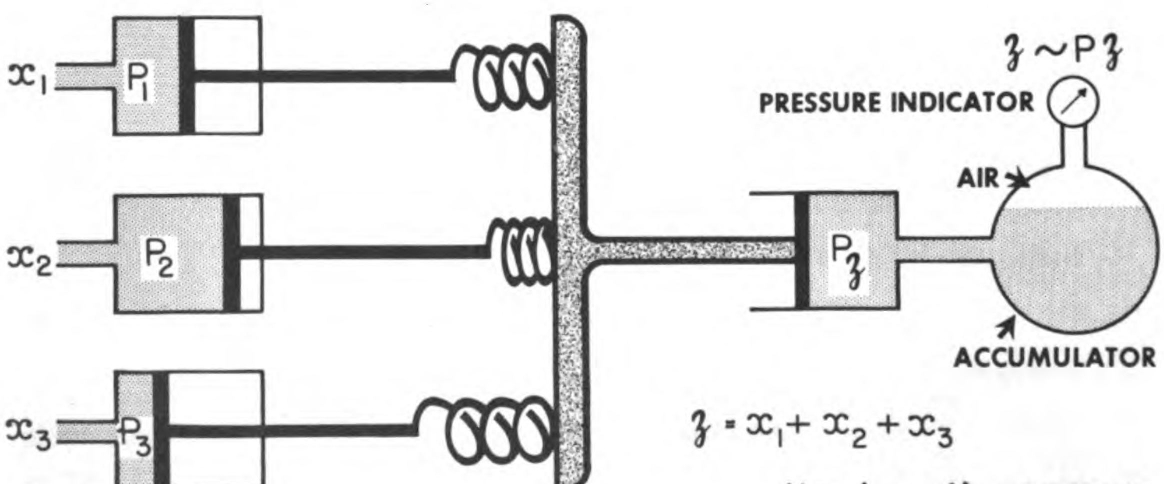


Fig. 3-3B ***Hydraulic* ADDER**

must equal the sum of the forces from the left. Therefore if the piston surface areas are the same, $P_z = P_1 + P_2 + P_3$ and $z = x_1 + x_2 + x_3$.

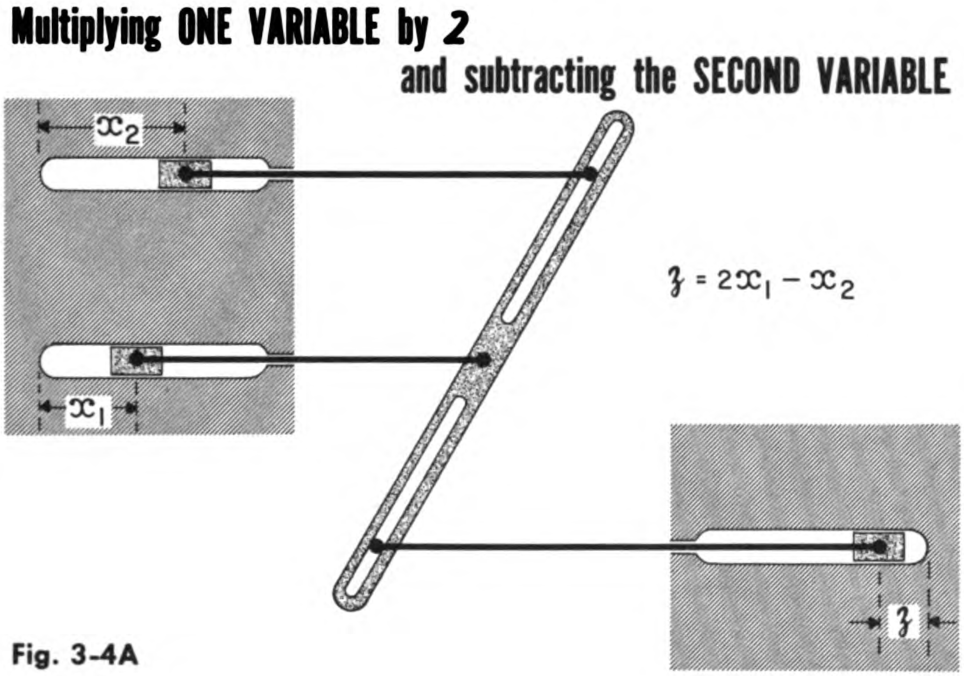


Fig. 3-4A

Adders and Inverters

The mechanical device illustrated in Fig. 3-4A multiplies one variable by two and subtracts the other variable.

There are many, many combinations of rods and slides which produce the analog of various simple arithmetical operations. The flexibility of such devices can be further extended by the use of pulleys and ropes.

Hydraulic Adder with Sign Inverters

It is clear that the x_2 and x_3 forces act in the opposite direction from x_1 and therefore subtract from P_z. Hence, for pistons with equal surface areas

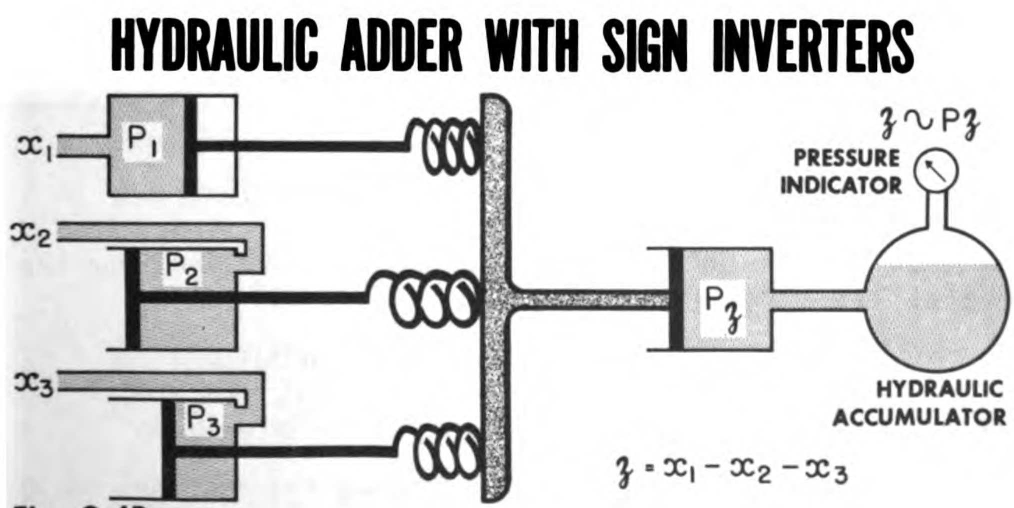


Fig. 3-4B

A common DIFFERENTIAL GEAR ASSEMBLY performs the operation of ADDITION of SHAFT speeds

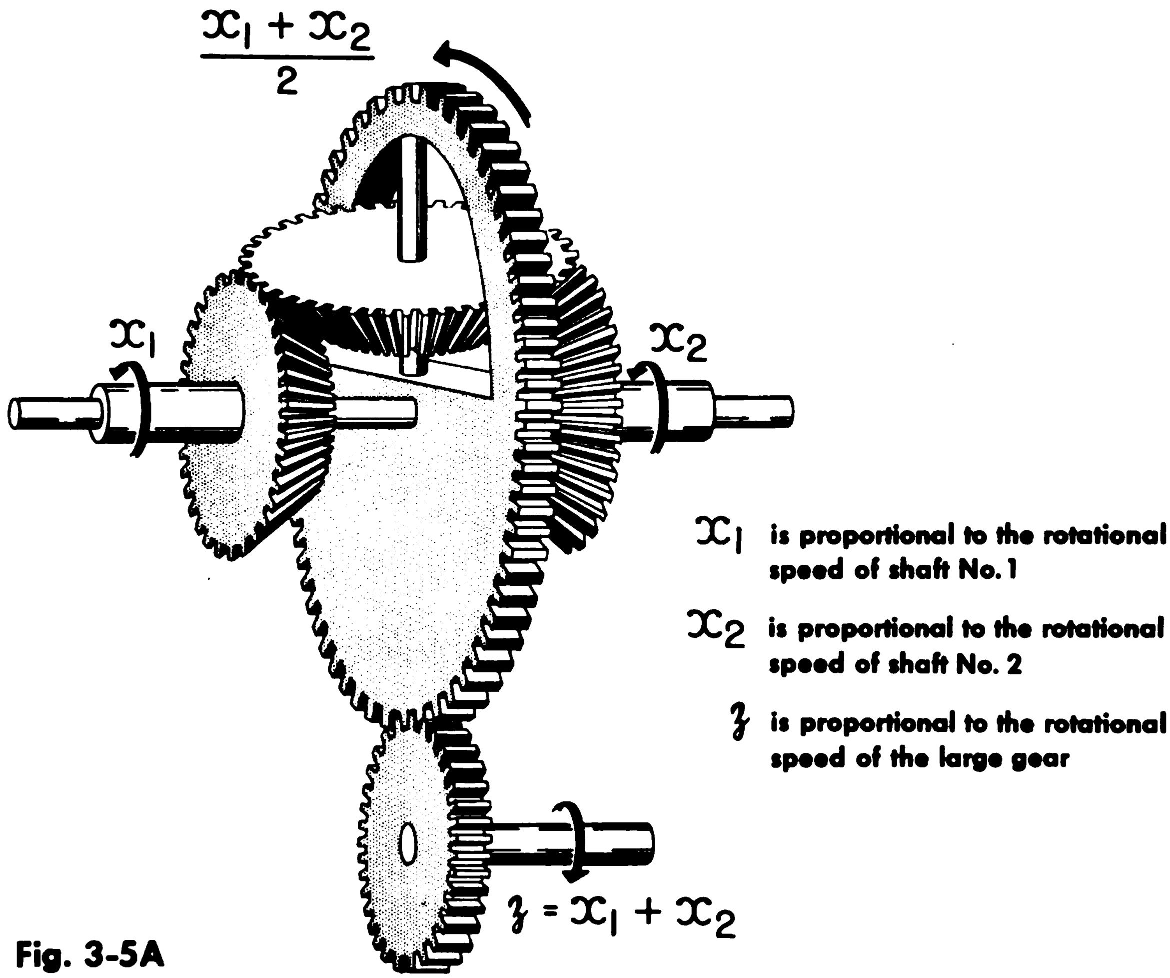


Fig. 3-5A

$P_z = P_1 - P_2 - P_3$ and the analog variables which are proportional to the actual pressures are related by $z = x_1 - x_2 - x_3$ (Fig. 3-4B).

Mechanical Adder with Rotating Shafts

The common differential gear assembly (Fig. 3-5A) performs the operation of addition (of shaft speeds).

x_1 is proportional to the rotational speed of shaft No. 1.
x_2 is proportional to the rotational speed of shaft No. 2.
z is proportional to the rotational speed of the large gear.

Subtraction is easily accomplished by reversing the direction of x_1 or x_2 with a pair of gears.

AN ELECTRIC CURRENT ADDER

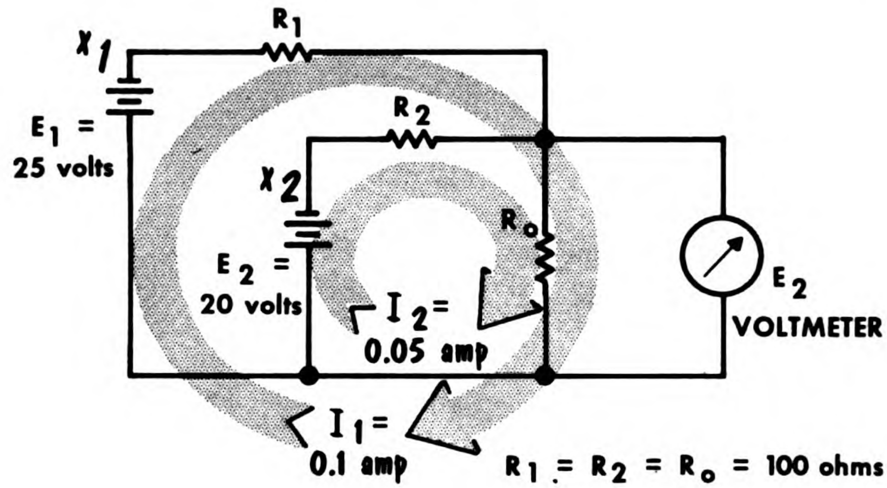


Fig. 3-5B

Suppose now that the variables are d-c voltages (Fig. 3-5B). The battery voltage x_1 causes 0.1 ampere to flow through R_1. The other battery voltage, x_2, causes 0.05 ampere to flow through R_2. Thus the total current in R_o is 0.15 ampere and the voltage drop across it (E_z represented by z) is 15 volts.

$$E_z = \frac{R_o}{3}\left(\frac{E_1}{R} + \frac{E_2}{R}\right)$$

$$E_z = \frac{25 + 20}{2} \text{ volts}$$

$$z = \frac{x_1 + x_2}{3}$$

The factors of one-half and one-third that occur in some of these devices are easily compensated for by multiplying by a constant.

Multiplication by a Constant

The constant multiplier is usually the simplest building block.

- *Mechanical Devices:* The constant is determined by gear ratios or ratios of rod lengths (Fig. 3-6A through C).

A. x = shaft rotational speed or angular position

$$K = \frac{\text{number of teeth on gear No. 1}}{\text{number of teeth on gear No. 2}}$$

B. x = rod position

$$K = \frac{a}{b} \text{ for } K \text{ less than unity}$$

C. $K = \dfrac{a}{b}$ for K greater than unity

MULTIPLICATION BY MECHANICAL DEVICES

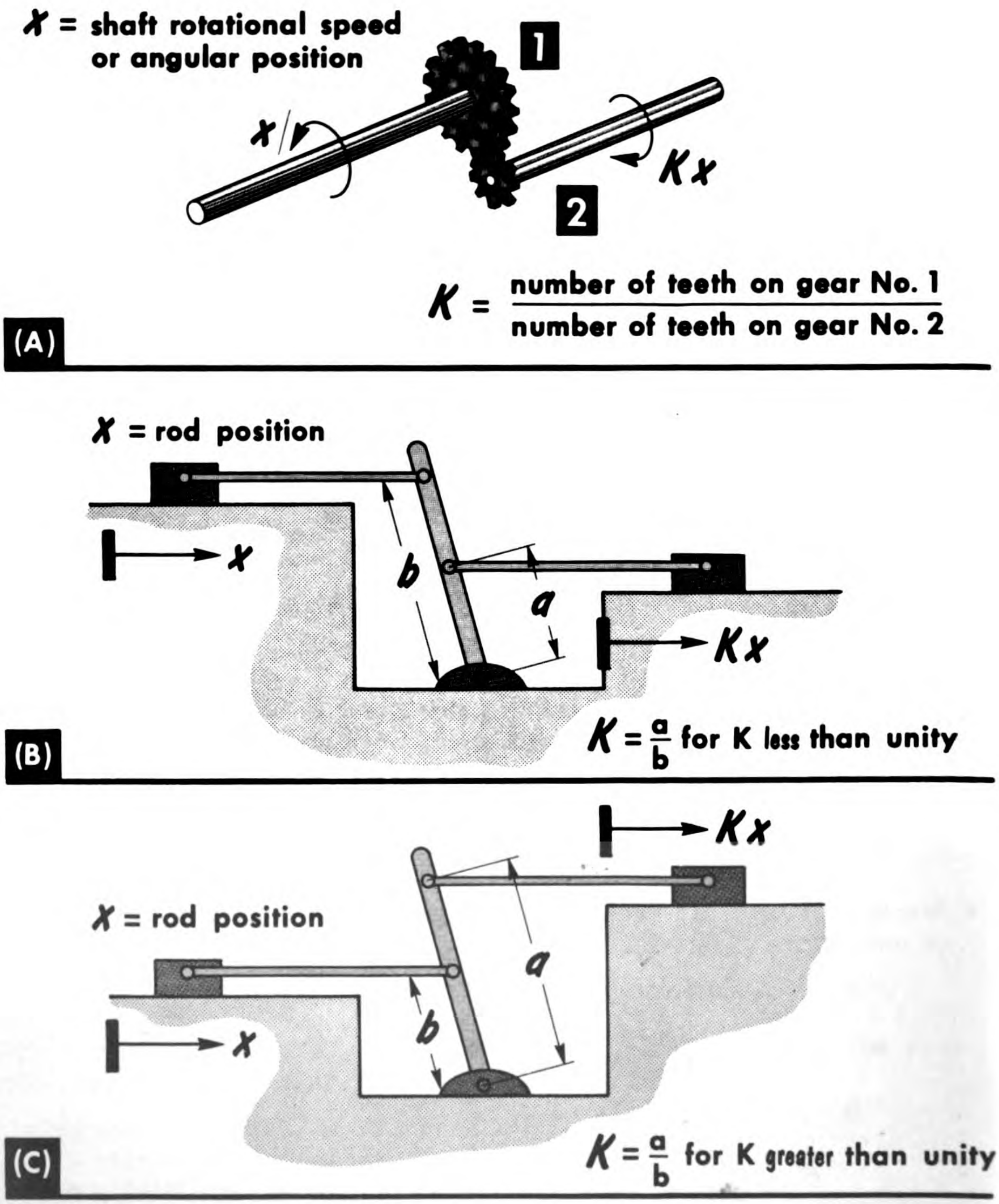


Fig. 3-6

- *Hydraulic Devices*: The constant is determined by the relative cross-sectional areas of two cylinders (Fig. 3-6D and E):

D. x = position of piston

$$K = \frac{a}{b}$$

E. x = hydraulic pressure

if pressure No. 1 = $P_1 = x$

then pressure No. 2 = $P_2 = Kx$,

where $K = \frac{a}{b}$

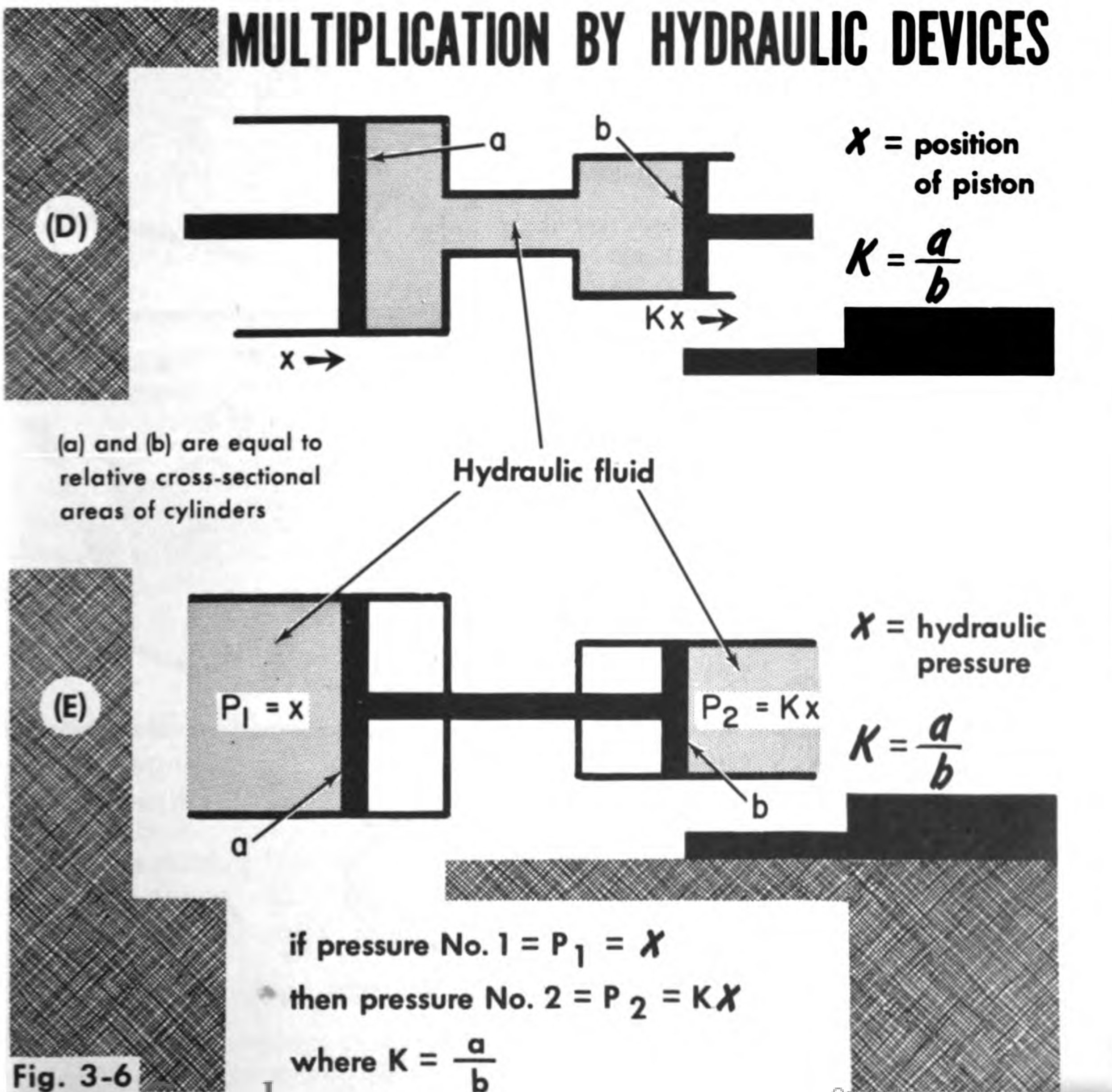


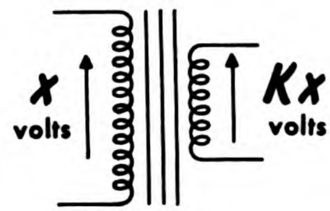
Fig. 3-6

MULTIPLICATION BY ELECTRICAL DEVICES

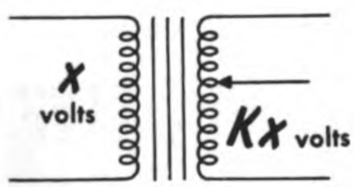
X = an a-c voltage

$$K = \frac{\text{number of turns on primary winding}}{\text{number of turns on secondary winding}}$$

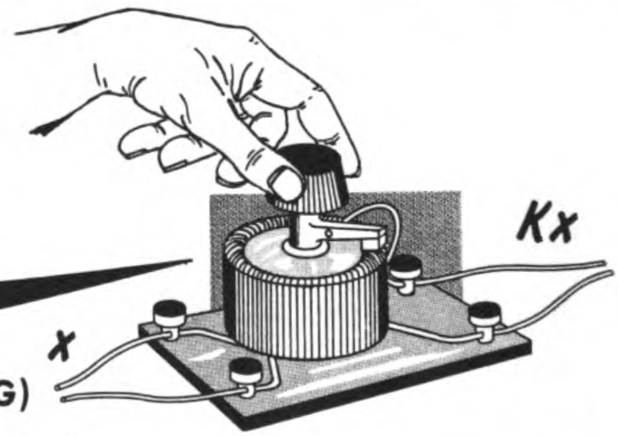
K is called the transformer *turns ratio*



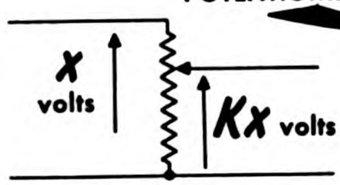
(F)



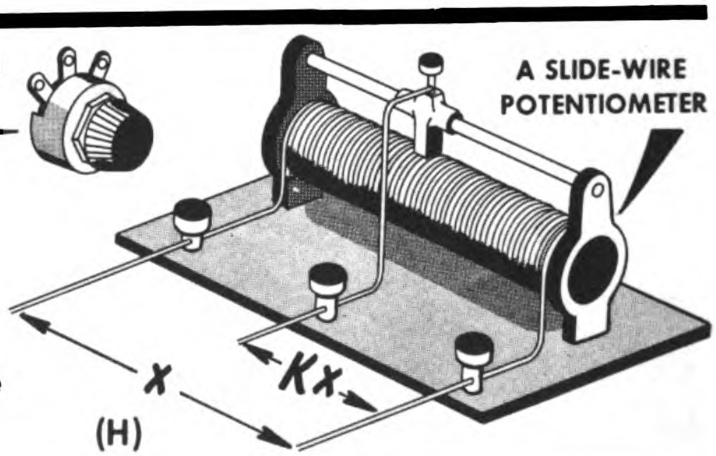
For an adjustable K use an adjustable transformer



(G)

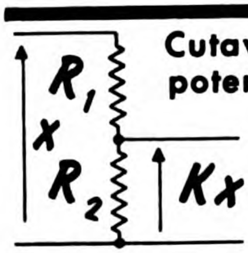


POTENTIOMETER

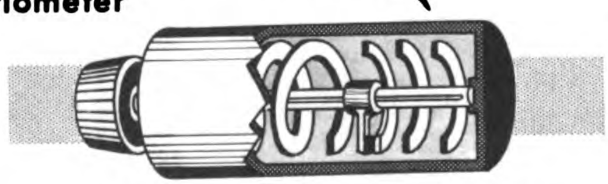


X = a d-c (or a-c) voltage

(H)



Cutaway view of a ten-turn potentiometer



Voltage divider for fixed K

$$K = \frac{R_2}{R_1 + R_2}$$

X = a d-c (or a-c) voltage

(I)

Fig. 3-6

- *Electrical Devices*: The constant is determined by transformer ratios or resistor ratios (Fig. 3-6F through I):

F. x = an a-c voltage

$$K = \frac{\text{number of turns on secondary winding}}{\text{number of turns on primary winding}}$$

K is called the transformer *turns ratio*

G. For an adjustable K, use an adjustable transformer

H. and I. x = a d-c (or a-c) voltage

K = a ratio of resistance values

MULTIPLIERS

Multiplication of a variable quantity by a constant number is the easiest operation we have to accomplish. Multiplication of a variable quantity by another variable quantity is often one of the most difficult operations to perform accurately.

Some multipliers are certainly simple in principle. Since multiplication by a constant is accomplished with a lever, gears, transformer, or a potentiometer, surely multiplication of two variables can be implemented with a variable lever, a variable gear ratio, a variable transformer turns ratio, or a variable potentiometer. Such multipliers are used and work well, although they are limited in operating speed because of the need for a mechanical system to bring about the *variation* of lever, gear ratio, turns ratio or potentiometer. That is to say, all mechanical systems are limited, by the inertia of the components, to following only slowly-varying computer variables.
A multiplier of two variables (Fig. 3-7) is a device that continually provides an output proportional to the instantaneous product of two input functions of time. For example:

$$e_3(t) = e_1(t) \times e_2(t)$$

where $e_1(t)$ and $e_2(t)$ are voltages that vary with time. Moreover, $e_3(t)$ is also a voltage function of time which at every instant of time is equal to the product of $e_1(t)$ and $e_2(t)$ *at that instant*. The t in parenthesis is shown merely to emphasize the time dependence of a variable quantity.

In an electronic computer both electromechanical and all-electronic multipliers are available. Let us look first at an interesting mechanical multiplier which has found considerable use in special purpose mechanical systems such as naval fire-control computers.

Mechanical Multipliers

This multiplier employs the principle of similar triangles to obtain a product. Different size triangles with identical angles are called *similar tri-*

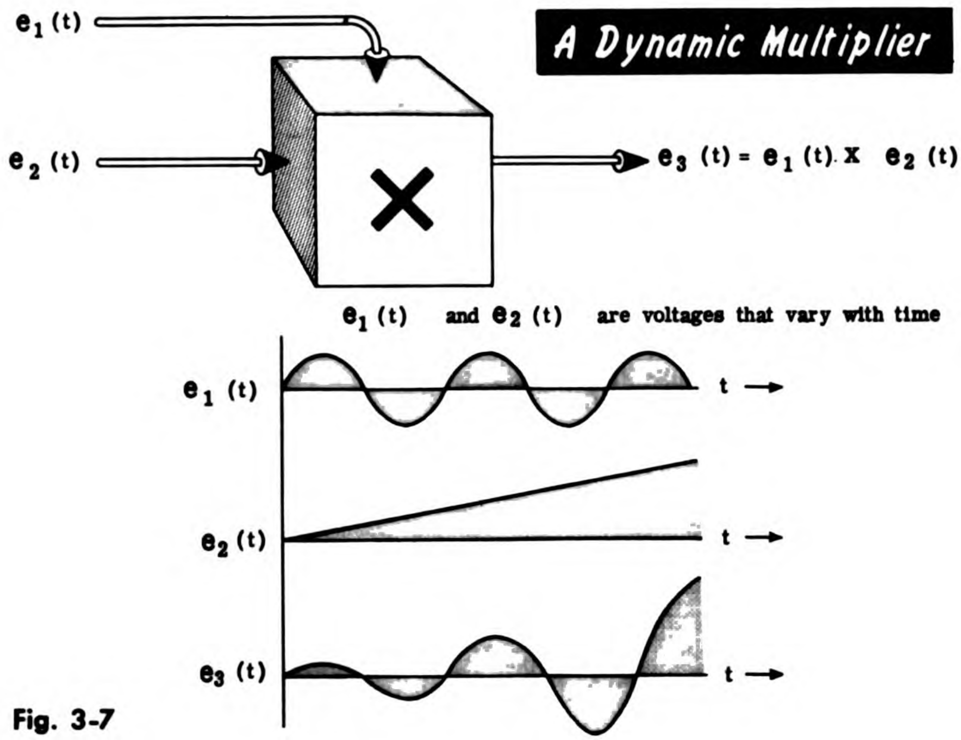


Fig. 3-7

angles. The *ratios* of corresponding sides of similar triangles are equal (Fig. 3-8A).

In the multiplier one side of each triangle is determined by an input variable, [$x_1(t)$ and $x_2(t)$]. The output [$z(t)$] is the vertical rod forming a

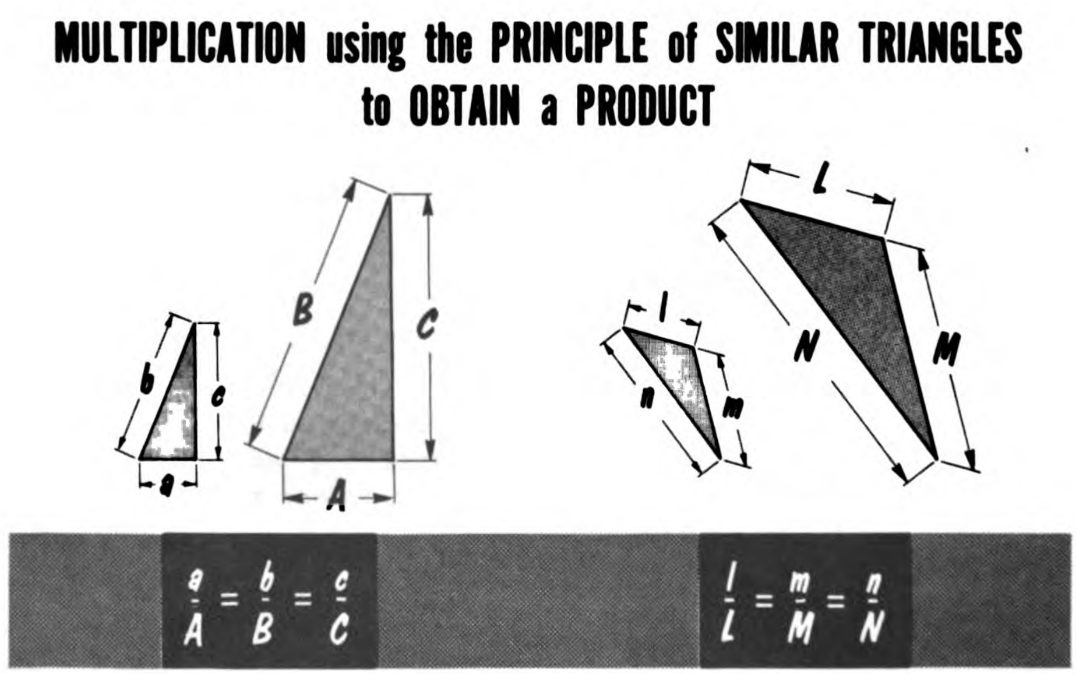


Fig. 3-8A

side of the upper triangle. Motion of this rod can be converted to shaft rotation by a rack and pinion. The hypotenuses of the triangles vary as the

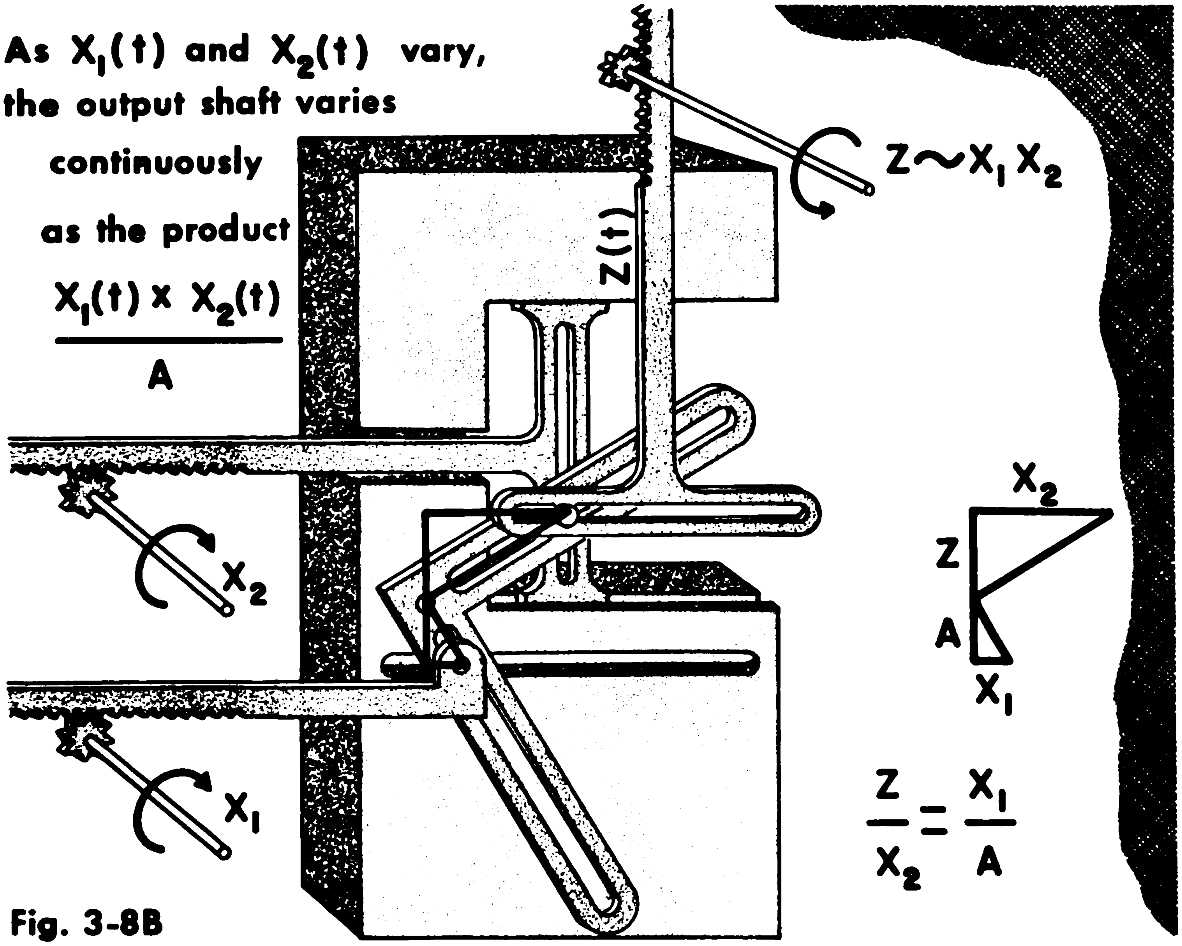


Fig. 3-8B

pins move back and forth in their slots, and the vertical side of the lower triangle is fixed (A). Thus we have the ratios:

$$\frac{z(t)}{x_2(t)} = \frac{x_1(t)}{A} \quad \text{or by multiplying by } x_1(t) \text{ we have}$$

$$z(t) = \frac{x_1(t) \times x_2(t)}{A} \quad \text{where } A \text{ is a constant of proportionality.}$$

Then as $x_1(t)$ and $x_2(t)$ vary, the output shaft $z(t)$ varies continuously as the product $\dfrac{x_1(t) \times x_2(t)}{A}$ (Fig. 3-8B).

Electromechanical Multiplying Building Blocks

It was seen before, that multiplication of a variable voltage by a constant could be accomplished with a simple voltage divider (provided the constant is a fraction less than unity) (Fig. 3-9A).

Multiplying by a CONSTANT

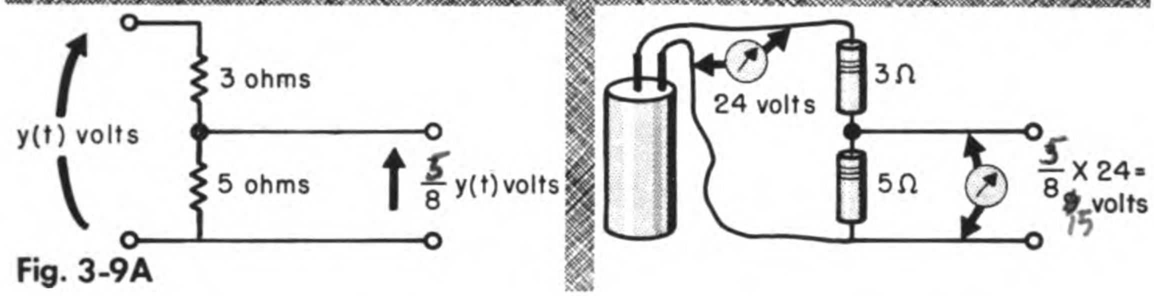


Fig. 3-9A

Multiplying by a VOLTAGE VARIABLE

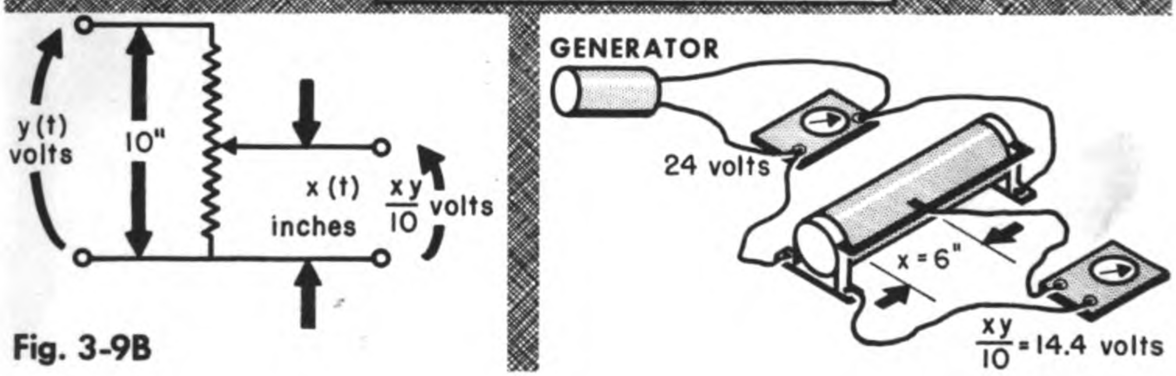


Fig. 3-9B

Multiplying by a MECHANICAL VARIABLE

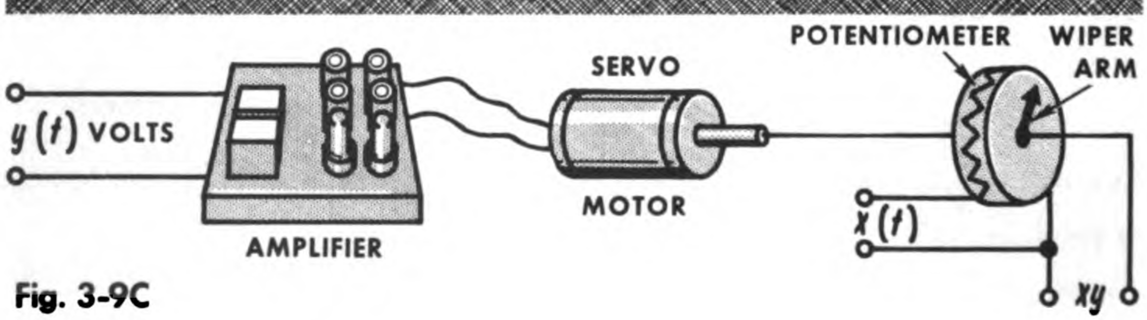


Fig. 3-9C

To multiply $y(t)$ volts by a variable, say $x(t)$, instead of the constant ratio, it is only necessary to construct a variable voltage divider such that the output tap is moved with the changes in $x(t)$ (Fig. 3-9B).

The wiper arm might be positioned by a mechanical variable moving as $x(t)$, or if $x(t)$ is a voltage variable a *servo amplifier and motor* are used to position the wiper arm (Fig. 3-9C).

All-Electronic Multipliers

When x and y are the same voltage variables, that is when it is desired to square the voltage variable, $x(t)$, two schemes are available which use no mechanical parts.

1. Certain diodes (a device which passes a current in one direction only) exhibit the property that the current through the device is proportional to the square of the applied voltage (Fig. 3-10A).
   This device is useful when the range of x is small and always positive, and when great accuracy is not required.
2. If a sequence of triangular-shaped pulses is fed into a device which terminates each pulse after a period of time proportional to the magni-

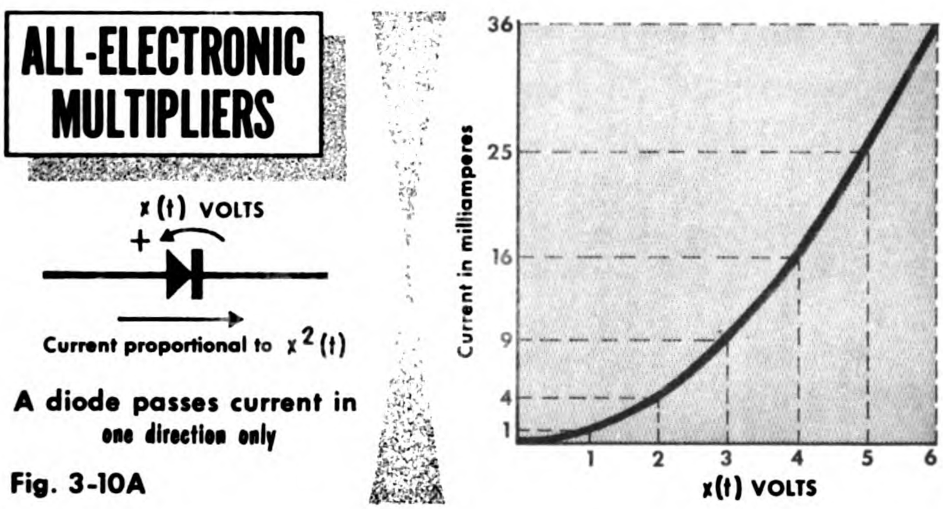


Fig. 3-10A

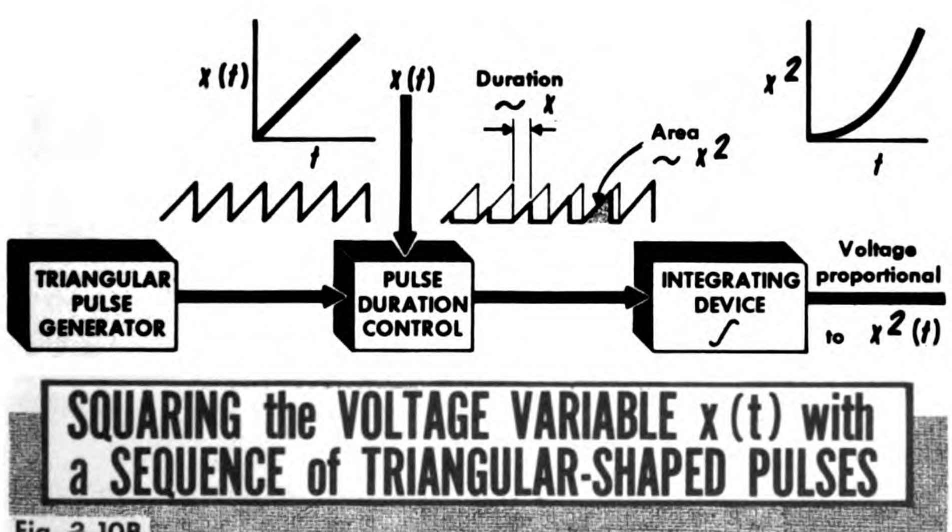


Fig. 3-10B

tude of the voltage variable $x(t)$, the result is a sequence of triangular pulses each having an area proportional to the square of $x(t)$ at a particular instant. This sequence of pulses is sent to a circuit which continually measures the area of the pulses (integrates) and has a voltage

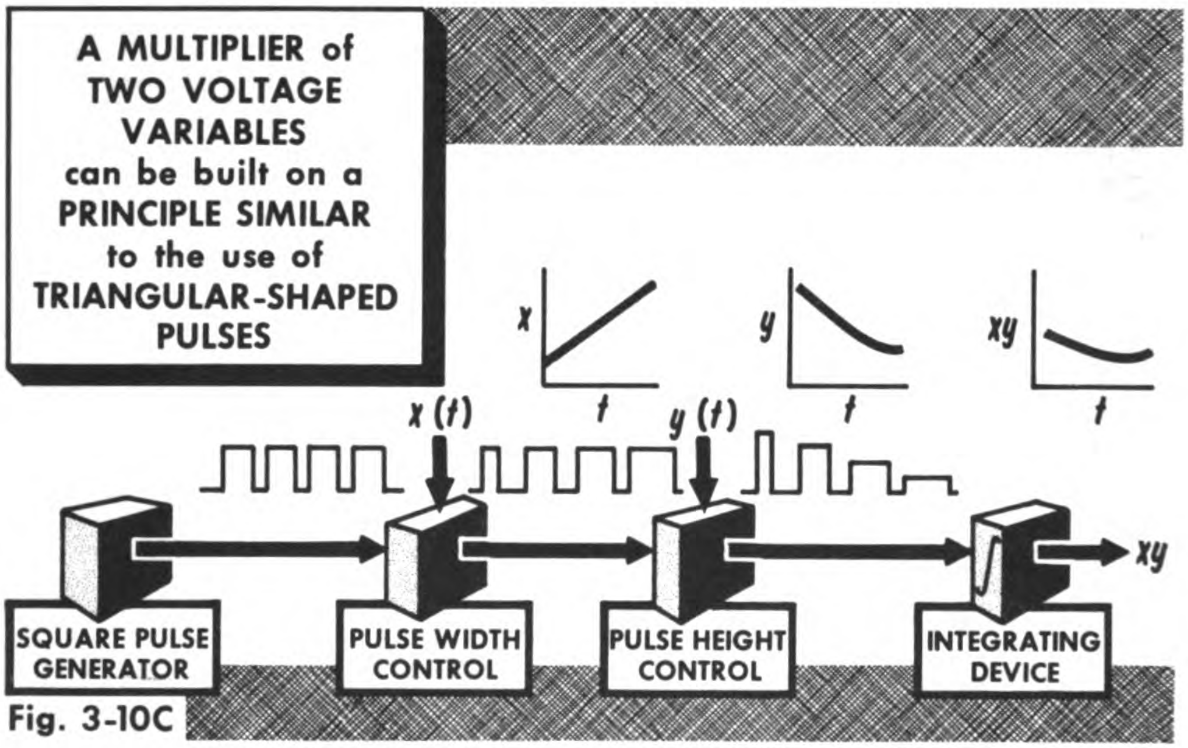


Fig. 3-10C

output proportional to the area or to $x^2(t)$. If the number of pulses per second is large compared with the variations of $x(t)$, the ripple in the output due to the pulses is negligible (Fig. 3-10B).

A multiplier of two voltage variables, $x(t)$ and $y(t)$, can be built on a similar principle. That is, the $x(t)$ voltage controls the height of a train of square pulses while the $y(t)$ voltage determines the width of the pulses — the area and thus the integral of the pulses is proportional to the product $x(t)y(t)$ (Fig. 3-10C).

Dynamometer or Wattmeter Multiplier

The instantaneous power flowing at a point in an electrical circuit, $P(t)$, is the product of the current, $i(t)$, and the voltage, $e(t)$, at that point. $P(t)$ is a function of time which is the product of two other functions of time. Thus any device capable of measuring $P(t)$ may be classified as a multiplier. Clearly any wattmeter falls in this category. To understand how a typical wattmeter works, consider the dynamometer (Fig. 3-11A) which is used as a wattmeter.

It must first be assumed that the high resistance of the voltage coil and the lower resistance of the current coil are such that the presence of the

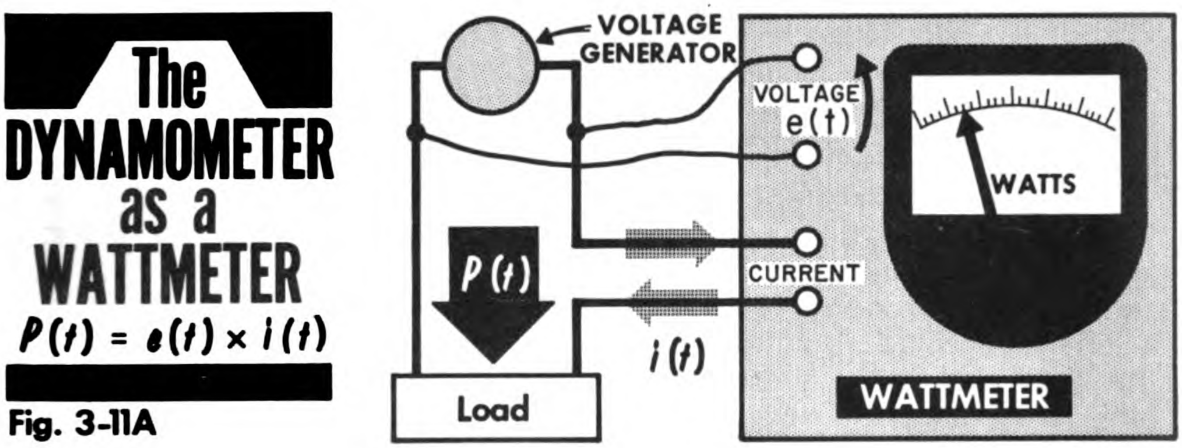


Fig. 3-11A

wattmeter does not affect the operation of the circuit being measured. The current in the voltage coil is proportional to the voltage, $e(t)$, in the circuit. The current in the current coil is the same current flowing in the circuit. A torque is developed between the two coils proportional to the product of the magnetic fluxes in the coils. But the flux in each is proportional to the current in each coil. Thus the torque varies as the product of $e(t)$ and $i(t)$ (Fig. 3-11B).

The torque acts against the restoring force of the spring connected to the needle on the moving coil. If the coil currents are constant then the position of the needle is constant. By proper calibration of the meter scale to account for the several constants of proportionality, the needle can be made to indicate the number of watts of power flowing in the circuit. If the

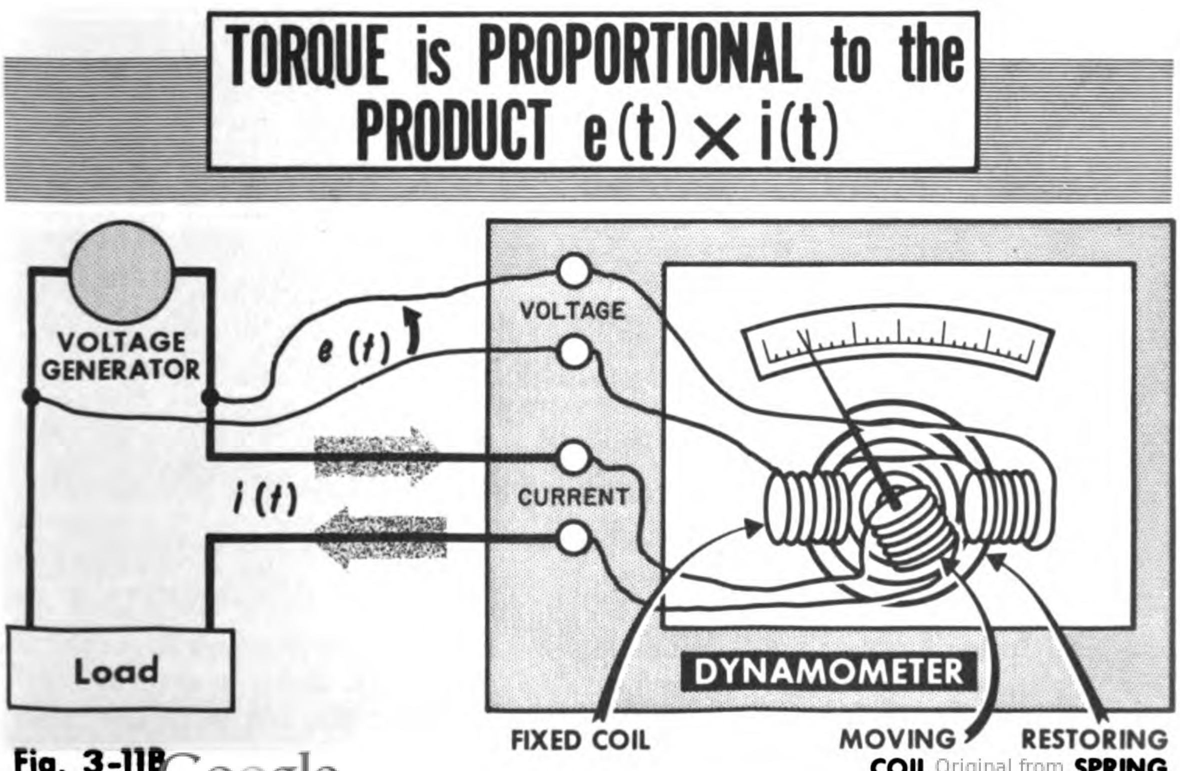


Fig. 3-11B

wattmeter is used on a 60-cps alternating current the inertia of the needle prevents it from following the a-c oscillations. The scale can be calibrated, however, to indicate the *average* a-c power which is a more useful quantity, anyway, than the instantaneous a-c power.

As a general purpose multiplier the dynamometer or wattmeter can be used to multiply two different voltage functions by supplying one voltage $e_a(t)$ to the top pair of terminals and a second voltage $e_b(t)$ to the other. The resultant currents will be proportional to the two voltages and thus the needle shaft rotation will be proportional to their product. This technique has been used in many special applications, though not often in general purpose analog computers.

The familiar watt-hour meters in our homes perform another operation in addition to multiplying $i(t) \times e(t)$. This unit is an *integrating multiplier* for it continually sums up the amount of power consumed. The integrating wattmeter was used in one of the first analog computers.

Division

Division is a mathematical operation which is the inverse of multiplication (Fig. 3-12A). This would suggest that the computer building block called

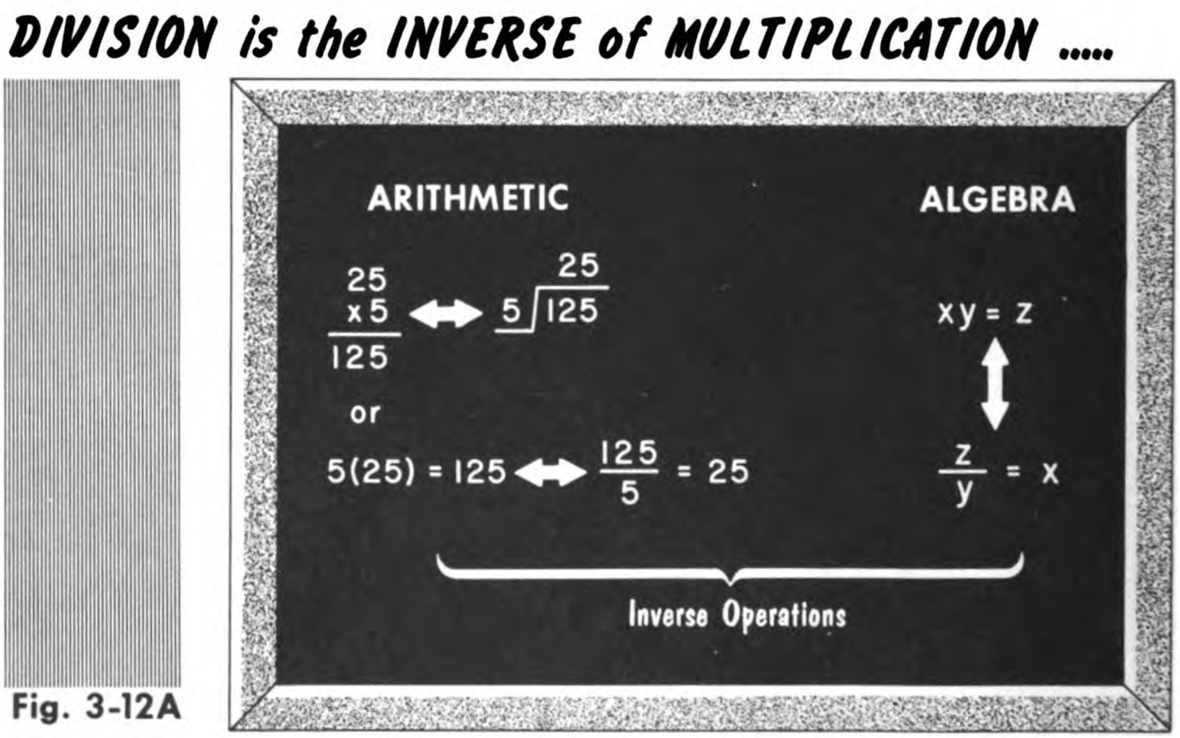


Fig. 3-12A

the multiplier might somehow be caused to operate *backwards* in order to accomplish this operation of *division* (Fig. 3-12B). Such a reversal will work with the mechanical multiplier (Fig. 3-12C), but not with the other multipliers described. We will see how such multipliers can be used in

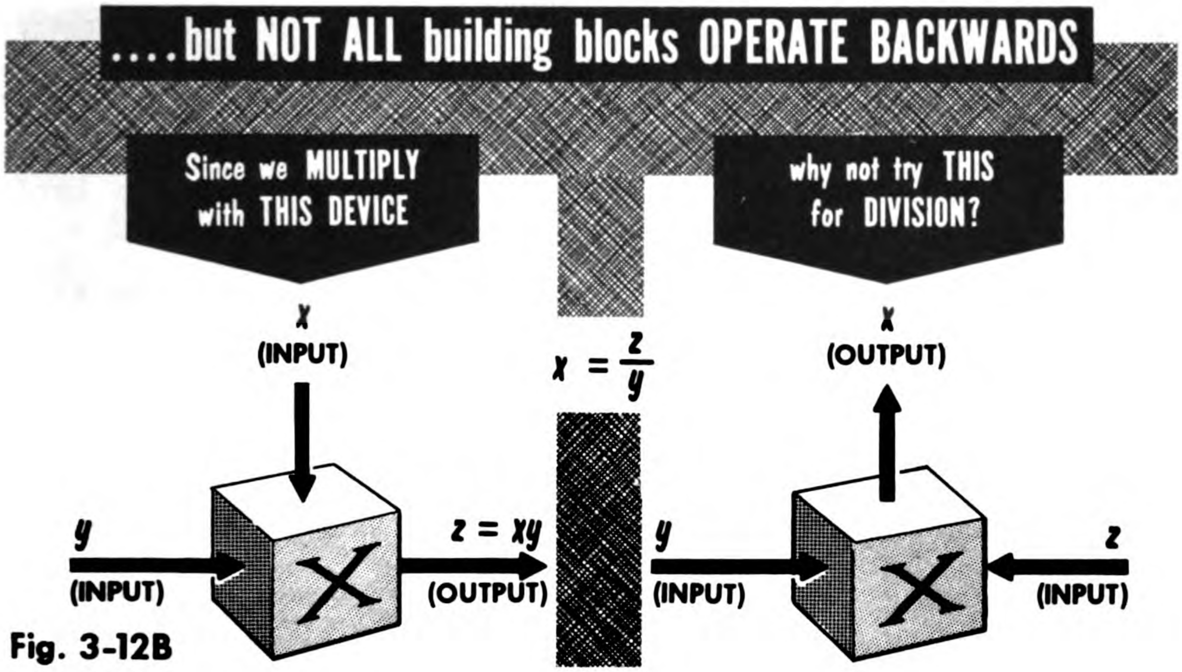


Fig. 3-12B

conjunction with a feedback amplifier to perform division. Suffice it to say here, that an implicit technique is used whereby a value of x is assumed which satisfies the equation $z = xy$. The error in the latter equality is

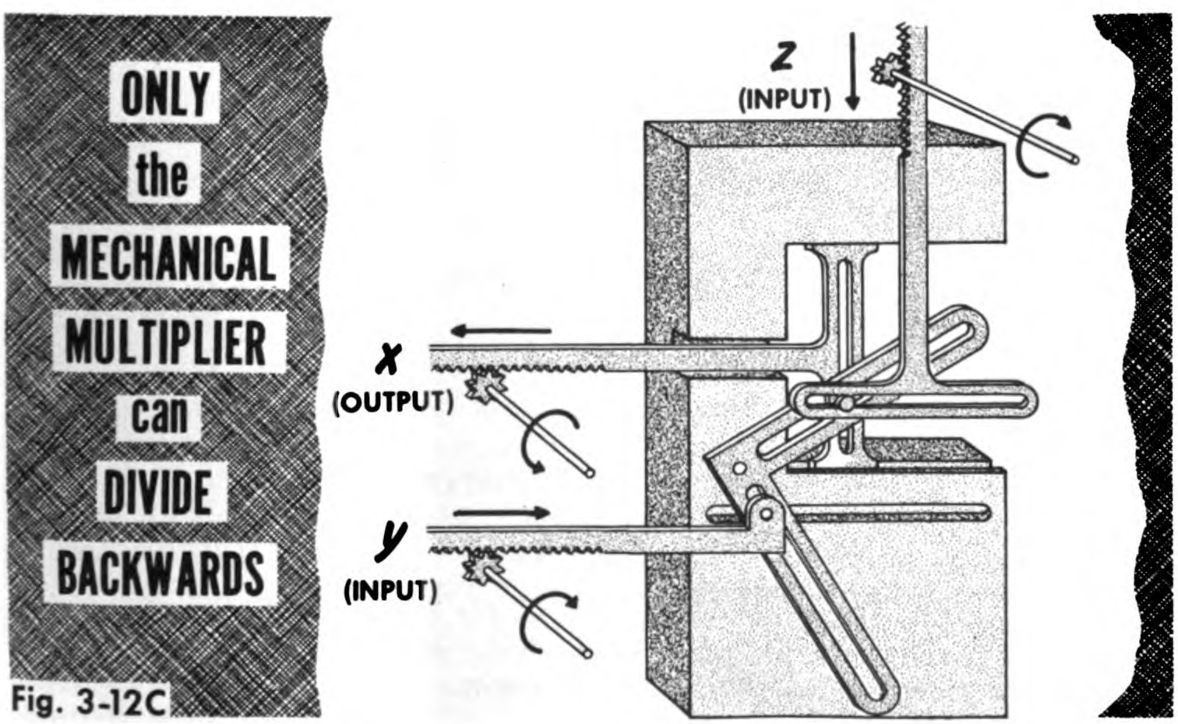


Fig. 3-12C

used to change the assumed value of x in a direction which reduces the error.

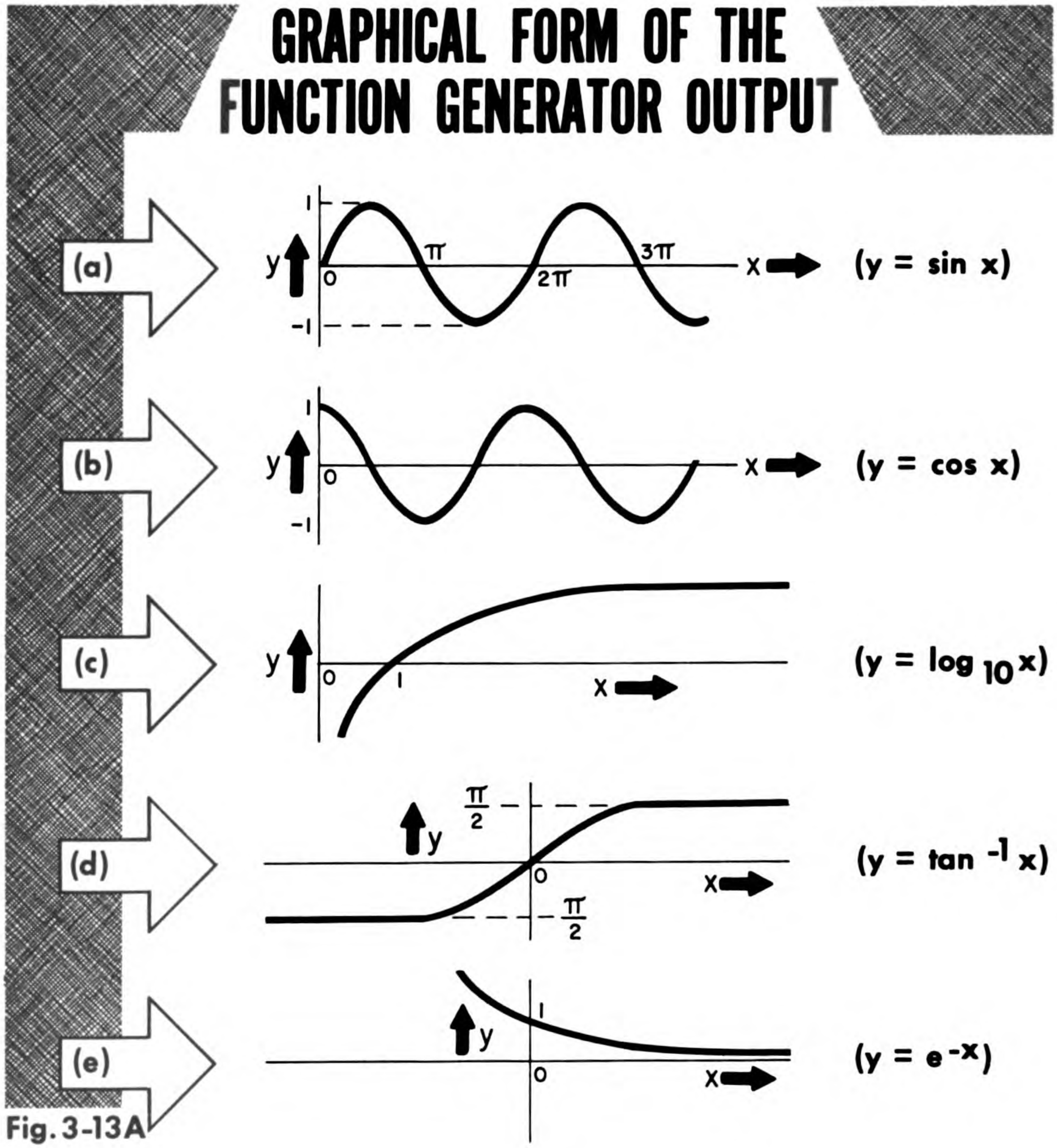


Fig. 3-13A

BUILDING BLOCKS: FUNCTION GENERATORS

Function Generation

An important category of building blocks for the general purpose analog computer is that of function generators. In the mathematical formulation of the dynamic behavior of physical systems, terms of the following form frequently appear in the equations. The graphical form of each of these functions is indicated in Fig. 3-13A.

(a) $y = \sin x$
($y =$ sine of x)

(b) $y = \cos x$
(y = cosine of x)

(c) $y = \log_{10} x$
(y = logarithm, to the base ten, of x)

(d) $y = \tan^{-1} x$ or arctan x
(y = the inverse tangent of x, that is, x = tangent of y)

(e) $y = e^{-x}$
(y = the number "e" raised to the minus x power; $e = 2.7183...$)

Each of the curves illustrated is described by a concise mathematical term (sin, cos, log, etc.). However, there often occurs a need for "arbitrary"

Graphical Forms of ARBITRARY Functions

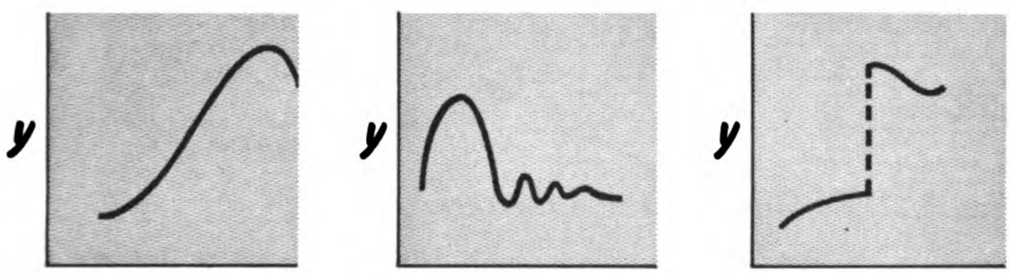


Fig. 3-13B x x x

functions, that is, functions which have no precise functional description but can readily be described graphically. Such functions usually result from experimentation with the physical system and represent the actual behavior of some part of the system. The curves shown in Fig. 3-13B are graphs of experimental data.

Function Generators

To solve serious engineering problems with an analog computer it is almost always necessary to generate certain functions. Thus we must have a computer building block that provides a computer-variable output proportional to some function, $f(x)$, when the input computer-variable is proportional to x. In some cases it is necessary to construct a building block that generates only one particular function, for ever and ever, whether an arbitrary function or one with a name. Such a function generator we call a *fixed function generator* (Fig. 3-14A).

On the other hand it is often possible to construct a device which can be adjusted to produce any function, or an approximation of any function, and can be changed from time to time by the operator. Thus we have the *variable function generator* (Fig. 3-14B).

Function generators of each type are illustrated on the following pages.

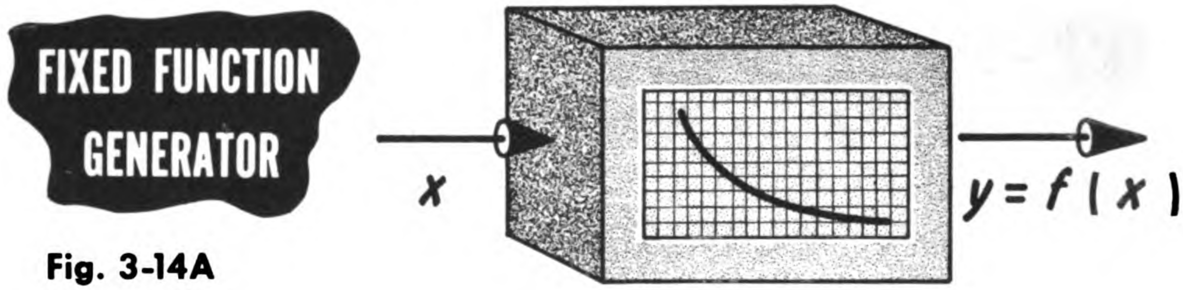


Fig. 3-14A

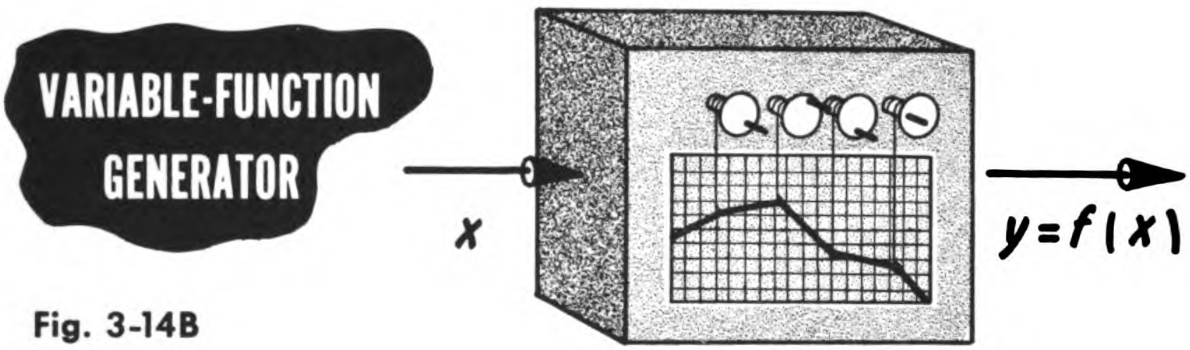


Fig. 3-14B

Mechanical Logarithmic Function Generators

In this device [Fig. 3-15 (A)] the input and output variables are the angular position of shafts. The input shaft position is proportional to the variable x. The position of the cam followers, and hence the output shaft position is proportional to the logarithm of x.

Other functions could be cut into a cam, but care must be taken to see that the cam follower will track smoothly without jamming, as might happen with a function with sharp discontinuities. An example is shown in Fig. 3-15 (B).

Two logarithmic generators [Fig. 3-15 (C)] can be used to form a multiplier. The vertical displacements of the cam followers are proportional to the logarithm of the input shaft positions, x and y. The vertical displacement of the pivot point is proportional to the sum $\log x + \log y$. The drum is an inverse logarithmic function generator. Hence the output shaft position is proportional to the product xy.

$$xy = \text{inverse logarithm of } (\log x + \log y)$$

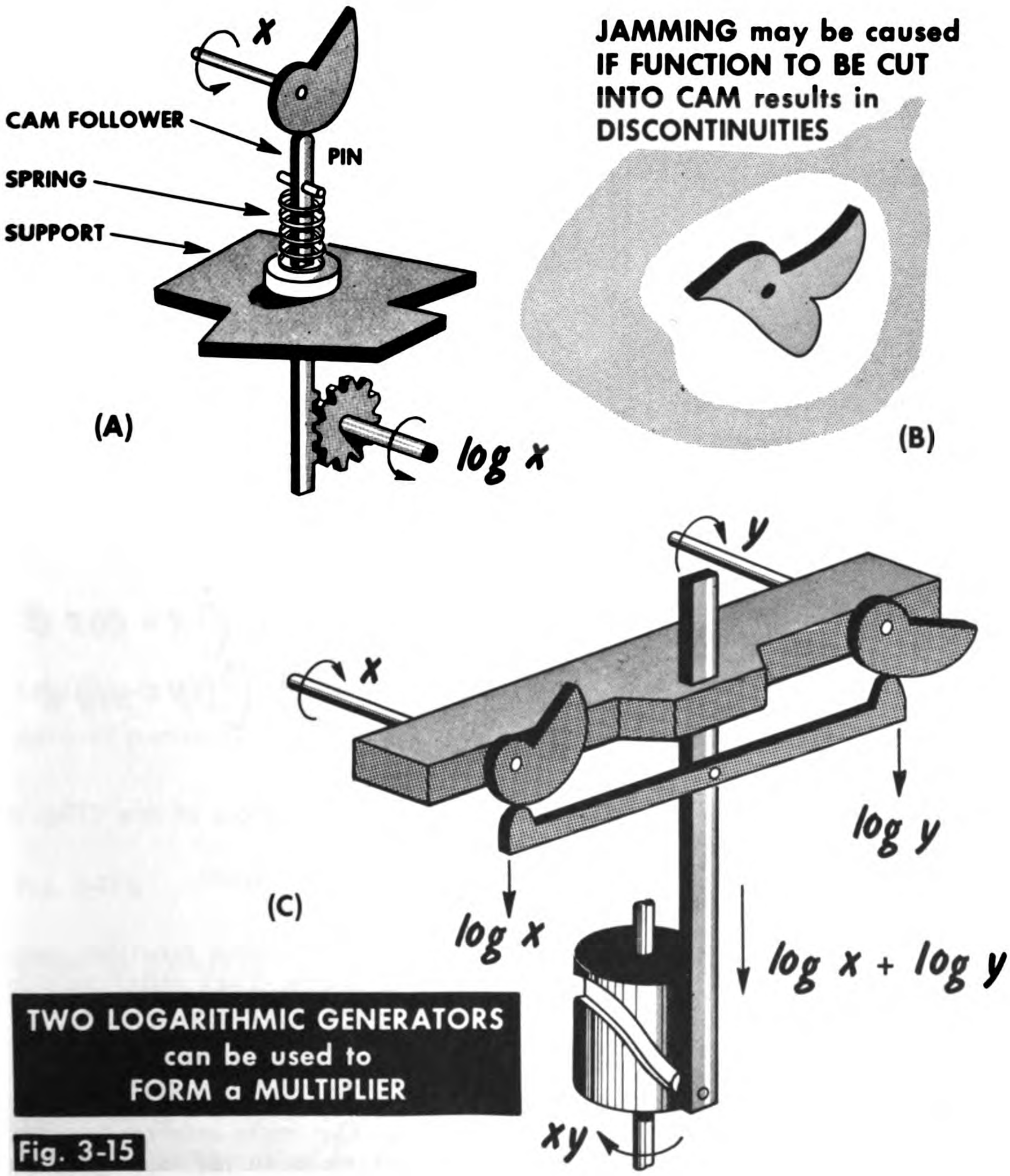
This operation is possible since the addition of the logarithms of two quantities is equivalent to the multiplication of the two quantities.

Trigonometric Function Generators: The Scotch Yoke

Here [Fig. 3-16 (A)] the input is a shaft rotation and the two outputs are rod or bar displacements (which can be easily converted to shaft rotations with rack and pinions). Let the wheel be turned, either by the handle or by the shaft whose angular position is proportional to the variable θ (theta),

MECHANICAL LOGARITHMIC FUNCTION GENERATORS

The INPUT and OUTPUT VARIABLES are the ANGULAR POSITION OF SHAFTS



TWO LOGARITHMIC GENERATORS can be used to FORM a MULTIPLIER

Fig. 3-15

then one scotch yoke is displaced horizontally a distance proportional to $r \cos \theta$ (let this be called x) and the other scotch yoke is displaced vertically a distance proportional to $r \sin \theta$ (call this y). Then if we choose

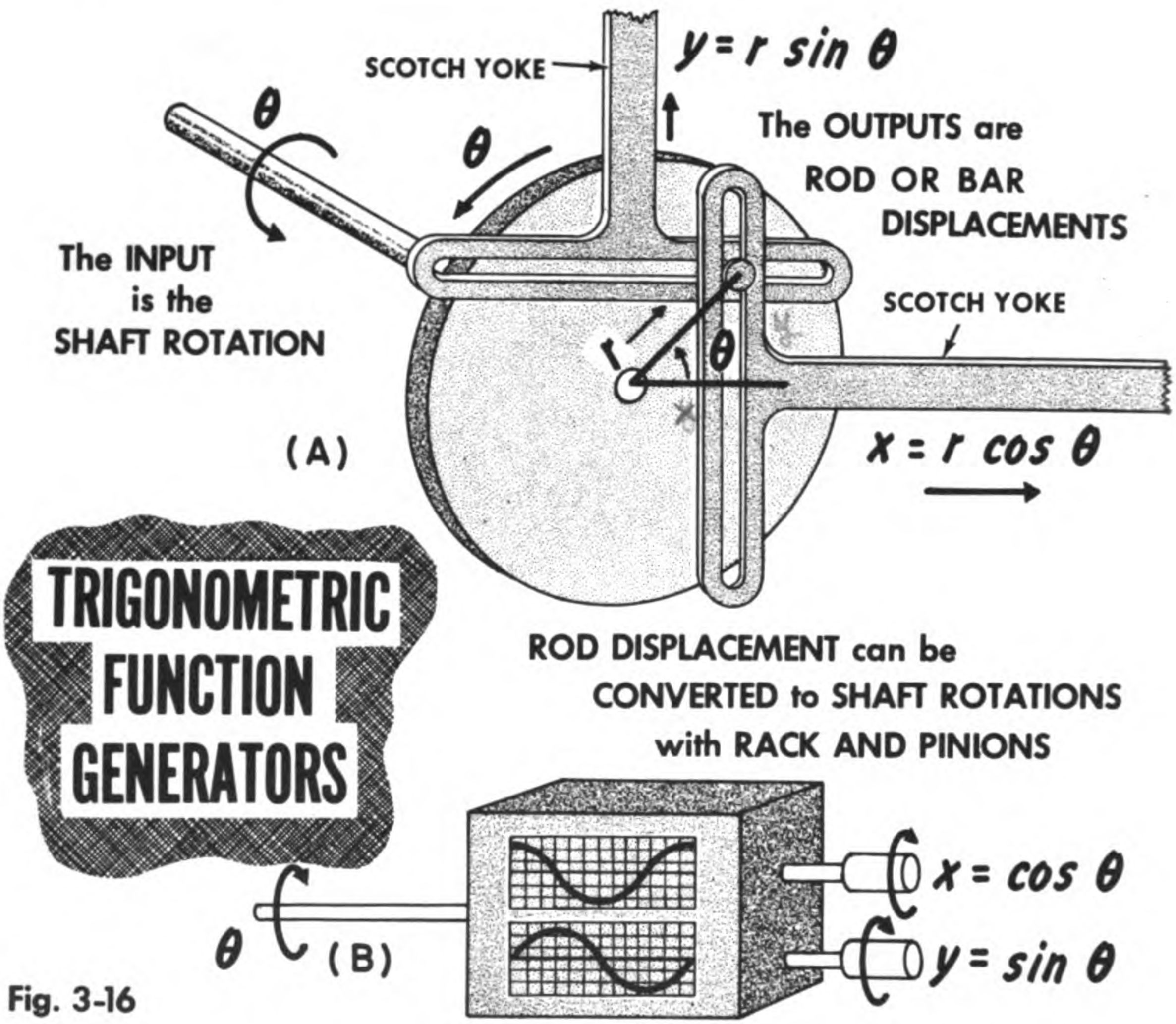
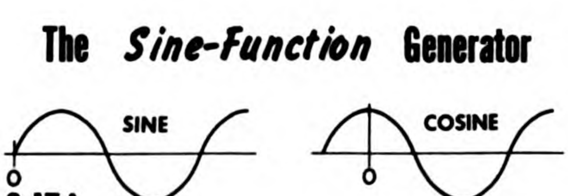


Fig. 3-16

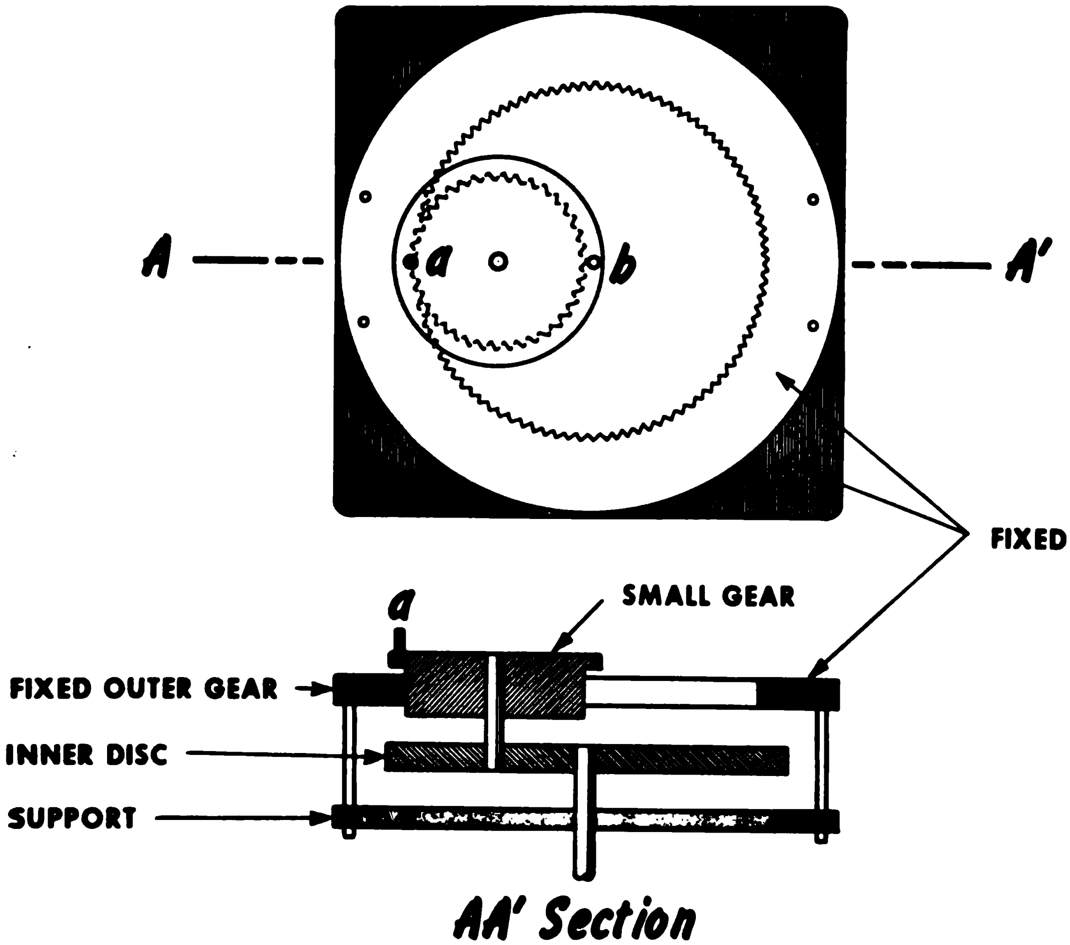
a scale such that $r = 1$, we have two function generators in one [Fig. 3-16 (B)].

Other Sine Generators



3-17A

The sine and cosine functions have identical forms. They differ only in which point on the abscissa is identified as the zero angle or argument (Fig. 3-17A). Any sine generator can also generate the cosine, by shifting the zero reference point. Consequently we will only consider sine generators. In the idler gear device (Fig. 3-17B) with the outer gear fixed and the inner disc center at B



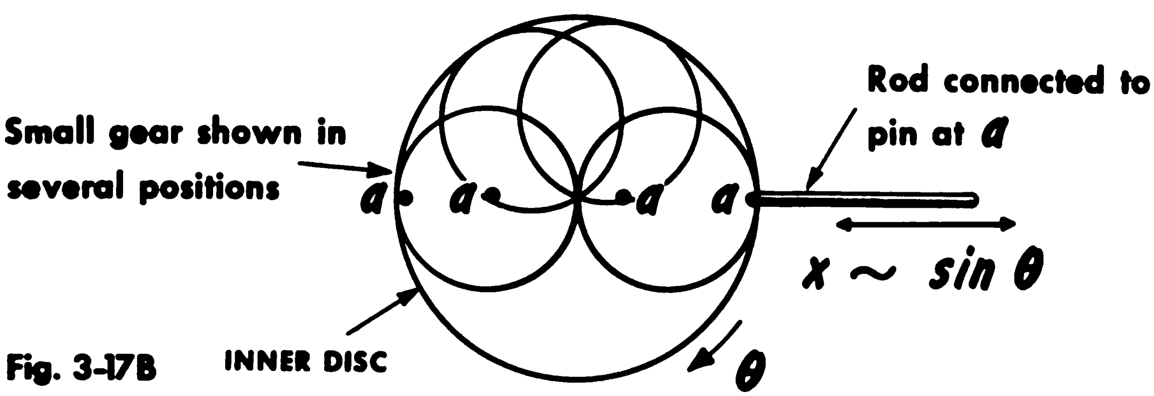


Fig. 3-17B

driven by a shaft position proportional to θ, the small gear (diameter one half the diameter of the inner disc), will roll around the other gear with the point a remaining on the horizontal line (A-A'). A rod connected to point a will be displaced a distance x, proportional to the sine of θ; $x = 2r \sin\theta$, where r is the radius of the smaller gear, and $x = 0$ when a is in the center.

An approximate method for sine generation is shown in Fig. 3-17C. If l is much larger than the radius of the wheel then x is proportional to $\sin\theta$.

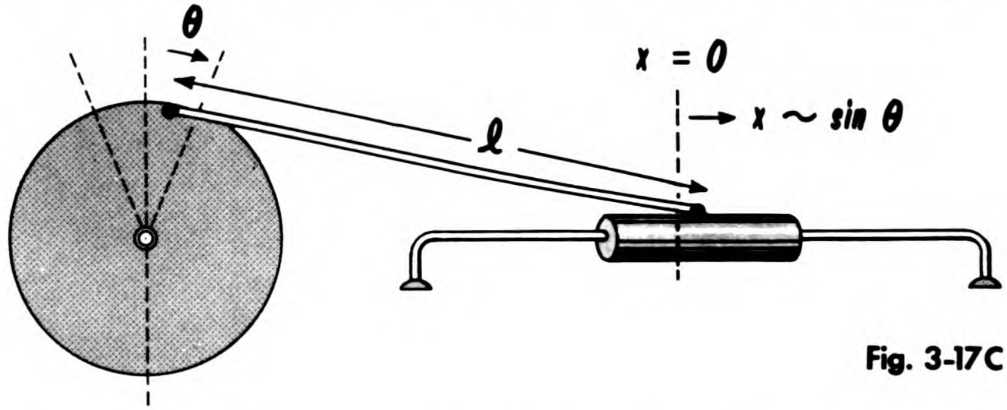
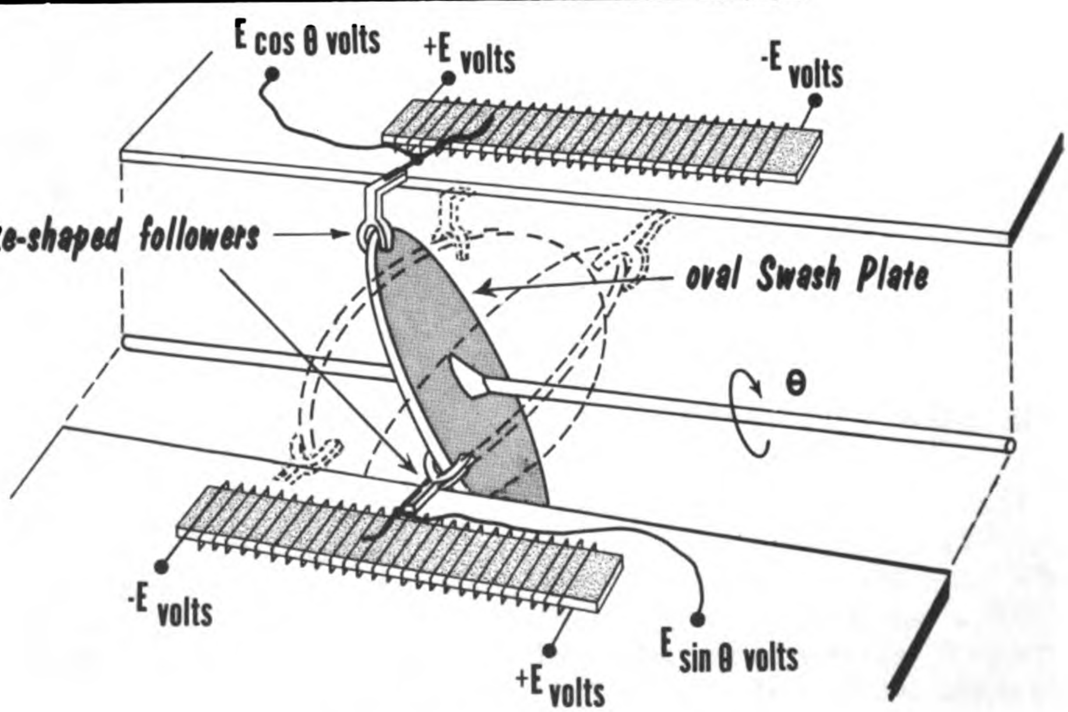


Fig. 3-17C

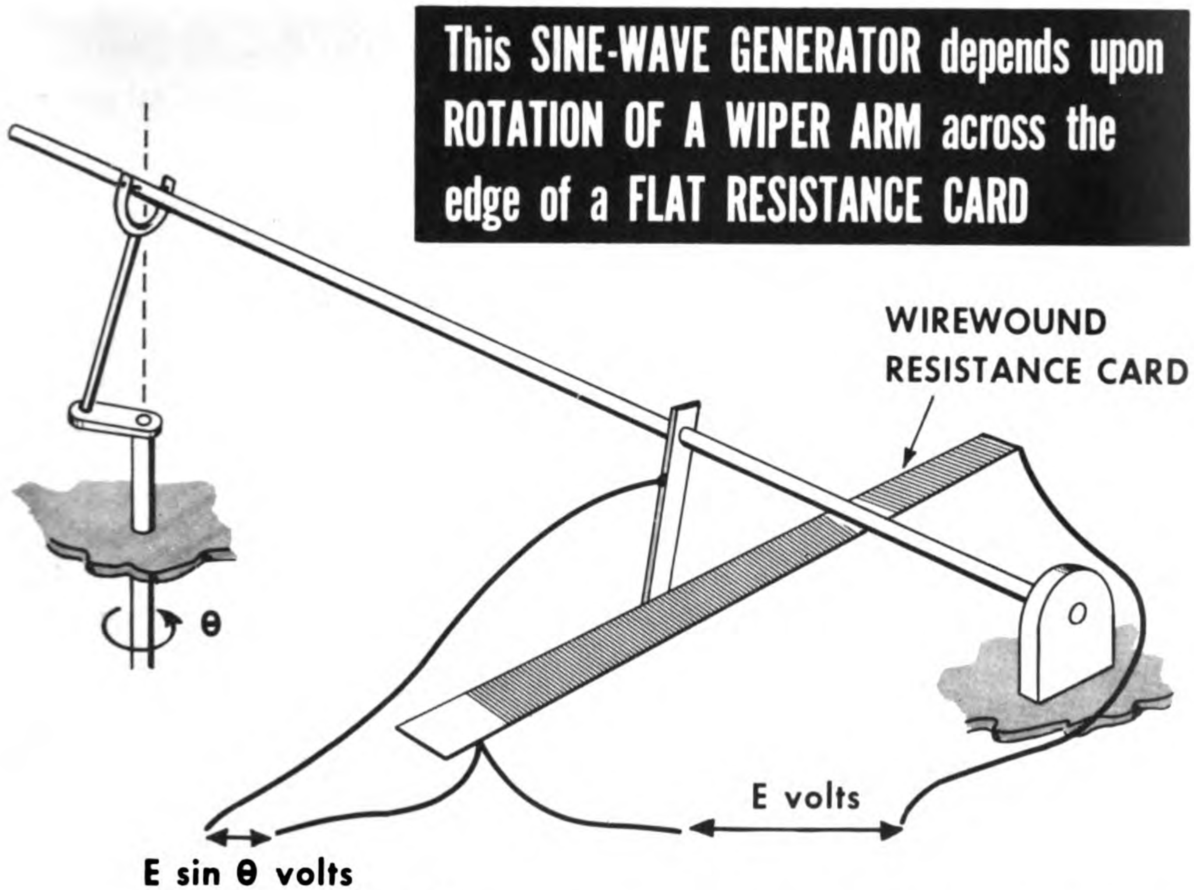
The "swash plate" (Fig. 3-18A) causes the yoke-shaped followers to move as the sine and cosine of the shaft position. The linear motion of the followers can be changed to angular motion by racks and pinions or to voltage variations by potentiometers as shown.

Another simple mechanical sine generator (Fig. 3-18B) depends upon rotation of a wiper arm across the edge of a flat resistance card. Due to the

The Swash-Plate Sine-Wave Generator



Rotation of oval plate causes
3-18A sinusoidal motion of follower yokes



The voltage on the wiper varies as the sine of the input shaft angle

Fig. 3-18B

eccentric fork and connecting link the long shaft rocks back and forth about its axis as the input shaft rotates. The voltage on the wiper varies as the sine of the input shaft angle.

Functions With Cams and Resistance Cards

It is apparent that cams can be used in several ways to obtain many functions (Fig. 3-19A). In a similar fashion, cards wound with resistance wire and provided with a movable connector or wiper arm, in place of a cam follower, can be used for arbitrary functions as well as functions "with a name". Both cams and resistance cards are fixed function generators. For example, consider the sine generator for each.

The logarithm cam was illustrated in Figs. 3-15B and C; the logarithmic resistance card is shown in Fig. 3-19B. The wiper arm is moved up and down with the variable x.

Arbitrary functions are illustrated in Fig. 3-19C.

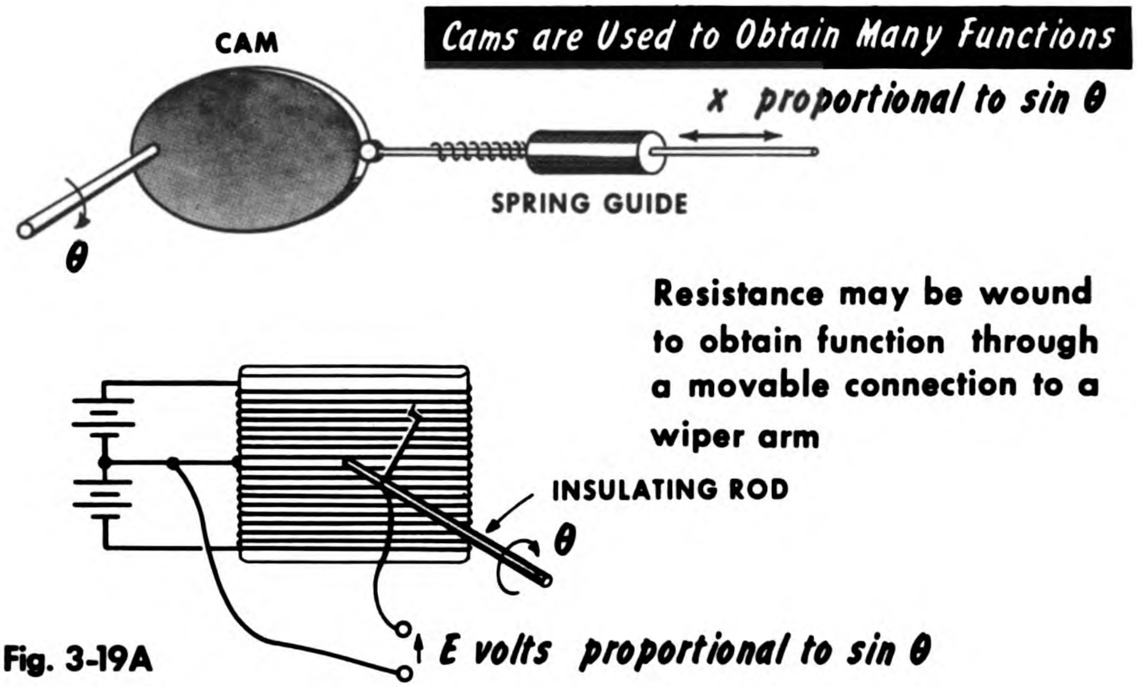


Fig. 3-19A

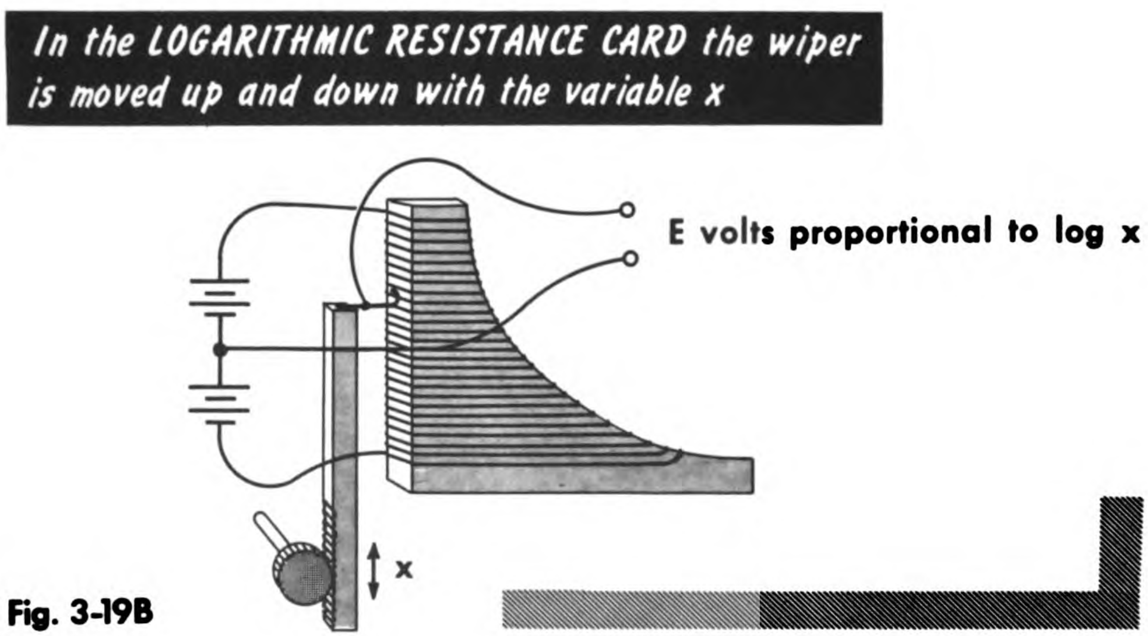


Fig. 3-19B

Trigonometric Resolver

Shaped resistance cards are often used to obtain high accuracy sine and cosine functions. The electromechanical resolver used in the d-c electronic analog computer employs shaped resistance cards and is described in detail later in this book.

When the sine and cosine functions are combined in one device with the additional feature of multiplying by another variable, the device is called

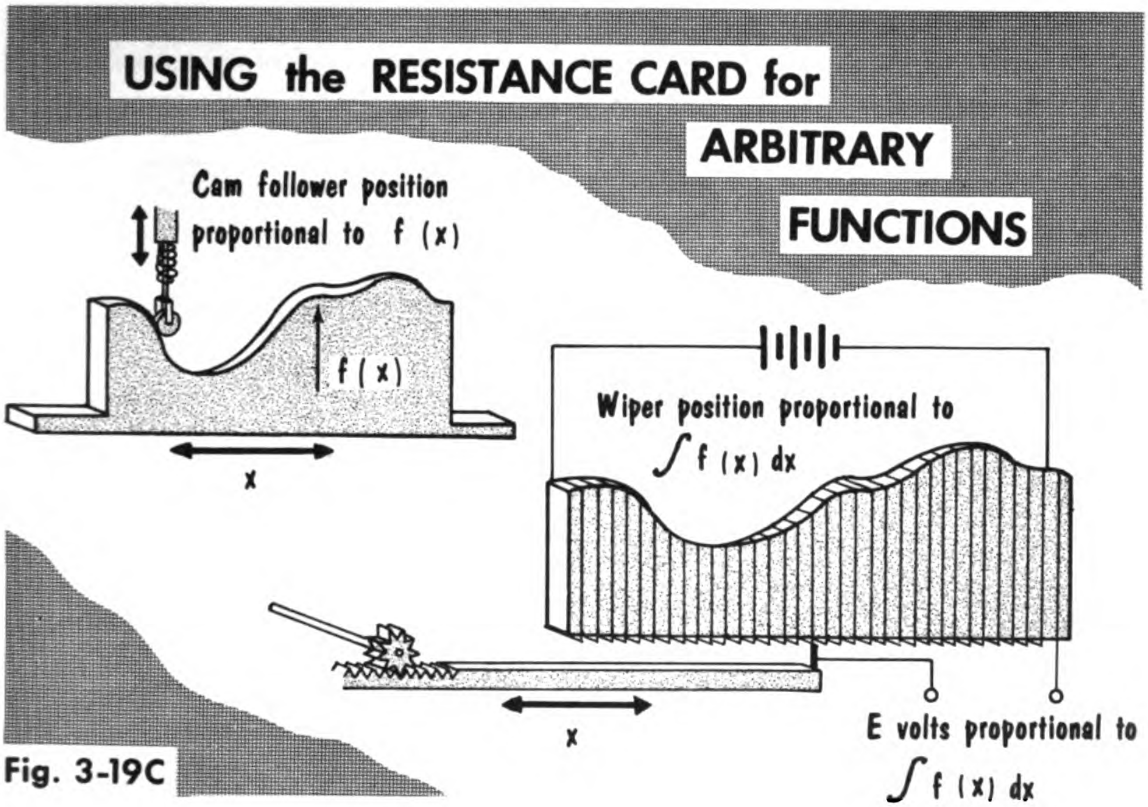


Fig. 3-19C

a *resolver* or *coordinate resolver*. The name derives from the operation of changing from one set of spatial coordinates to another.

To identify a point on a graph, with respect to some reference point called zero (Fig. 3-20A), two numbers are required: the distance to the right or

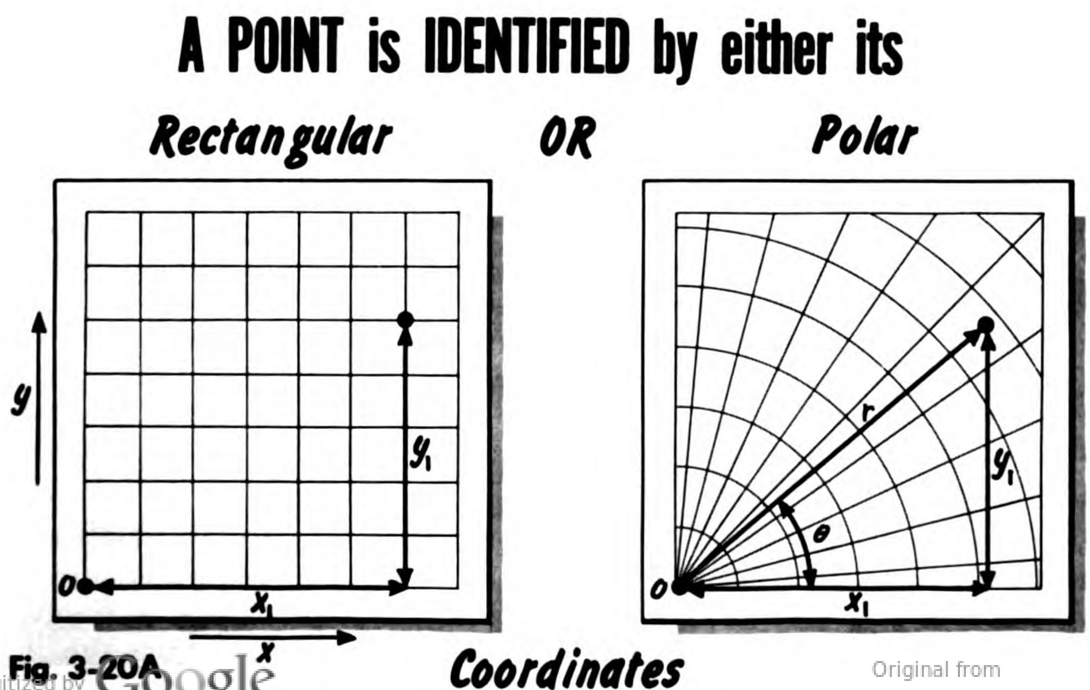


Fig. 3-20A

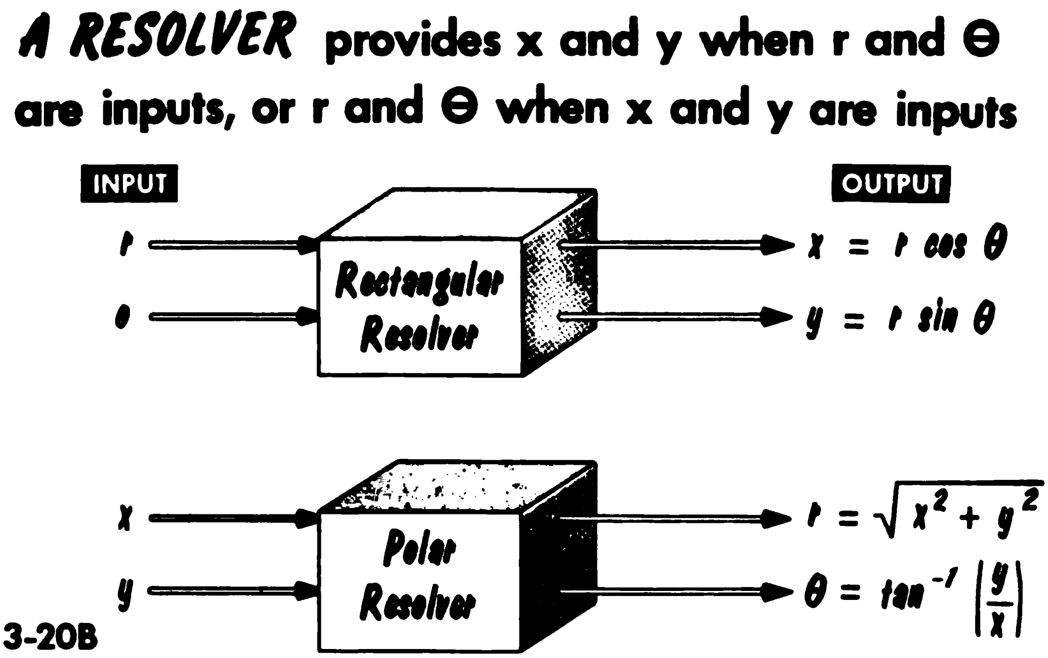


Fig. 3-20B

left of zero (x coordinate) and the distance above or below zero (y coordinate). These are known as *rectangular coordinates*. *Polar coordinates* could have been used by specifying the angle from the horizontal (θ coordinate) and the distance from the zero point (r coordinate).

Now the *resolution* of r, θ coordinates into x, y coordinates is accomplished by the following equations:

$$x = r \cos \theta$$

$$y = r \sin \theta$$

A resolver (Fig. 3-20B) is a building block which provides x and y when r and θ are inputs. Some resolvers can also provide r and θ when x and y are inputs.

Mechanical Function Generator for Arbitrary Functions (Manually Operated)

The desired function is drawn on a large piece of paper which is placed on the table of the device shown in Fig. 3-21A. The pointer is aligned with the starting point on the drawn curve, and when the computer is operated a change in the input shaft position (x) causes the pointer to move to the right or left. An operator is required to turn a hand crank which moves the pointer up and down across the table so that the pointer remains on the curve. Turning the crank also changes the output shaft position proportional to the function $y = f(x)$.

This device takes a different form when we let x be proportional to the total number of revolutions of the input shaft, or in other words, one revolution of the input shaft is an increment Δx in the variable x. A small displacement (rotation) is called a differential change in x; thus the input

MECHANICAL FUNCTION GENERATOR FOR ARBITRARY FUNCTIONS

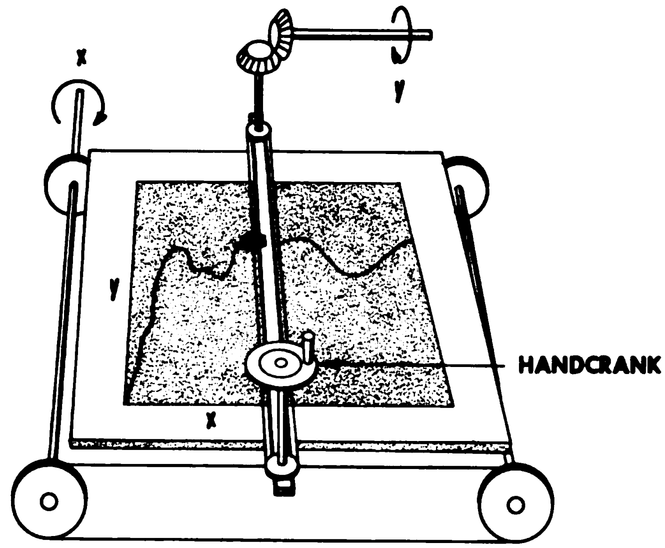
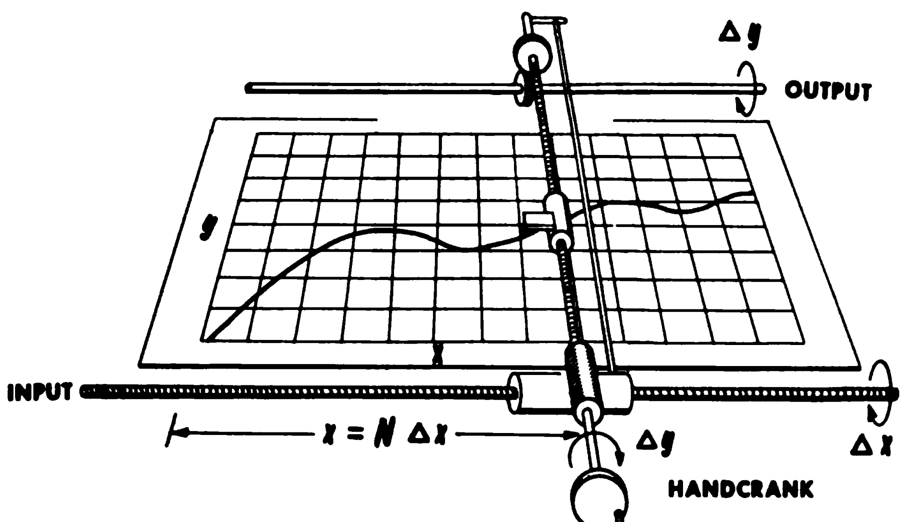


Fig. 3-21A

shaft motion is called "dx". Similarly the output shaft motion is a differential, dy. Turning the hand crank moves the pointer up and down by a worm gear or lead screw and turns the output shaft directly (Fig. 3-21B).

Provided the hand crank operation is quick enough the output shaft speed is proportional to the slope of the arbitrarily drawn curve. The quantity y



X is Proportional to the Number of Rotations of the Input Shaft

Fig. 3-21B

may be found by counting the number of Δy revolutions. This device is used in the Bush mechanical differential analyzer described later.

Electromechanical Device for Generating Arbitrary Functions (Automatic Curve Follower)

As in the previous example, a drawing of the desired function is placed on a table surface, over which a pointer rides. The pointer moves up and down on an arm, driven by a small motor (Fig. 3-22A). The arm moves right and left, driven by another small motor. The input variable is a voltage which is compared with the feedback voltage on the wiper arm of the resistance card. The latter voltage is proportional to the position of the arm. If the input voltage is greater than the feedback voltage, the *difference* between the two causes the arm drive motor to move the arm to the right until the

AUTOMATIC CURVE FOLLOWER

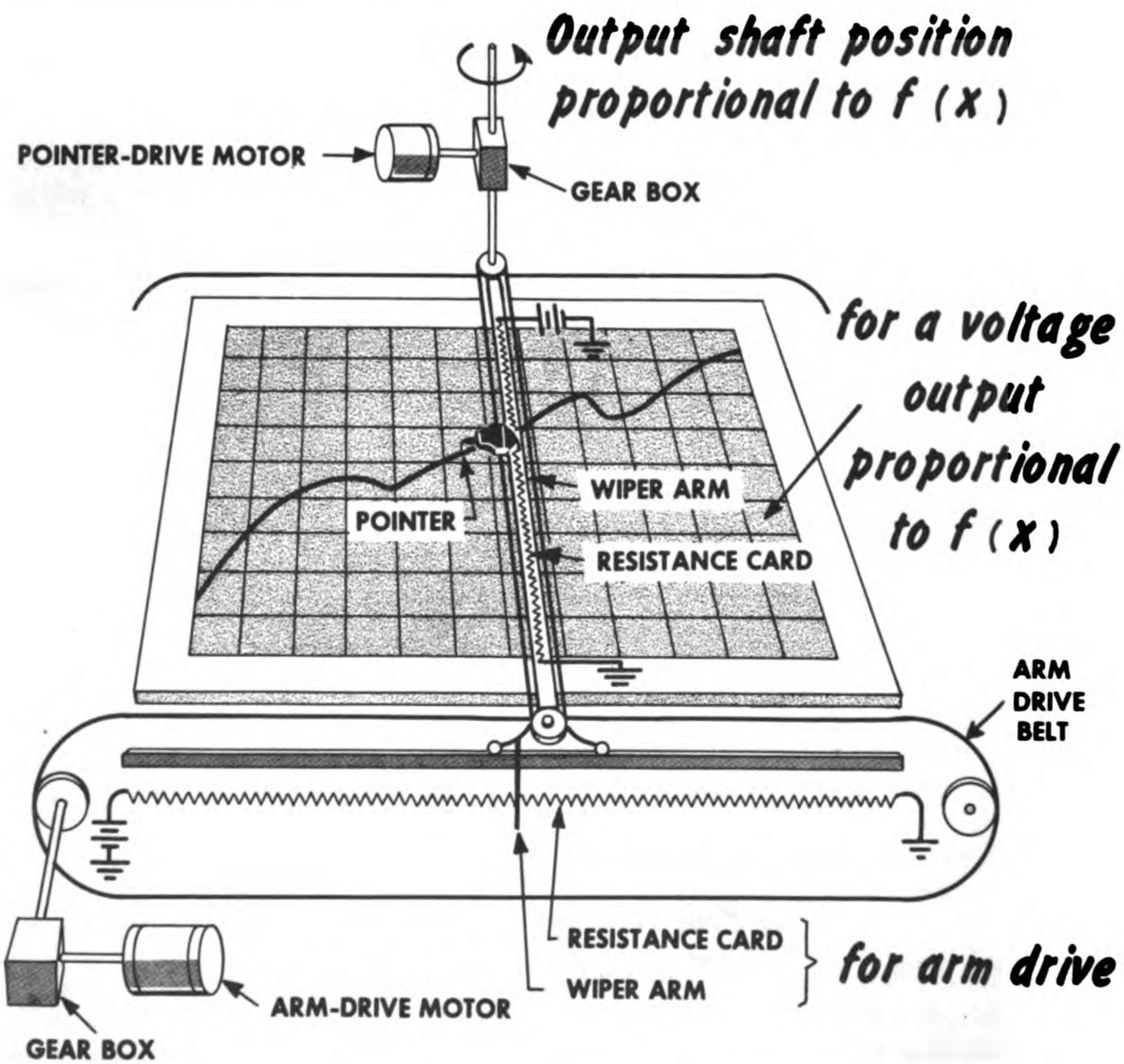


Fig. 3-22A

difference is zero. If the input voltage is less than the feedback, the arm moves to the left until the difference is zero. If the voltage at the end of the feedback resistance card is 100 volts, then the input voltage may vary

Detecting the Pointer Position with the Aid of Pickup Wipers

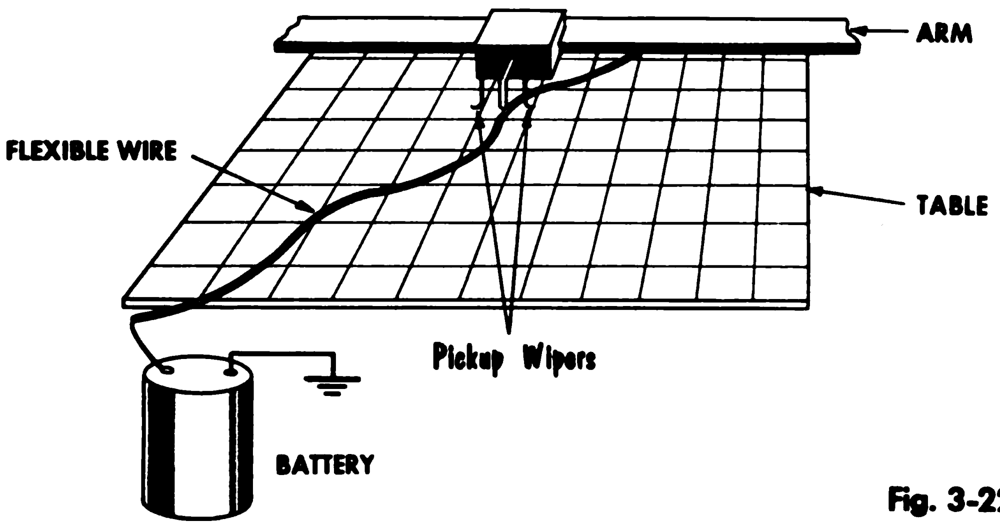


Fig. 3-22B

from zero to 100 volts and the position of the arm will always be proportional to the input variable. The arm drive system is termed a *servo-mechanism* or just a *servo.*

Now instead of relying on a human operator to position the pointer on the line, one of several possible methods is used to detect whether the pointer is above or below it. With this information the pointer drive motor can be connected to drive the pointer toward the line. Thus as the arm moves back and forth across the table, the pointer moves along the line — or at worst moves up and down across the line in very small steps, never departing from it by more than a small fraction of an inch (for example, 1/50 inch). The output shaft rotation is proportional to the vertical motion of the pointer.

One way of detecting the pointer position relative to the line is to secure a flexible copper wire over the line and energize the wire with a fixed voltage. Now if the pointer is equipped with two small wipers as shown (Fig. 3-22B), contact with one or the other will tell the motor which way to drive the pointer in order to get back onto the line.

An alternative method is to cover the table surface *above* the line with conducting metal foil energized with a fixed voltage (Fig. 3-22C) and then replace the pointer with a single pointer-wiper. When the wiper is in contact with the foil the motor drives the pointer down, when no contact is made the motor drives up. The pointer thus zigzags across the line. The stronger the motor the smaller the zigzags.

Energized Metal Foil and a Single Pointer-Wiper will also tell the motor which way to go

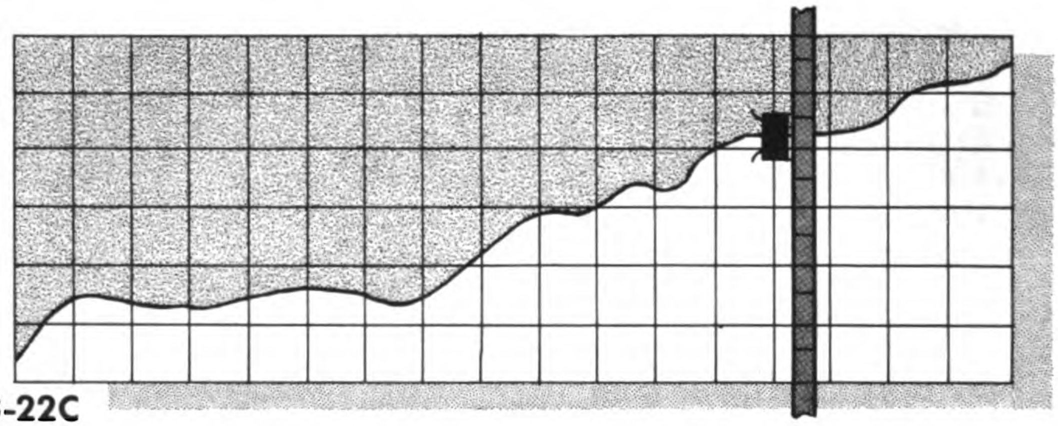


Fig. 3-22C

Another important electromechanical device for arbitrary function generation is known as the *pot padder*. The device is a servo-driven potentiometer with taps along the length of the resistance card to which fixed voltages are "padded". Detailed consideration is given to this unit in Volume 2.

The Photoformer

When a computer must work with and generate rapidly changing signals or computer variables, all mechanical devices become quite impractical.

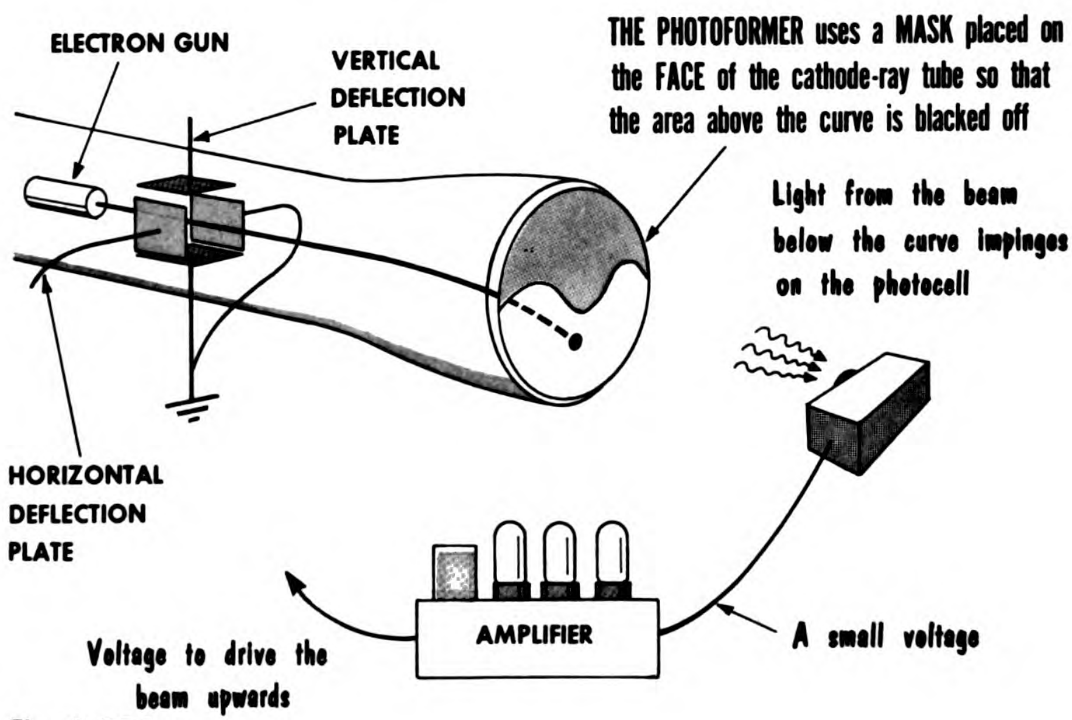


Fig. 3-23A

This, of course, results from their inherent sluggishness or the inertia of heavy parts. Consequently, even in an electronic analog computer all electromechanical function generators are ruled out when rapidly changing variables are expected to be encountered. Two ingenious electronic devices are

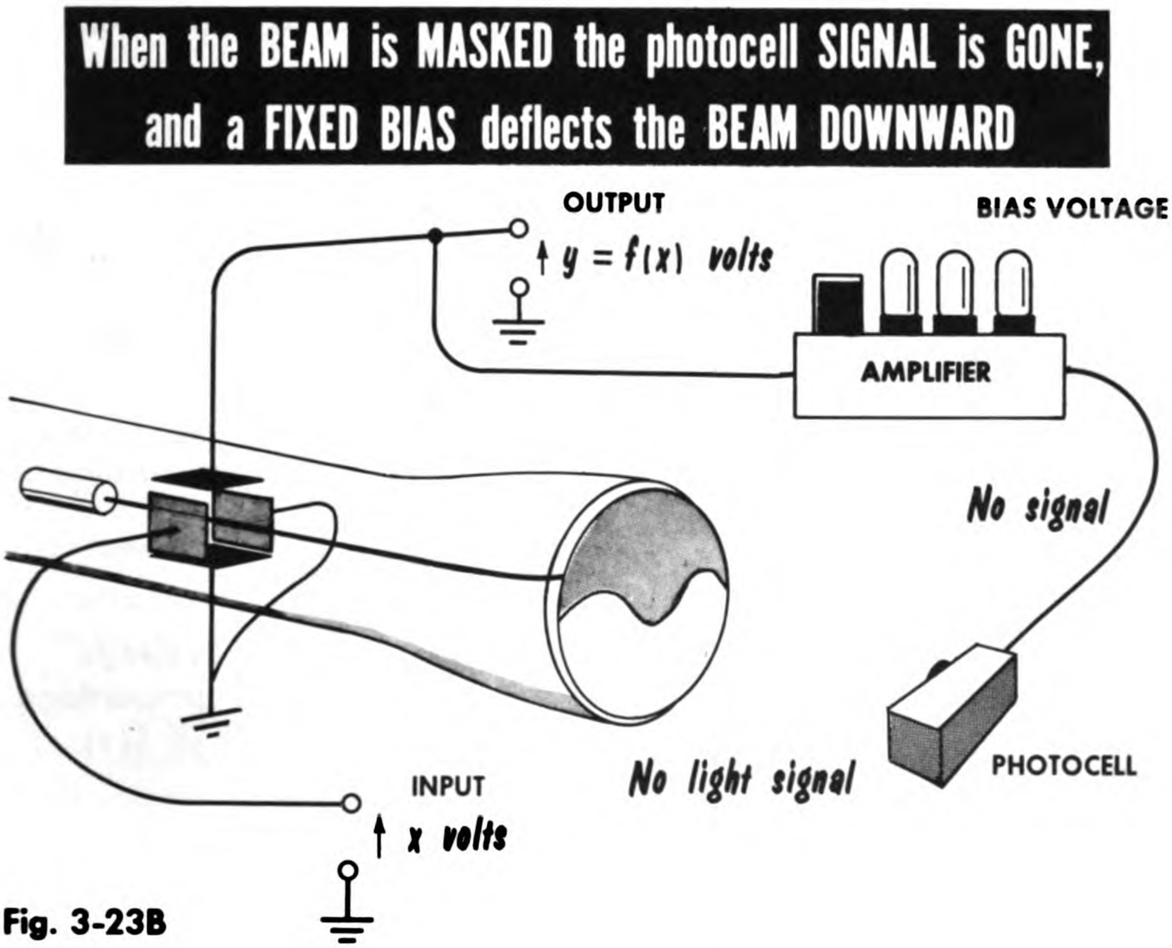


Fig. 3-23B

available in such instances: the diode function generator, usually designated the *dfg*, and the *photoformer*.

The photoformer (Fig. 3-23A) utilizes a mask similar to the metal foil mask suggested for use with the automatic curve follower. One edge of the mask is shaped according to the desired function. The mask is placed on the face of a cathode-ray oscilloscope so that the area above the curve (shaped edge) is blocked off. If the narrow oscilloscope beam strikes the screen (face of oscilloscope) below the curve, light is detected by a photocell. The signal from the photocell is amplified and returned to the vertical deflection plate of the oscilloscope to move the beam upwards. When the beam is hidden by the mask the photocell signal is gone, and a fixed bias deflects the beam downward (Fig. 3-23B). If the amplification of the signal amplifier is high the beam will oscillate rapidly about the edge of the mask with very little deviation from the curve, and furthermore, the fluctuating

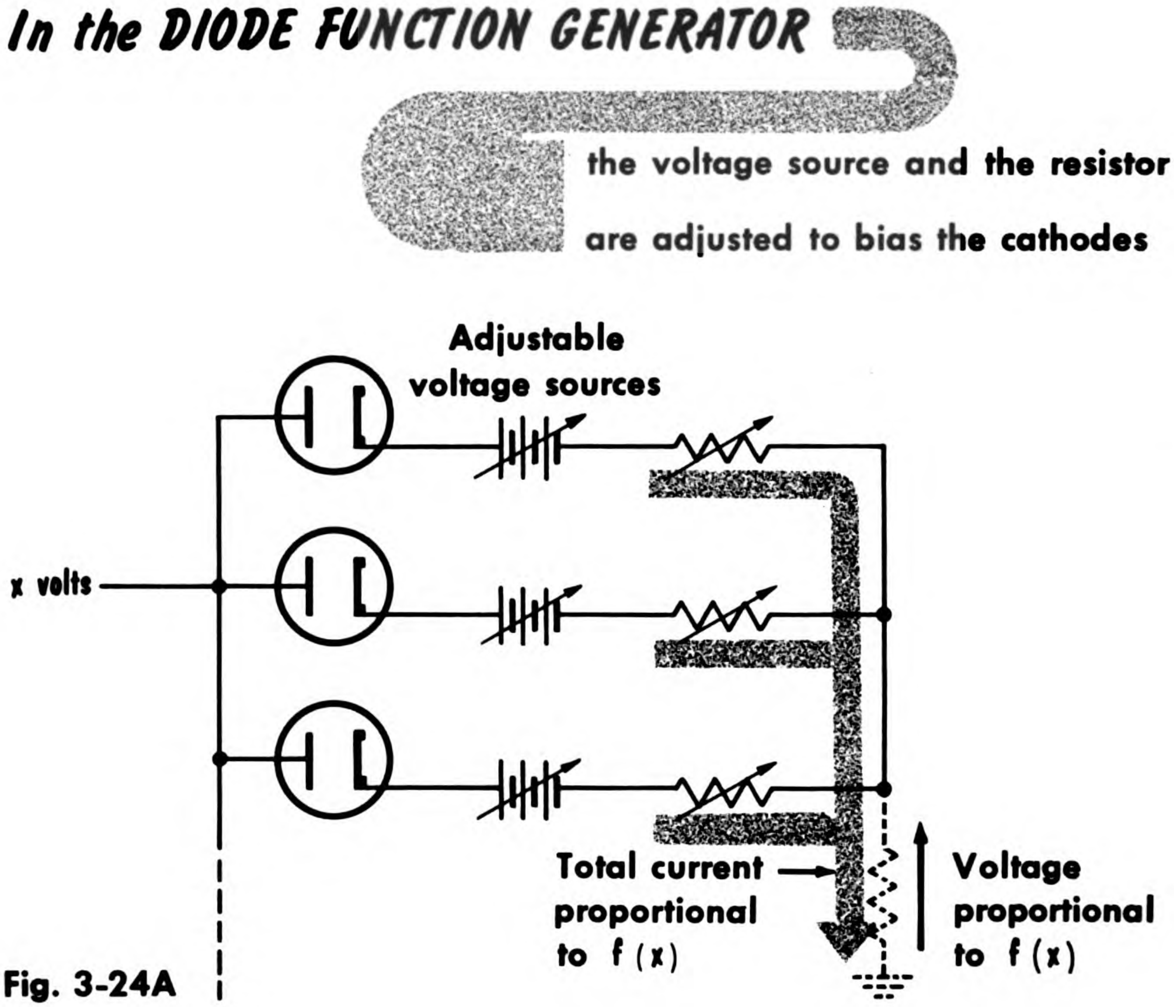


Fig. 3-24A

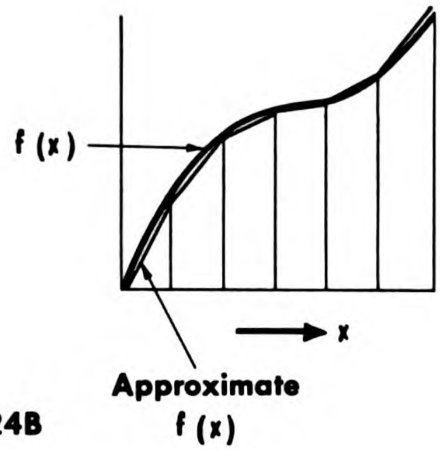
voltage on the vertical deflection plate which keeps the beam at the mask edge is exactly proportional to the desired function, $f(x)$. The *small* oscillations in the output voltage are easily filtered out. Thus as the horizontal deflection plates are energized to move the beam to the right and left across the oscilloscope face proportional to the input voltage variable, x, the beam follows along the curve and the output voltage changes according to $f(x)$.

The Diode Function Generator

The two function generators most often used with electronic computers are the dfg (diode function generator) and the padded servo-driven potentiometer. Both are discussed in considerable detail later in the book.

The diode function generator (Fig. 3-24A) uses a chain of adjustable resistors and voltage sources. In this device the voltage source and resistor are adjusted to bias the cathodes of a number of diodes. The bias prevents conduction by the diode when the input voltage is less than the bias. All the diodes are connected in parallel and the total current through them is summed. With the biases correctly adjusted the total current varies proportionally to $f(x)$ as the input voltage changes proportionally to x.

It should be noted that the dfg (as well as some other function generators) does not provide an exact reproduction of the desired curve (Fig.



The DFG does not provide an exact reproduction of the desired curve. It approximates only.

Fig. 3-24B

3-24B). It approximates the desired function with one consisting of many straight-line segments.

Generation of Functions of Two Variables

Occasionally in a computer study of a physical system it is necessary to generate a function of *two* variables. That is, we wish to generate a com-

GENERATING a FUNCTION of TWO VARIABLES

A combination of ordinary function generators and certain approximating techniques

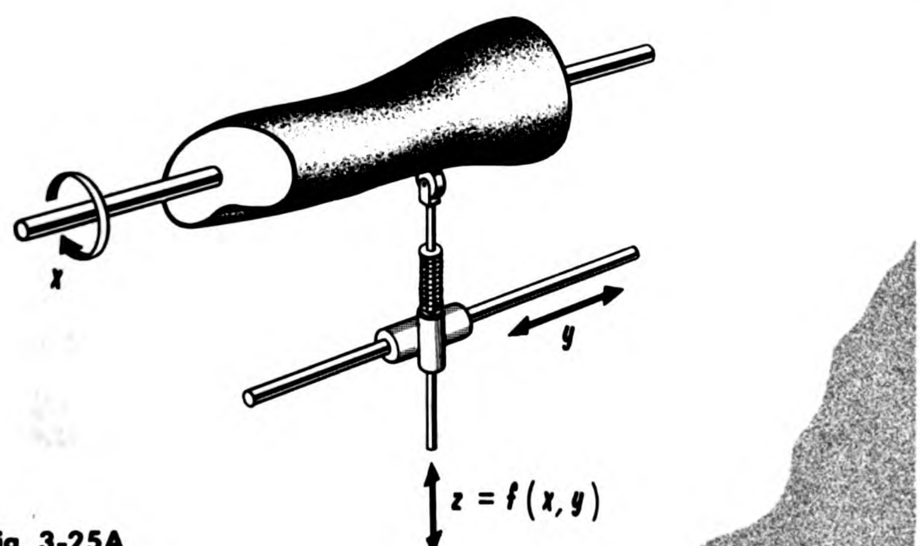


Fig. 3-25A

puter variable which varies as some physical quantity which depends upon *two* other quantities. For instance, in a chemical reactor the reaction rate may depend upon the temperature *and* pressure; the flow rate of a fluid

GOOD GENERATORS for FUNCTIONS of TWO VARIABLES are HARD TO FIND, but the RIGHT DEVICES SOLVE the PROBLEM

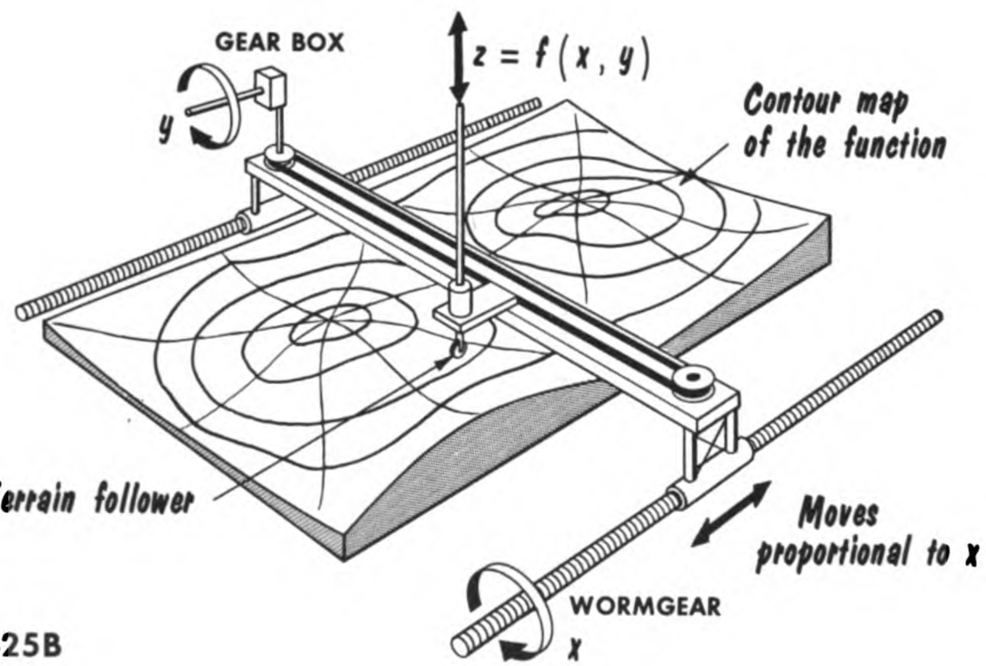


Fig. 3-25B

through a porous medium may depend upon the pressure and the degree of saturation of a particular constituent of the fluid. In a mechanical structure, forces, velocities and other variables may depend upon the three space dimensions as well as time. In an electric circuit the voltage across the terminals of a coil is a function at the rate of change of magnetic lines of flux linked by the coil, but this depends upon the current through the coil as well as its physical dimensions which may be varied independently.

Since a good generator for functions of two variables is hard to find, considerable effort is made to avoid the necessity for one by using a combination of ordinary function generators and certain approximating techniques. However, when the requirement cannot be simplified or avoided, the devices shown in Figs. 3-25A and 3-25B provide quite good results.

The two-dimensional conductive sheet device (Fig. 3-25C) is the electrical equivalent of that in Fig. 3-25B. The x and y input variables drive a wiper over a conducting sheet which is energized by several sources so that the voltage varies at the surface just as the height of the terrain map in Fig.

The TWO-DIMENSIONAL CONDUCTIVE SHEET DEVICE is the

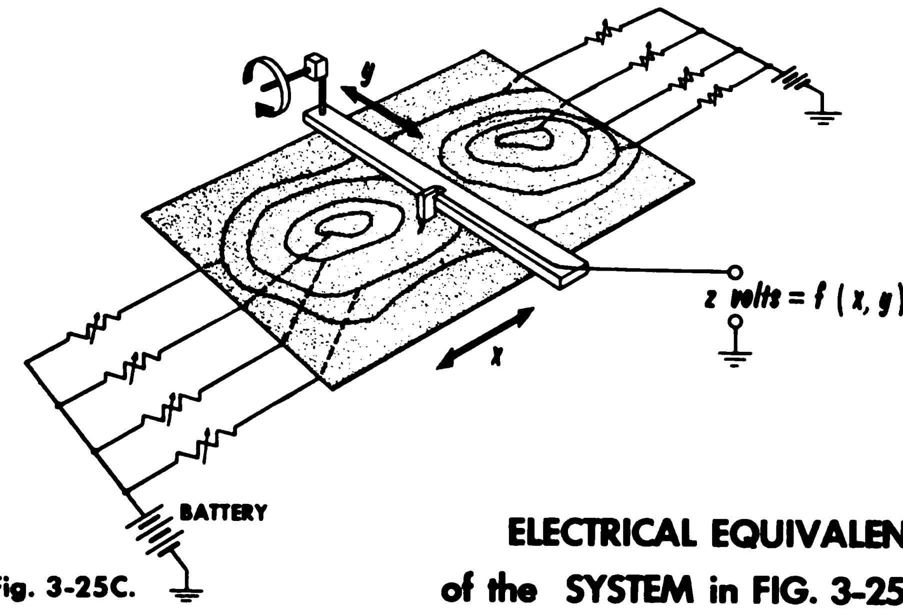


Fig. 3-25C. **ELECTRICAL EQUIVALENT of the SYSTEM in FIG. 3-25B**

3-25B. Each equipotential line is connected to a different voltage source. The lines are drawn with silver ink on a sheet whose resistance is equal in all directions, and sufficient to prevent excessive currents.

QUESTIONS

1. What is meant by "analog building blocks"?
2. Why not have a "subtractor" as one of the basic analog building blocks?
3. Describe the operation of the electromechanical multiplier.
4. What does "pulse-width, pulse-height modulation" mean? How is it used for multiplying?
5. What mathematical functions does the household watt-hour meter perform?
6. In analog model building in engineering design why are function generators almost always required?
7. What building blocks would you require to multiply two variables by logarithmic means?
8. Sine and cosine functions can be generated by a variety of rotary devices. What is the difference between a trigonometric resolver and a sine/cosine function generator?
9. Describe at least two "arbitrary function" generators.
10. When might a function generator of two variables be used? Describe the operation of such a generator.

Chapter 4

THE MATHEMATICS OF COMPUTING

What is a Variable?

Before proceeding with a description of the building blocks known as integrators and differentiators it is probably worthwhile to review what we mean by a variable.

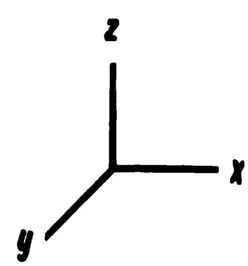
Mathematics and computing are intimately concerned with variables — variable quantities or variable functions — some are called independent and some are called dependent (Fig. 4-1). There are many, many classifications of variables and functions, so let us consider only those we need at present.

INDEPENDENT VARIABLES

Time .... t

The three space dimensions:

Length x
Width .... y
Height .... z



DEPENDENT VARIABLES

Position	Density
Velocity	Volume
Acceleration	Flow rate
Temperature	Electric charge
Momentum	Electric current
Energy	Voltage
Angular Momentum	Power
Revolutions per second	Flux density
Acidity	Pressure
Alkalinity	.
Weight	.
Mass	.

Fig. 4-1

The variables of interest are the physical properties of the primary system whose values are changing with time and whose behaviors are found to be determined by certain fundamental laws. For example: the velocity, altitude, and roll rate of a missile; the temperature, acidity, and pressure of

Displacement in the negative direction is a function of *x* and *y*

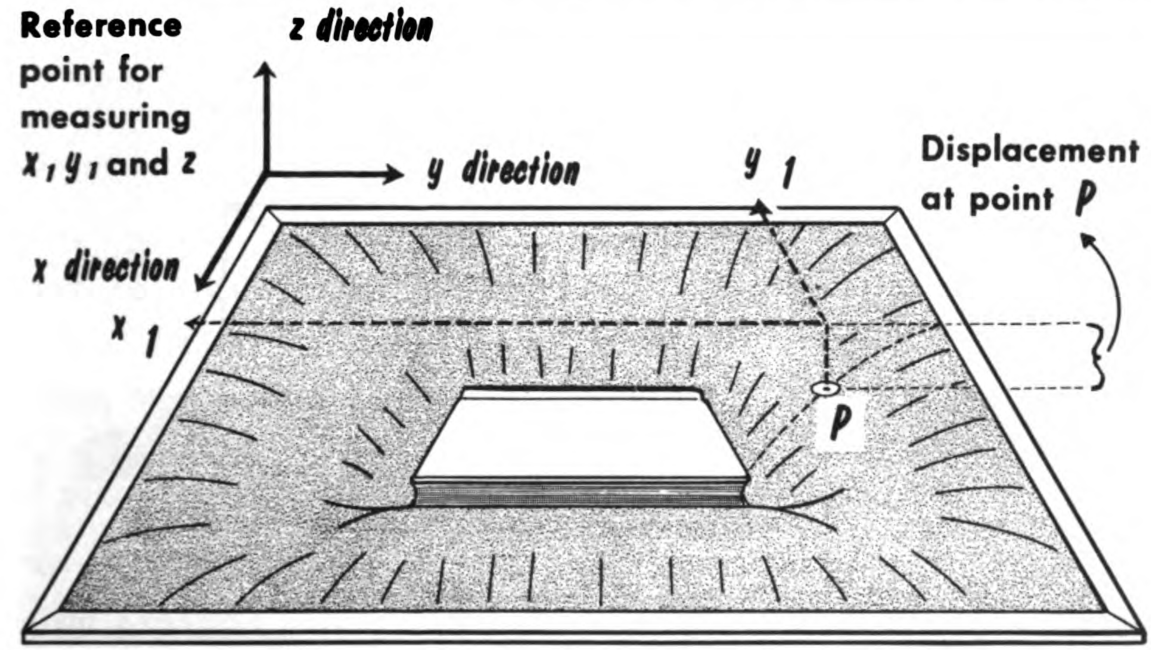


Fig. 4-2

a substance in a chemical reactor; or the forces exerted on the wheels of a car on a bumpy road and the forces on the passengers. These variables are all said to be *functions of time* or *time dependent variables.* That is, they are all *dependent* variables and primarily they are dependent upon *time.* They are dependent upon time in the sense that they vary with time, that is, the variables assume different values for each instant of time.

Functions and Variables

Now it is true that the variables just discussed depend upon many factors, *including the other dependent variables of the system.* Nevertheless, they are all classed as functions of time, since (for our present purposes) time is the only *independent* variable. Time varies (increases) in a very predictable way — it *marches on* completely *independent* of all other variables.

The other *independent* variables of importance are the space dimensions: length, breadth, height, or call them x, y, z. Consider placing a book on a thin rubber diaphram (Fig. 4-2). If the book is dropped onto the rubber sheet the strain and tension within the sheet will at first vary as a function of time, as well as varying from point to point across the sheet.

When the book is finally at rest upon the sheet, the tension and strain within the sheet are no longer functions of time. They vary only with the space dimensions x and y. In this two-dimensional example, the displacement in the negative z direction is a function of x and y. It has a particular value for each pair of values selected for x and y; for example, consider the point P, with coordinates x_1 and y_1.

Variables and Physical Laws

For the present we will consider only functions of time. We have said that the variables are the physical properties of the primary system, whose be-

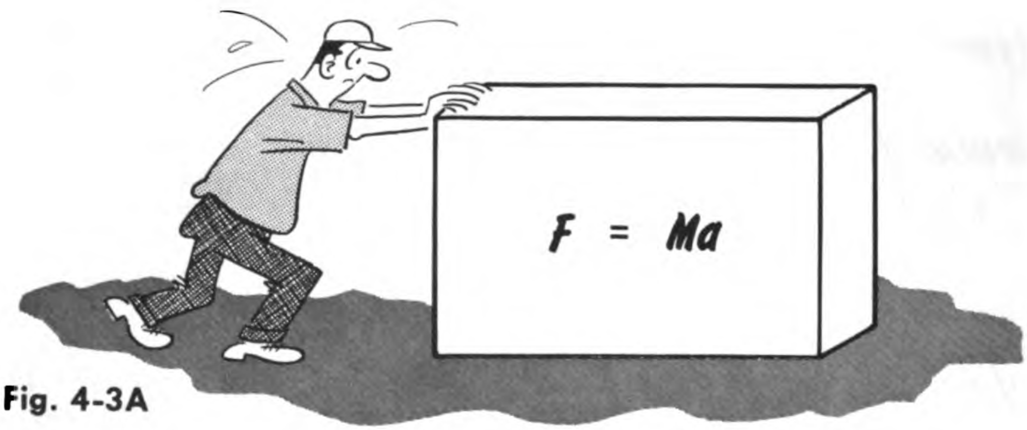


Fig. 4-3A

havior satisfies certain natural laws. The laws can be expressed mathematically and the variables will be the subjects of such expressions. For example:

- *Laws*: The net linear force (Fig. 4-3A) on a rigid body is equal to the mass of the body times its linear acceleration.
- *Mathematical expression:* $f = \mathrm{Ma}$

  a = acceleration = the rate of change of v
  v = velocity = the rate of change of x
  x = position of the body, measured as a distance from a fixed point
  M = mass of the body
  f = net applied force

- *The variables*: a, v, x, and f.

Hence the word variable implies not only the physical properties of the primary system but the mathematical symbol for the property. That is f is said to be a function of time and is usually written as $f(t)$. Likewise $a(t)$ is a function of time; but M is a constant. The mathematical functions $f(t)$ and $a(t)$ are also called mathematical variables, which assume numerical values as t varies (as time progresses). This may be expressed graph-ically as shown in Fig. 4-3B.

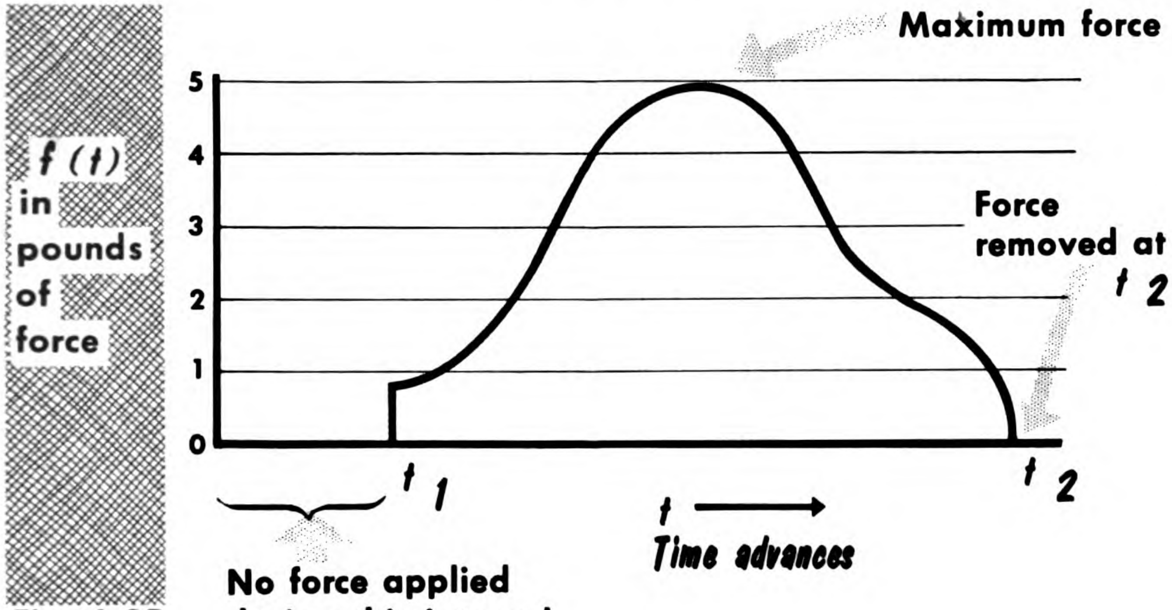


Fig. 4-3B

Physical, Mathematical, and Computer Variables

Now since an analog computer is programmed to solve a set of mathematical equations analogous to those which determine the behavior of the primary system, for every variable in the primary system there must be a corresponding *computer variable*. If the physical variable is force (with a graph as shown in Fig. 4-3B), then somewhere in the computer there should be a voltage (assuming an electronic computer) which varies as shown in Fig. 4-4A. That is:

$$e(t) \text{ volts} = K\, f(t) \text{ pounds of force}$$

where K is a constant scale factor with units of "volts per pound" relating how many volts correspond to one pound.

Hence we find that to speak of a variable may simultaneously mean *three*

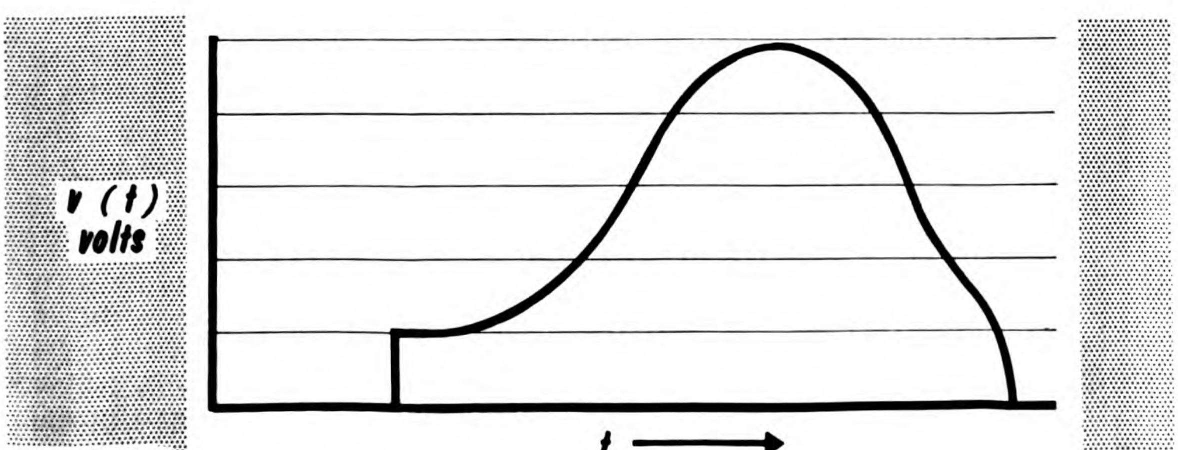


Fig. 4-4A

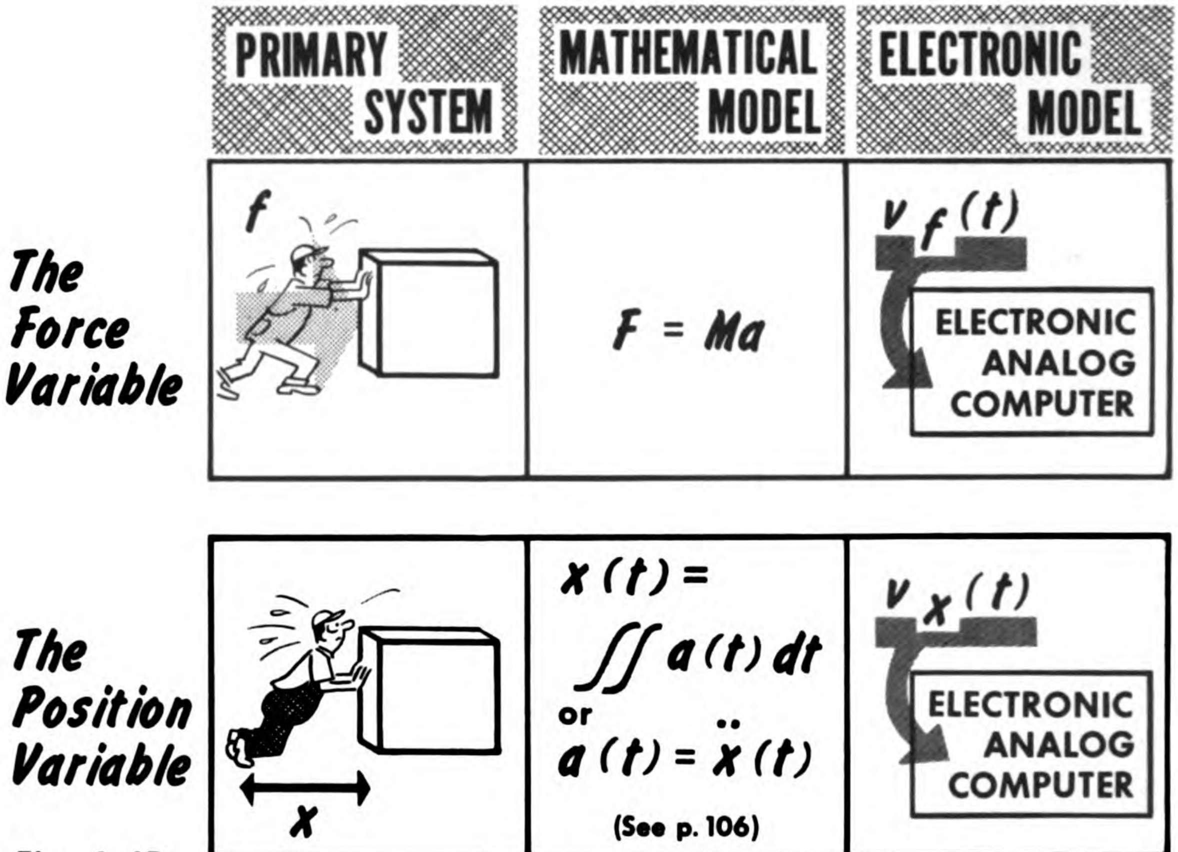


Fig. 4-4B

things: *the physical force f, the mathematical function f(t)*, or its *voltage equivalent* $e(t)$ (Fig. 4-4B). It is usually not necessary to distinguish between the three, for actually that is one of the key features of any analog computer study — that there exists this correspondence between the computer model, the mathematical model, and the physical primary system. (Double integration and second order differential equations are dealt with later. See p. 106.)

INTEGRATION

In Chapter 3 we described and discussed all the basic analog building blocks, except two: the integrator and the differentiator. Introduction of these devices is being left until we discuss the mathematical operations of integration and differentiation. We will find that the differentiator plays only a minor role in analog computing while the *integrator is probably the most important single element in any general purpose analog computer*. Since this is the case, it seems appropriate that we should make an extra effort to fully understand the mathematical operation of integration and its realization in an analog building block. To that end the remainder of this chapter is devoted to the idea of integration and to the solution of equations arising from physical laws which require integration for solution. A short discussion of differentiation is included.

Integration may be described in a nutshell (Fig. 4-5)). The simple process of counting can in some cases be considered equivalent to an integration. For example, the counting of revolutions of a revolving disc or the counting of miles traveled as you move down the highway or railway; each of

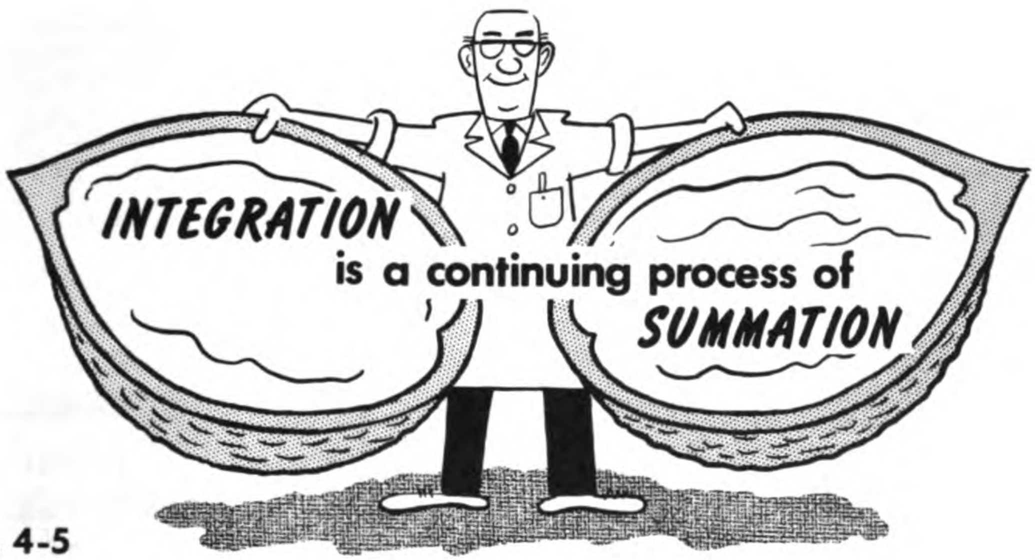


Fig. 4-5

these processes amounts to a continual addition, at a variable rate (of revolutions or miles), to an up-to-date accumulated total. In one case the rate is the revolutions per minute (rpm) and in the other the miles per hour (mph). Mathematically, it is said that the total count is equal to the *integral* of the *rate* with respect to *time*. Integration is indicated by a special symbol — an elongated "S" — which is suggestive of the summation process.

$$\text{Total count} = \int (\text{RATE})\, dt$$

where the symbol *dt* indicates the summation is performed over an interval of *time*, and *with respect to time*.

Examples of Integration

Consider a stop watch (Fig. 4-6A) which continually adds up the number of seconds elapsed since the beginning of a race and indicates the total. Also, an odometer (or mileage recorder) in a car continually adds up the miles traveled and presents the total. That is, it "integrates the speed of the car over its lifetime" yielding the total distance traveled at any time.

If we were to record on graph paper as a function of time the speed of a car during some short trip, it might look as shown in Fig. 4-6B.

Now assuming our odometer is not functioning and we wish to know the distance traveled (in fact the distance traveled at any instant of time during the trip), we can find the distance by integrating the above function with respect to time. This can be done graphically or by a computer. Let

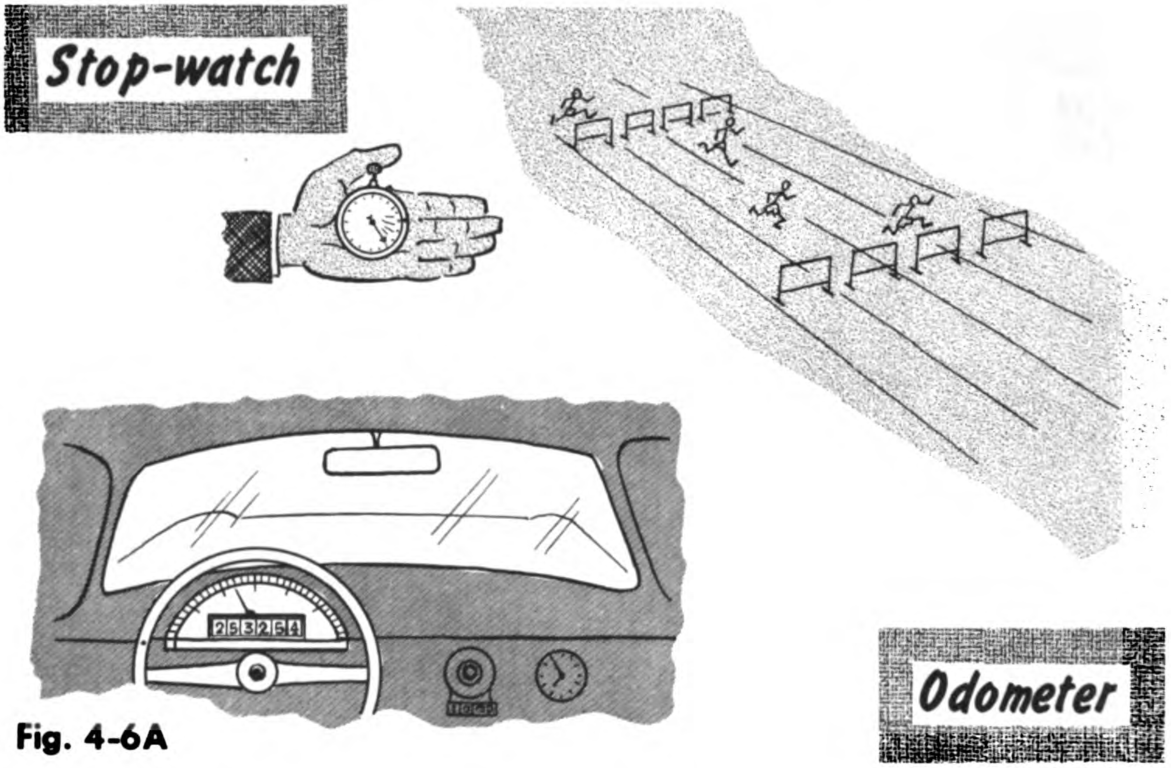


Fig. 4-6A

us do it graphically to illustrate how simple integration is. First we start by noting that for a constant speed the distance traveled is the product: speed × time spent at that speed. But the speed does not remain constant, so to obtain a measure of the distance we might, to begin with, approximate

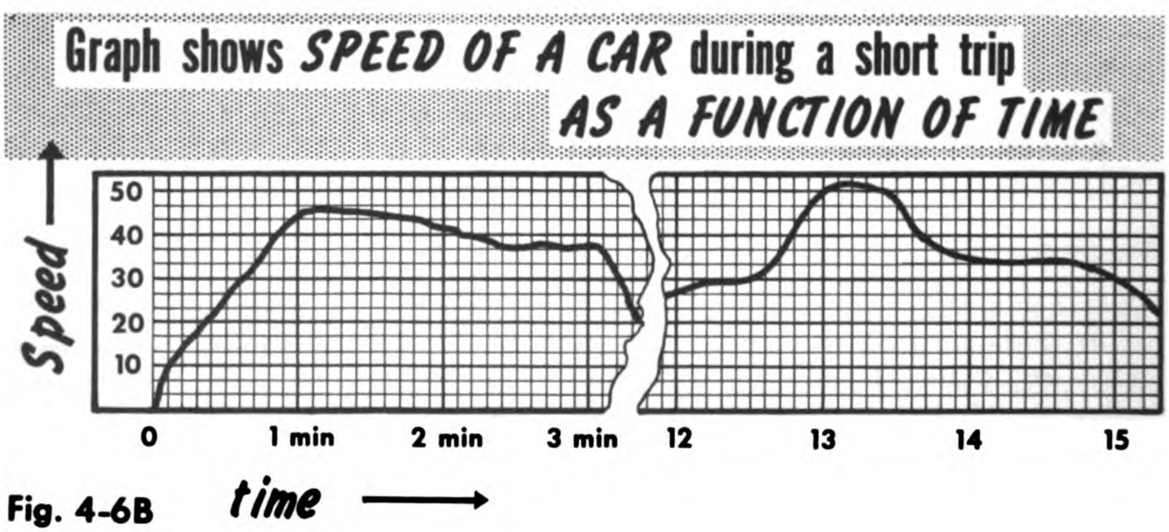


Fig. 4-6B

the speed curve by many small segments of constant speed (Fig. 4-6C). Each segment lasts for a short interval of time, which we will call an increment Δt.

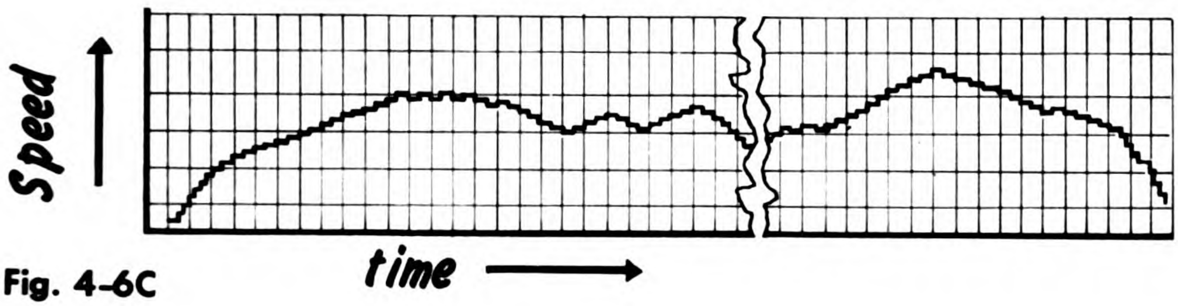


Fig. 4-6C

Now the distance traveled in the first increment, Δt, (say, $\Delta t = 1/10$ second) is 1/10 times the speed during that increment (Fig. 4-7A), or:

$$\text{distance (at } t = 0.1 \text{ second)} = (0.1 \text{ second})\ (1.2 \text{ mph}) = 0.176 \text{ feet}$$

In the next increment the distance traveled is (Δt) (2.4 mph) = 0.352 feet, or the total:

$$\text{distance (at } t = 0.2 \text{ second)} = 0.176 + 0.352 = 0.528 \text{ feet}$$

But now we note that each of these increments of distance is equal in numerical value to the area of a small rectangle under the speed curve (Fig. 4-7B).

If we continue this procedure for the 150 increments of Δt necessary to get to the point $t = 15$ seconds, we will find that we have done nothing more than calculate the area under the segmented curve. Since this conclusion would be true irrespective of the size of Δt, we can see that by using smaller increments (and more of them) to approximate the curve up to $t = 15$ sec.) we would calculate the area under a segmented curve which is itself a better approximation of the original curve (Fig. 4-7C). If we were to make the size of Δt smaller and smaller the work required to calculate the area under the segmented curve would increase. But now, since

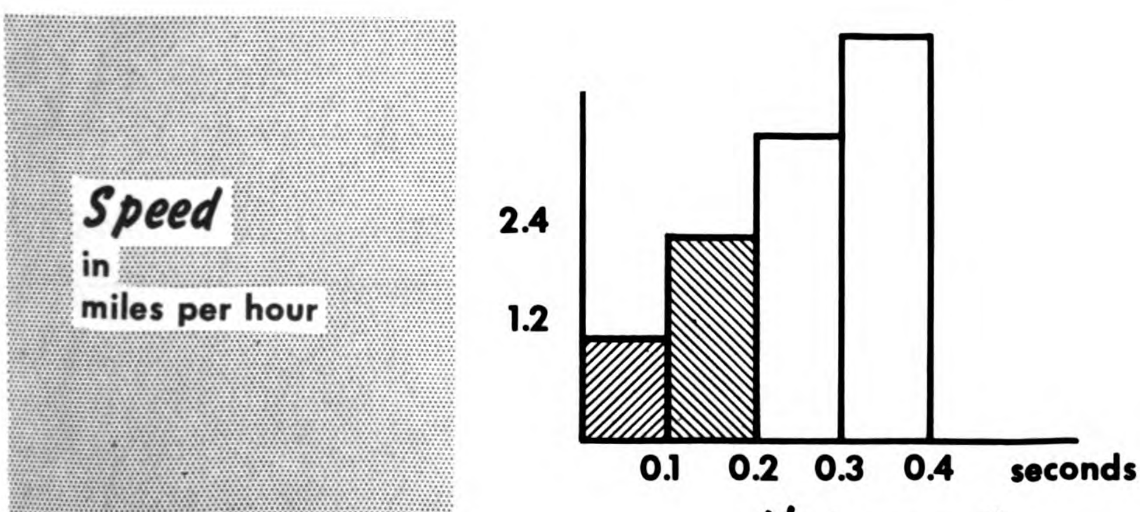


Fig. 4-7A

as Δt becomes smaller the segmented curve approaches more nearly the speed curve, we may conclude that the desired integral (or distance actually traveled as a function of time) is proportional to the area under the speed

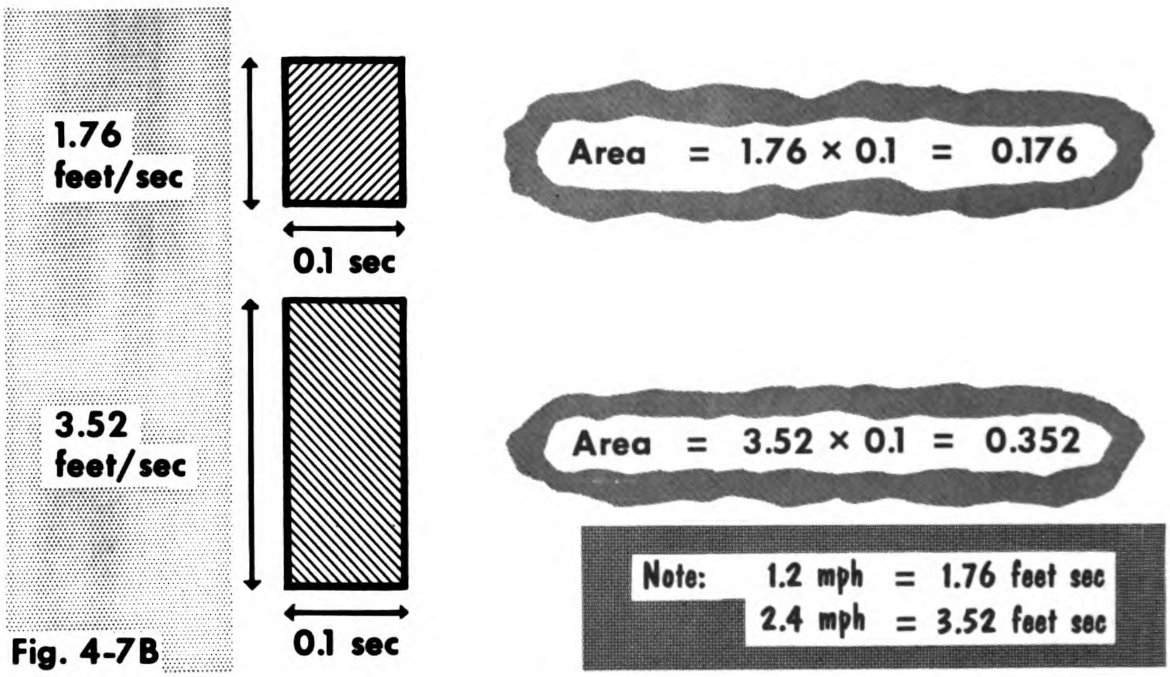


Fig. 4-7B

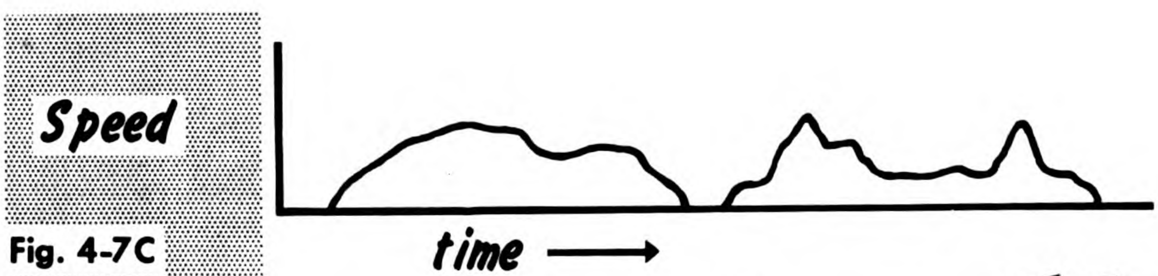


Fig. 4-7C

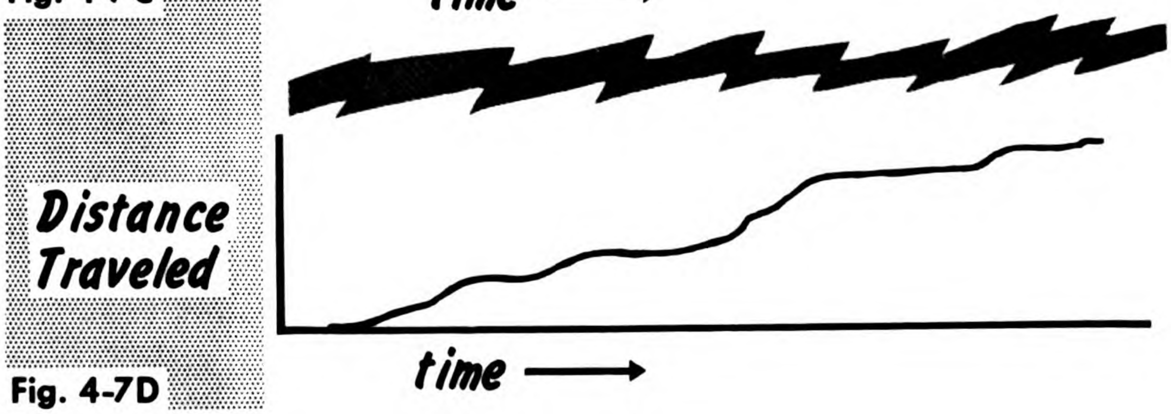


Fig. 4-7D

curve. In fact, we can say that the integral of any curve is equal to the area under the curve. Thus the graph of distance traveled is as shown in Fig. 4-7D.

A Mechanical Integrator

From the foregoing we see that to integrate a function of time we need only to find the area under the curve representing that function — but now comes the rub! How to do it.

To integrate with pencil and paper or even with an electronic *digital* computer, one must go back to the method we started with, that is, measuring the area of small rectangles or counting the small squares of the graph paper under the curve. Of course, the digital computer could count thousands of little squares per second, and thus, by using a "fine mesh", would produce a very accurate answer.

But we wish to integrate by analog means.

Specific integrators will be described later, but the principle of the analog integrator building block is illustrated here by three examples.

1. If a small shaft is caused to rotate at a speed proportional to the function to be integrated, the revolutions can be counted or a shaft can be positioned to record the integral of the function. Just such a system is built into your car, of course, and is called the odometer (Fig. 4-8A).

HYPOTHETICAL ANALOG INTEGRATOR

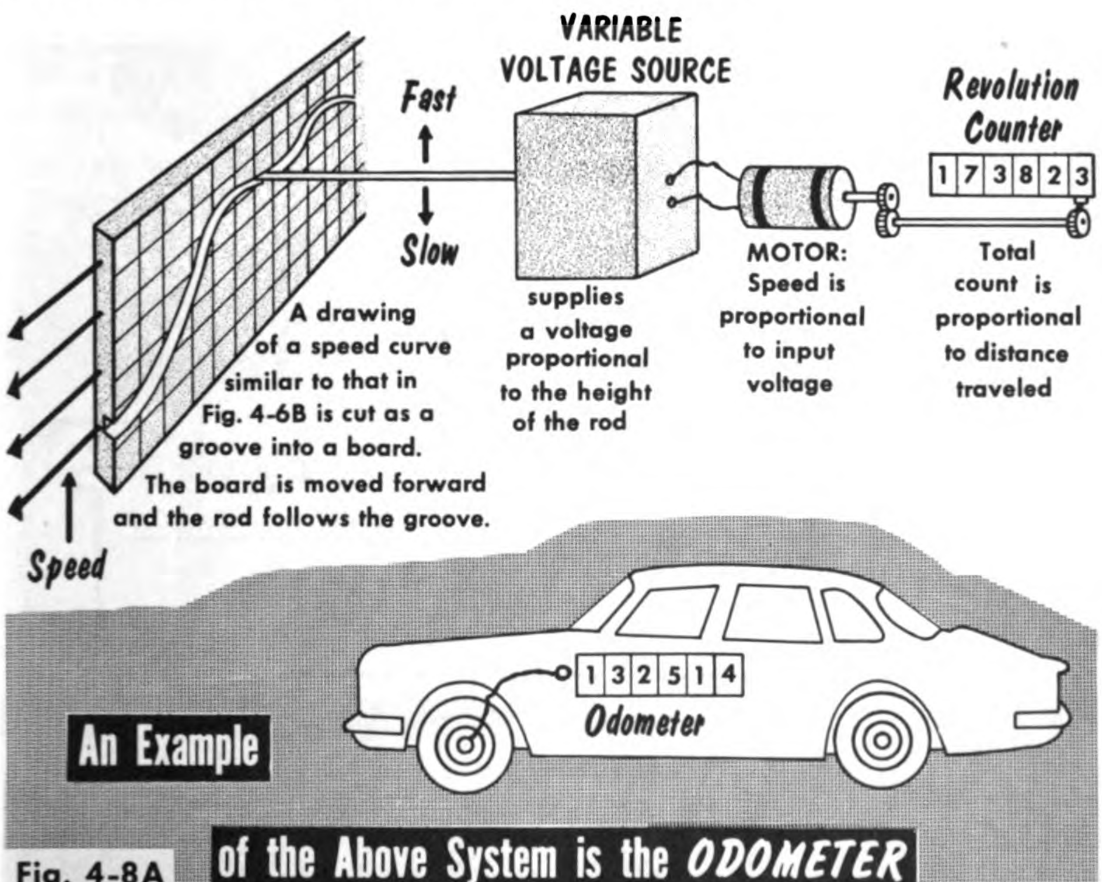


Fig. 4-8A

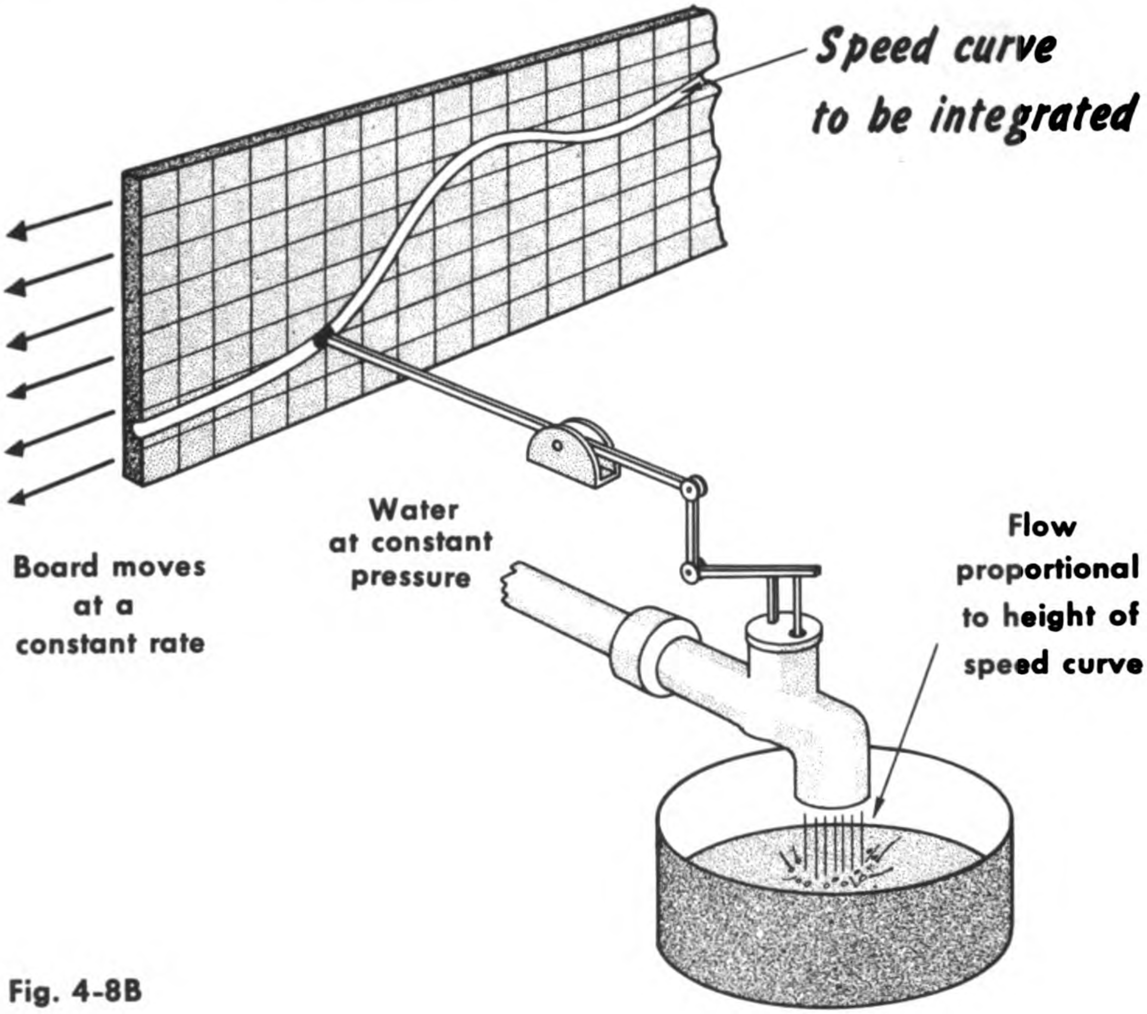
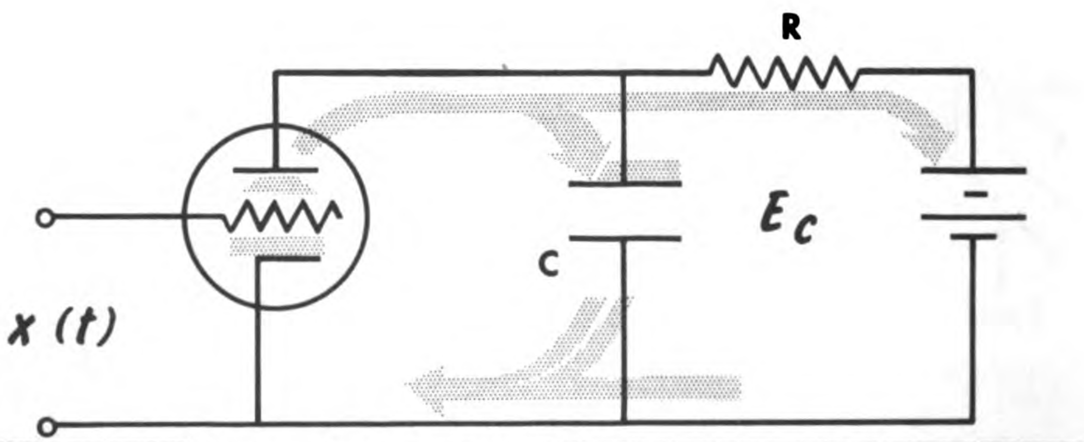


Fig. 4-8B



If R and C are large
E_C is approximately equal to the integral of
$x\ (t)$ with respect to time

$$E_C \cong \int x\,(t)\,dt$$

Fig. 4-8C

2. Let water flow through a valve at a rate proportional to the input function, and collect all the water in a bucket (Fig. 4-8B). The volume of water held by the bucket is then proportional to the integral of the input function. Furthermore, if the container has a constant cross section (vertical sides) the depth of water is also proportional to the integral.
3. The electrical equivalent of the second example is an electronic valve (tube) controlling the flow of electrons into a storage capacitor (Fig. 4-8C). The electron flow is controlled by the input voltage function.

Definite Integrals

Unless specified otherwise, any reference to integration will be understood to be *integration with respect to time*. That is, the independent variable is time. Graphically this means the horizontal axis represents time. Later on we will want to integrate a function with respect to some other variable, such as a distance, x. In such a case it is necessary to resort to approximating methods or to let the variable x be represented by time for the purpose of the computer calculation of the integral of the function of x:

$$I = \int_{x_1}^{x_2} f(x)dx \longrightarrow \int_{t_1}^{t_2} f(t)dt$$

The above says that the numerical value of the integral, I, defined as the area under the curve $f(x)$ for values of x between x_1 and x_2, is given by the integral with respect to t, of $f(t)$, from t_1 to t_2, when we let t assume the same numerical values of x. I is called a *definite integral*, for the limits of integration, x_1 and x_2, are specified.

In general we shall speak of the integral of $f(t)$, written $\int f(t)dt$, where the limit t_1 is taken to be *zero* (the time at which the analog computer is

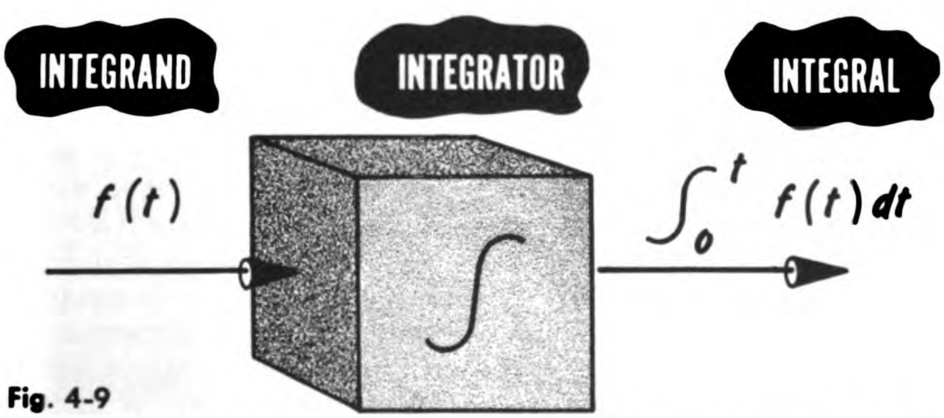


Fig. 4-9

placed into operation), and the limit t_2 is the variable t, the time at which the output is observed or recorded.

Symbolically, this is usually indicated by a "black-box" (Fig. 4-9) whose inputs and outputs are shaft revolutions, voltages or some other analog variables.

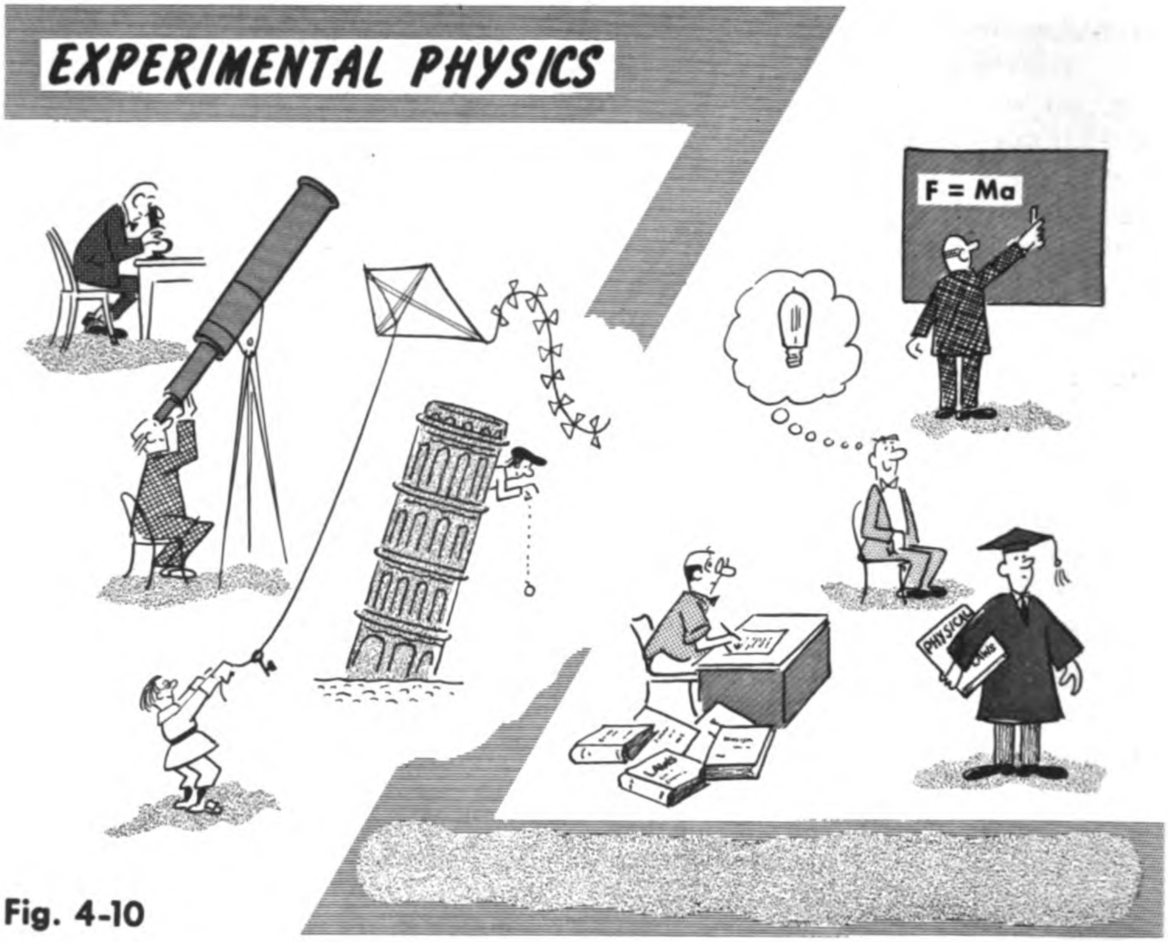


Fig. 4-10

DIFFERENTIATION AND INTEGRATION

We have spoken of acceleration as being the "rate of change of velocity", and of velocity as the "rate of change" of position or of distance traveled both with respect to time. Later we described "distance traveled" as the integral of the velocity. One word used to replace "rate of change" is "derivative"; we say that acceleration is the *derivative* of velocity and velocity is the *derivative* of position. The two mathematical operations, *integration* and *differentiation*, which are otherwise spoken of as "forming the *integral* or *derivative* of a function", are essential for a very large portion of advanced mathematics. The advanced applications of these operations require considerable experience with the disciplines of the calculus and differential equations. The operations themselves, however, and the basic principles of their application are quite simple. Furthermore, the concepts of integration and differentiation are so intimately related to the dynamic behavior of physical devices and systems that a knowledge of either is helpful in understanding the other.

All physical systems behave, or react to stimuli, in ways determined by nature, or by some universal order (certainly not by any man-made laws). However, through centuries of careful observation and experimentation man has compiled a set of so-called "physical laws" which within the limits

of sensible measurement *describe* the behavior of physical systems (Fig. 4-10). Certainly these are not laws in the sense that they dictate what the behavior ought to be, but rather they are rules from which one can predict the behavior of a particular system in a particular environment. The most concise formulation of these laws is in terms of mathematics which requires the use of the familiar operations of addition, subtraction, multiplication, division, *differentiation* and *integration.*

Learning by Analogs

Mathematical equations which express physical laws and which contain the operation of differentiation, as well as the simpler algebraic operations, are called *differential* equations. Equations containing integration are called *integral equations.* There are some equations which include both operations. We shall be concerned only with the more frequently required differential equations. The analog computer is primarily intended as a machine for solving differential equations. In fact, the analog computer is often called a *differential analyzer.*

To obtain a good grasp of the concepts of integration and differentiation, it is only necessary to consider the dynamic behavior of some simple, familiar devices. Such a learning process is consistent with the primary justification for this book and analog computation. That is, if we can find a connection between the behavior of a known model or analog and some

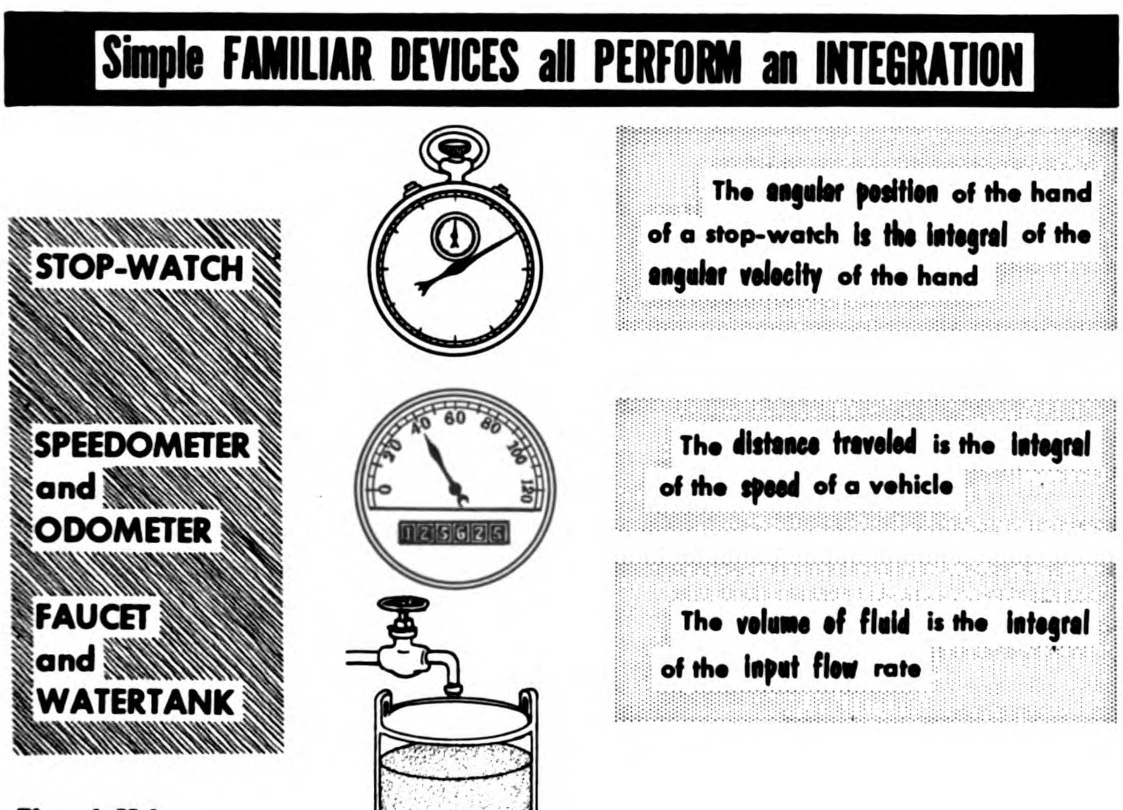


Fig. 4-11A

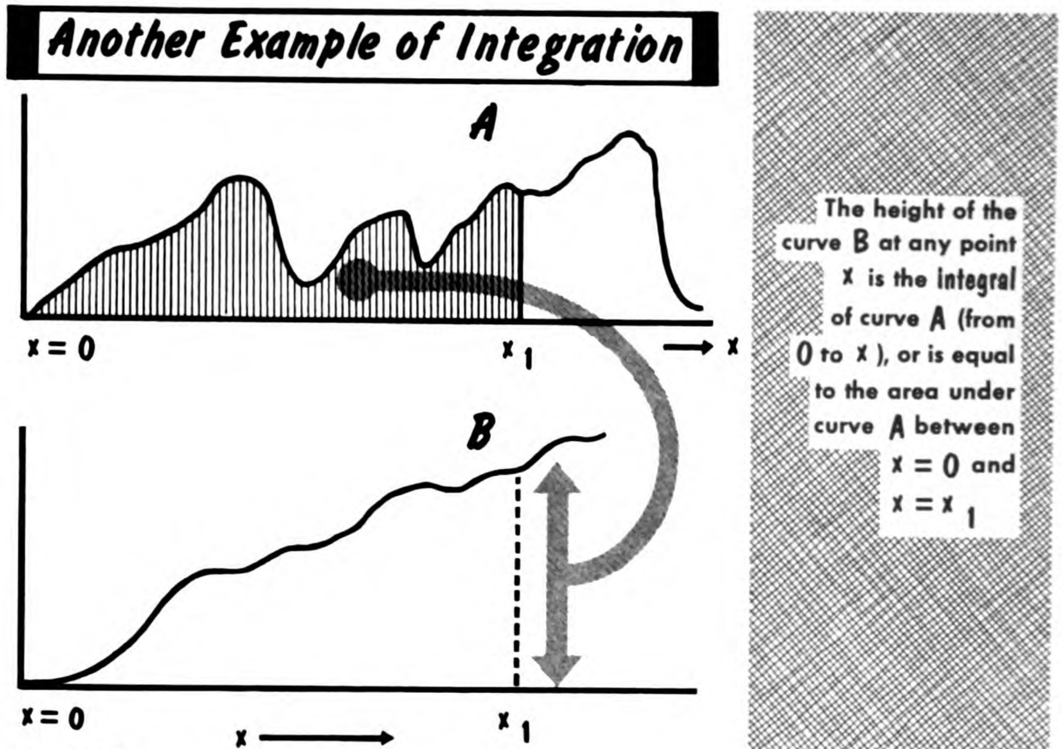


Fig. 4-11B

system we wish to study (be it physical or purely mathematical), then we can learn about the latter from the former. Never should we let the mathematical trappings and double-talk keep us from understanding the elementary and commonplace experiences of every day.

To be sure, we have already observed that integration is simply a continuous summation of potentially variable quantities. A clock, an odometer, and a tank with provision for filling, each perform an integration (Fig. 4-11A). Yet another example is illustrated in Fig. 4-11B.

Differentiation

Differentiation is the inverse operation of integration. For example:

1. The height of curve A at any point x is the *derivative* of curve B at x, the rate of change of curve B at x, or the slope of curve B (Fig. 4-12A).
2. The flow rate is the *derivative* of the volume of fluid in the tank (Fig. 4-12B). That is, it is equal to the rate of change of the volume. In Fig. 4-12C the flow rate is the negative of the rate of change of volume (since the volume is decreasing).
3. Speed is the *derivative* of distance traveled (speed = rate of change of distance) (Fig. 4-12D). Furthermore,

$$\text{acceleration} = \text{rate of change of speed}$$

Therefore, acceleration is the derivative of speed (Fig. 4-12E).

DIFFERENTIATION is the INVERSE OPERATION of INTEGRATION

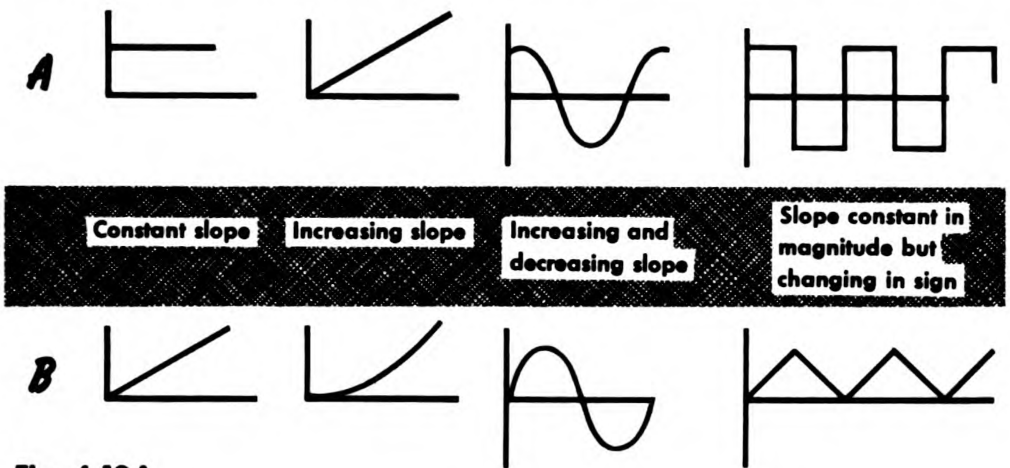


Fig. 4-12A

The FLOW RATE is the derivative of the volume

SPEED is the derivative of distance traveled

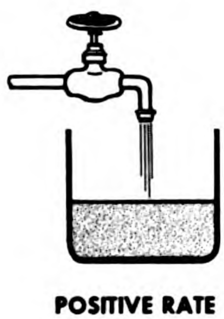
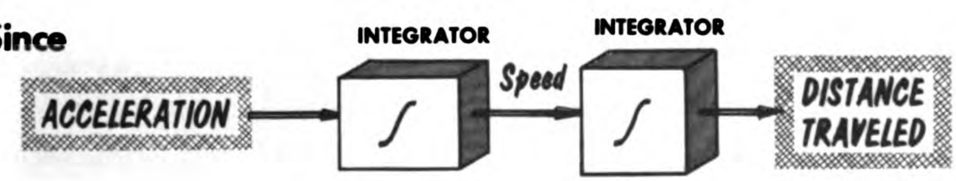


Fig. 4-12B

Fig. 4-12C

Fig. 4-12D

Since



we have

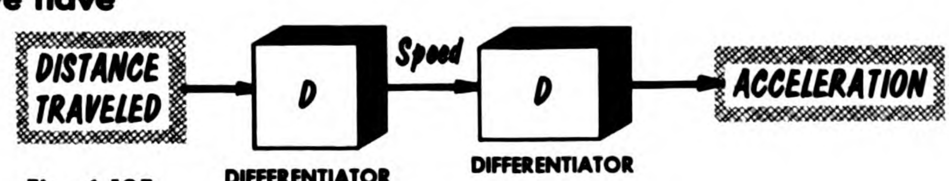


Fig. 4-12E

Differentiating Devices

Integrators have been discussed briefly but no mention has been made of differentiators. Two common differentiators are the tachometer and speedometer. These may be thought of as converting the angular velocity of a crankshaft or vehicle wheel into an angular position of a needle on a dial.

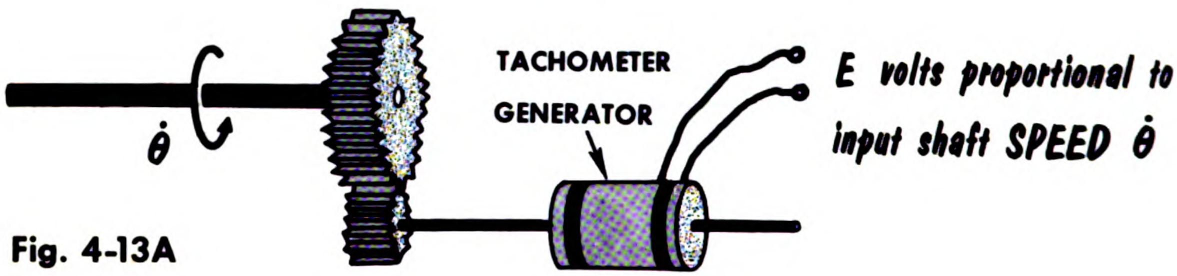


Fig. 4-13A

Hence the needle position is the derivative of the angular position of the crankshaft or wheel. In other applications the output quantity might be a voltage proportional to the derivative of the shaft position, such as is produced by a tachometric generator (Fig. 4-13A).

An example of a speed differentiator is an accelerometer, a device that provides an indication of the acceleration of the object to which it is attached. In an aircraft it tells a pilot how many "g's" he is "pulling". Similar devices are used in shock testing. In its simplest form, an accelerometer consists of a weight, a spring, and an indicator (Fig. 4-13B).

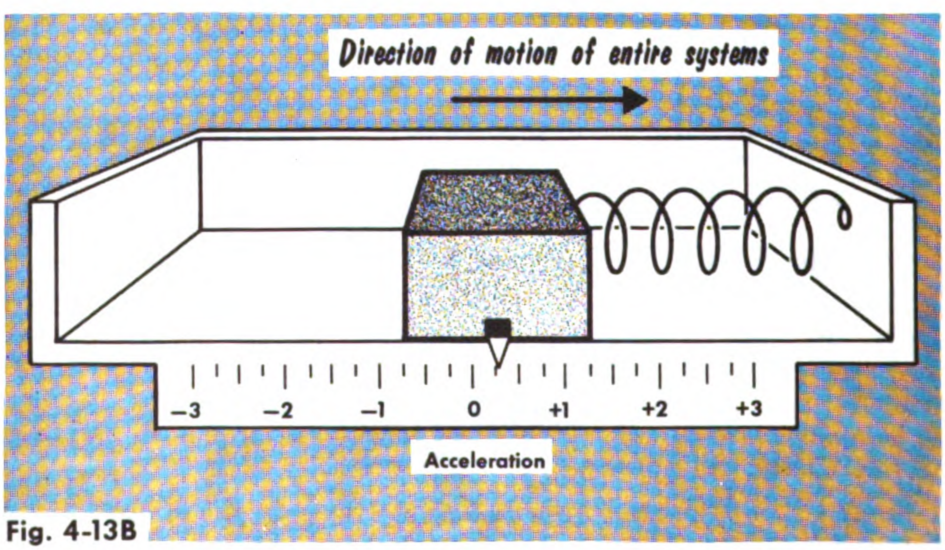


Fig. 4-13B

If the speed in the direction indicated changes, the weight moves forward (for a decrease in speed) and the rate of change of the speed, or the derivative of the speed, is indicated by the position of the weight. Hence the pointer on the scale measures acceleration.

Noise in a Differentiator

In a computing device the "real" variables (distances, speeds, accelerations) just discussed are represented by computer variables (shaft rotations, lever displacements, voltages or hydraulic pressures). All computer variables are subject to small high-frequency variations. In a mechanical system we would call them *vibrations;* in an electronic system, *electronic noise*. Generally these variations are negligible, and they in no important way affect the

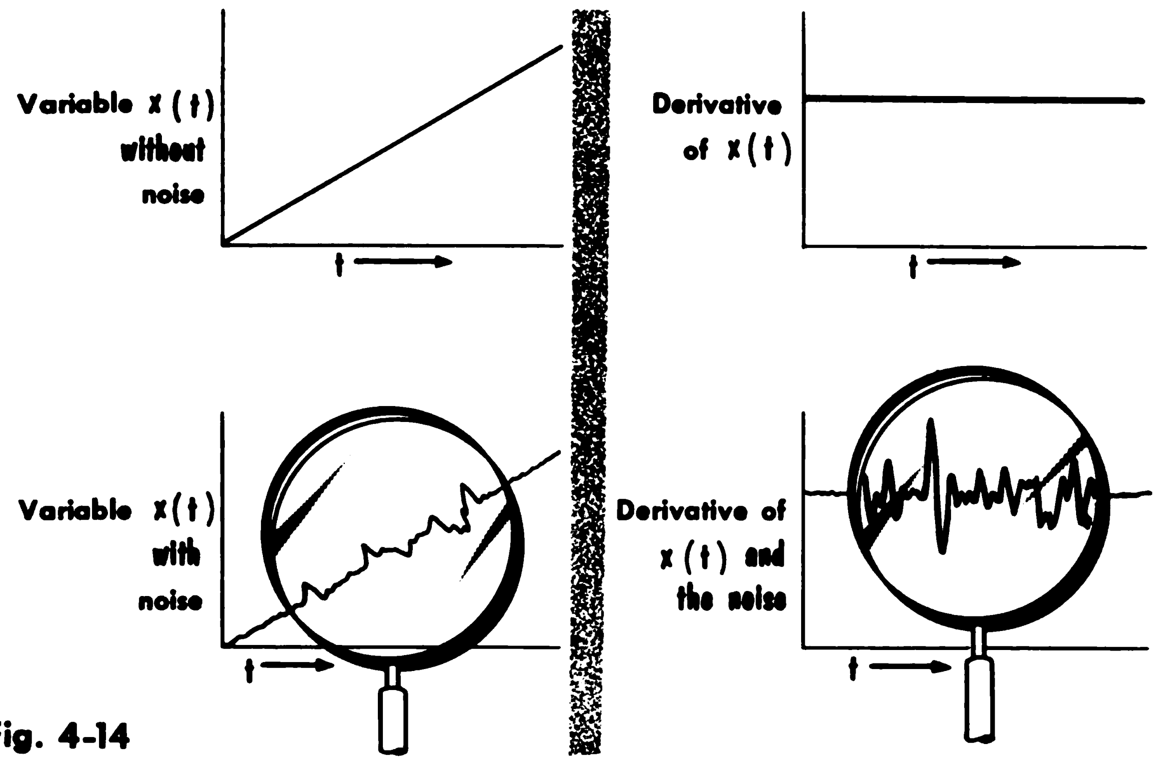


Fig. 4-14

computation. However, consider what happens when you differentiate or take the derivative of a variable (Fig. 4-14) that contains an additional noise component.

Note that since the derivative of $x(t)$ is a plot of the *slope* of $x(t)$, every time $x(t)$ experiences a bump, the derivative of $x(t)$ has a double spike.

Thus differentiation accentuates and magnifies any noise present in the variable $x(t)$ and although this noise in $x(t)$ may have been negligible, the resultant noise in the derivative of $x(t)$ will frequently be quite objectionable.

Differentiation of a Piecewise Continuous Function

Consider now a second characteristic of differentiation which makes its accomplishment in an analog computer difficult. Assume that the variable x is free of noise and has the form shown below — first increasing at a constant slope and then decreasing at a constant slope [Fig. 4-15 (A)]. It is

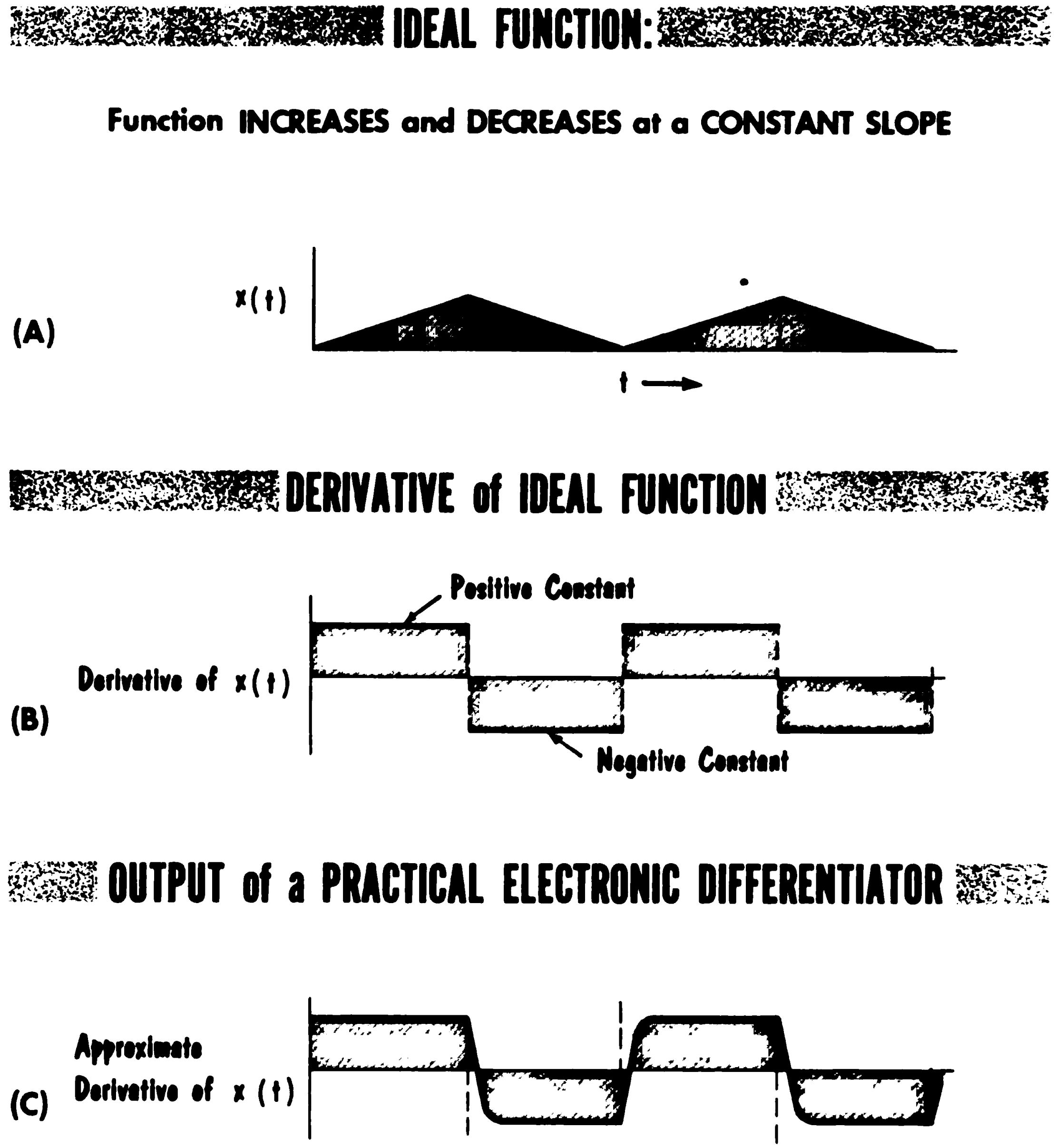


Fig. 4-15

desired to form the derivative of *x(t)*. It is apparent from the graph that the derivative of *x(t)* should have the form shown in Fig. 4-15 (B). The function $\dot{x}(t)$ is first a positive constant and then a negative constant.

Now consider the problem of constructing a mechanical or electrical device which will switch *instantaneously* from a positive to a negative value, a requirement that makes a perfect differentiator impossible. A practical electronic differentiator may be possible. It would give almost the correct graph for the derivative of $x(t)$ and it would require some means of filtering the accentuated noise [Fig. 4-15 (C)]. A practical mechanical differentiator would have very limited use, for it could differentiate only very small changes in the variable $x(t)$, due to the inertia of the parts of the device.

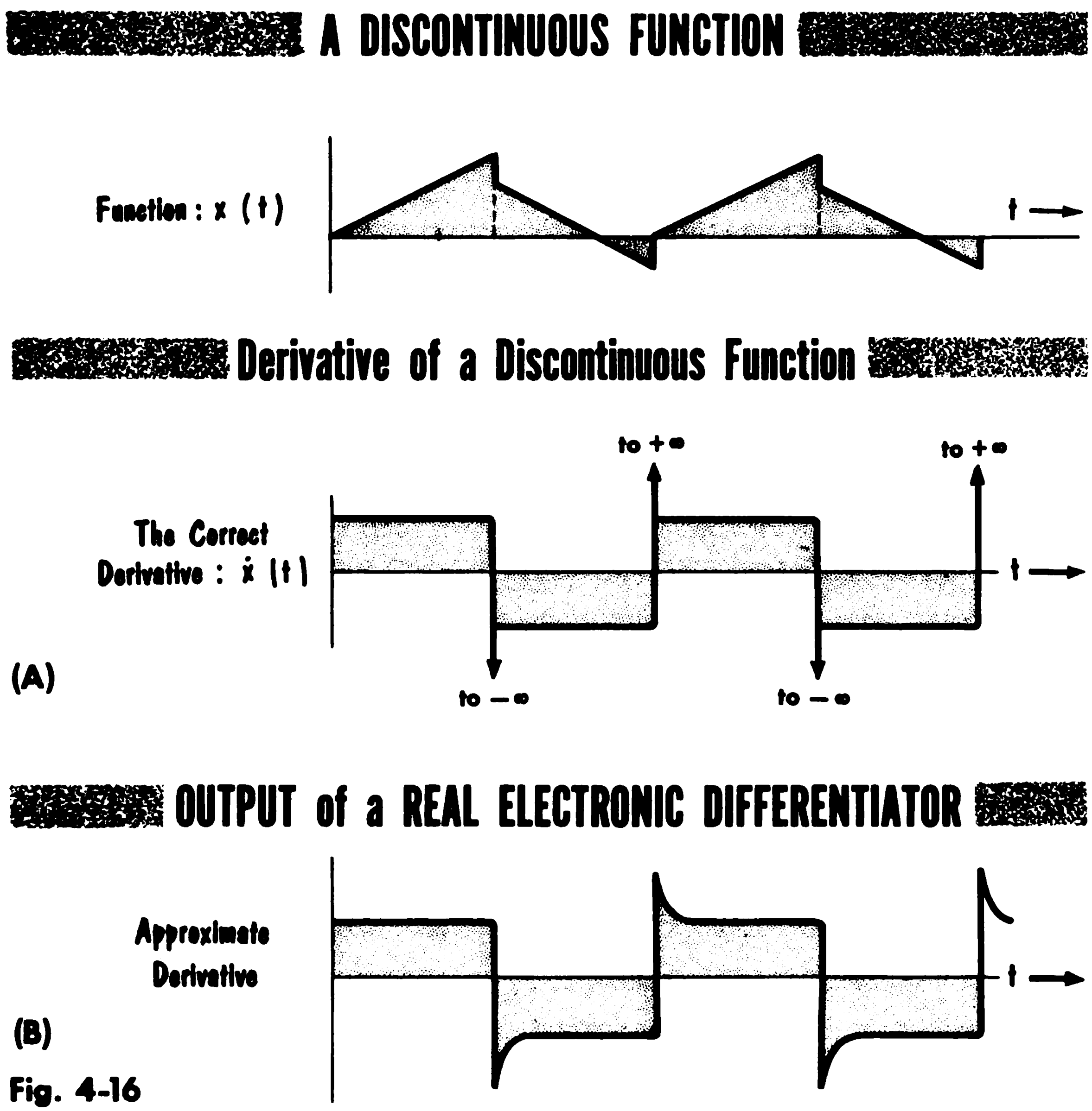


Fig. 4-16

Differentiation of Discontinuous Functions

Consider, however, how the above practical electronic differentiator would respond if *x(t)* had the form shown in Fig. 4-16 (A). In this example *x(t)* is known as a *discontinuous* function, and at the discontinuity the derivation of *x(t)* (or slope of the curve) is infinite. The infinite spikes are only theoretically possible and any real electronic differentiator would produce only an approximation to the proper derivative function [Fig. 4-16 (B)].

The foregoing discussion is intended to show that a good differentiating device is hard to come by, and that even if we had one we would be plagued by accentuation of noise. As a result, the mathematical operation of differentiation is not often performed on most analog computers.

You might reasonably ask, at this point, if the absence of a differentiator

does not place a serious limitation upon the scope of application of analog computing techniques. Surprisingly, perhaps, and fortunately, the answer is *definitely no*. Only rarely are we confronted with the need to find the derivative of a function. Almost invariably we know the derivatives and wish to find the function itself, and hence must integrate. Furthermore, since integration and differentiation are inverse operations, when differentiation is required it can be obtained with an integrator using the implicit techniques to be discussed later. This is similar to the method for performing division with a multiplying unit.

The reason that integration is a frequent operation in the analog simulation of physical systems is simply that the mathematical forms of the laws which describe much of the behavior of physical systems are obtained from considering differential motions. The laws are expressed by equations containing for the most part various order *derivatives* of one or more variables. If the laws are given in terms of derivatives then it is only necessary to integrate each derivative to find the variables themselves.

Notation for Derivatives

While discussing derivatives and differential equations it will be easier to identify differentiated quantities if we adopt a shorthand notation for the

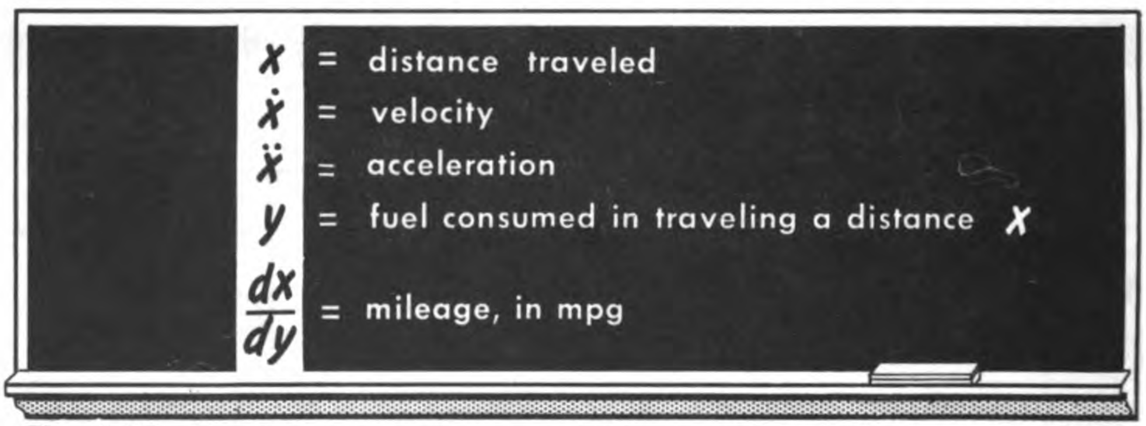


Fig. 4-17 ***Simple Notation for Derivatives***

first and second derivatives of a function. The ordinary notation used by mathematicians is that the first derivative of a function $x(t)$ with respect to time is indicated by $dx(t)/dt$, which in itself also is a function of time. The *second* time derivative of $x(t)$ is indicated by $d^2x(t)/dt^2$.

If x is a function of some other variable, then the derivative of x with respect to that variable is similarly indicated: for example, dx/dy, d^2x/dy^2, etc.

In this book we are interested primarily in functions of time and their time-derivatives. We will therefore adopt a simpler notation which is com-

monly used among engineers, whereby a dot over a variable will indicate the first derivative with respect to time of that variable. Two dots will indicate the second time derivative (Fig. 4-17).

Thus $\dot{x}(t)$ is the first time derivative of $x(t)$ [say "x dot"]

and $\ddot{x}(t)$ is the second time derivative of $x(t)$ [say "x double dot"]

DIFFERENTIAL EQUATIONS

The existence of physical devices and phenomena predates man's formulations of physical laws and their mathematical embodiment. Hence man's laws are said to *describe* (not govern) the behavior of the physical systems (which are actually *governed* by the absolute laws of nature, to which man's laws are approximations). Furthermore, the laws exist independently of the mathematics; the mathematical forms only describe the laws and are not laws in themselves. Hence the meanings of the laws are more important than their mathematical shorthand formulation. Also, it is more important to be familiar with typical forms of physical behavior than to be able to solve the complicated equations. Thus by studying the meanings of the fundamental physical laws and the types of behavior associated with each, one can obtain an understanding of scientific analysis of physical systems without any explicit mathematical training.

If the use of analog computers in scientific analysis is to be presented within a reasonable number of pages, however, it will be necessary to employ some of the mathematician's shorthand. It has been stated that the mathematical forms of the laws which describe much of the behavior of physical systems contain derivatives of the physical variables; thus they are differential equations. The solution of differential equations we shall leave to mathematicians and computers, but since they occur so frequently we must explore the meaning of the equations and the terms thereof. That is, we shall study the meaning of the physical laws through their mathematical embodiment in differential equations. For simplicity we shall restrict our attention to the example of linear motion of rigid bodies as described by Newton's force law: $F = M\ddot{x}$.

As simple as the Newton force law may appear, the degree of its universality is astounding. With a few words (or with only four symbols) Newton has propounded a relation between forces and accelerations which must hold for all rigid bodies known to, and subject to measurement by man, for all time — past, present, and future. Note that the law involves the rate of change of velocity of a body, with no mention of the actual velocity or of the trajectory of the body. The law states a *differential principle*— a relation between external forces and derivatives or rates of changes, but with no information about the integrals of those rates. Again, Newton was able to say that if we observe a moving rigid body the forces on the body from other objects, air friction, etc., maintain a precise relation to the instantaneous rate of change of the velocity of the body (Fig. 4-18A). On the other hand, had Newton attempted to establish an *integral principle*

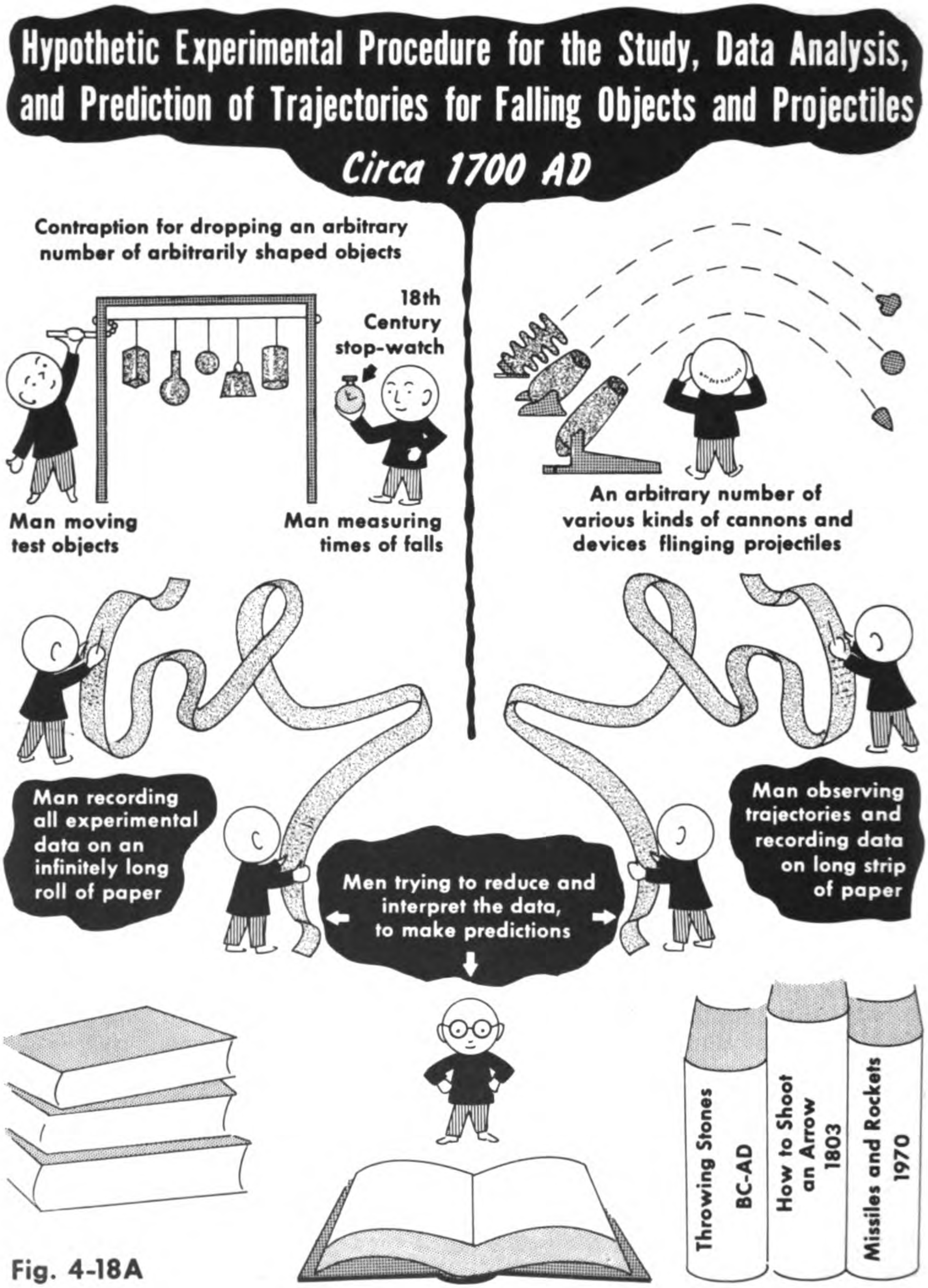

Hypothetic Experimental Procedure for the Study, Data Analysis, and Prediction of Trajectories for Falling Objects and Projectiles
Circa 1700 AD
Contraption for dropping an arbitrary number of arbitrarily shaped objects
18th Century stop-watch
Man moving test objects
Man measuring times of falls
An arbitrary number of various kinds of cannons and devices flinging projectiles
Man recording all experimental data on an infinitely long roll of paper
Men trying to reduce and interpret the data, to make predictions
Man observing trajectories and recording data on long strip of paper
Throwing Stones BC-AD
How to Shoot an Arrow 1803
Missiles and Rockets 1970

Fig. 4-18A

which would predict the actual trajectories of all moving bodies for all time he would surely have failed, even after the most exhaustive experimental study of falling apples and flying projectiles.

Given this far-reaching differential principle now, if one wishes to determine the velocity of or distance traveled by a projectile, he must first determine all the forces acting on the object. Since these forces may depend upon the motion of the projectile itself, they will ordinarily be expressed by terms in the equations which are more complicated than just a set of numbers. For example, the force of gravity on a missile decreases as the missile departs from the earth. If h is the height of the missile from the surface and R the radius of the earth then the gravitational force is proportional to:

$$\frac{1}{(h+R)^2}$$

As a second example, the air friction or viscous drag on a cannon ball is proportional to the square of the velocity of the ball.

Following the determination of the forces, the velocity and position are obtained by dividing by the mass and integrating once for velocity and twice for position. The integration is generally difficult, if not impossible, to perform with paper and pencil.

Note that Newton's force law equates $M\ddot{x}$ to the net sum of the external forces and that this means the "vector sum", or the algebraic sum, of forces

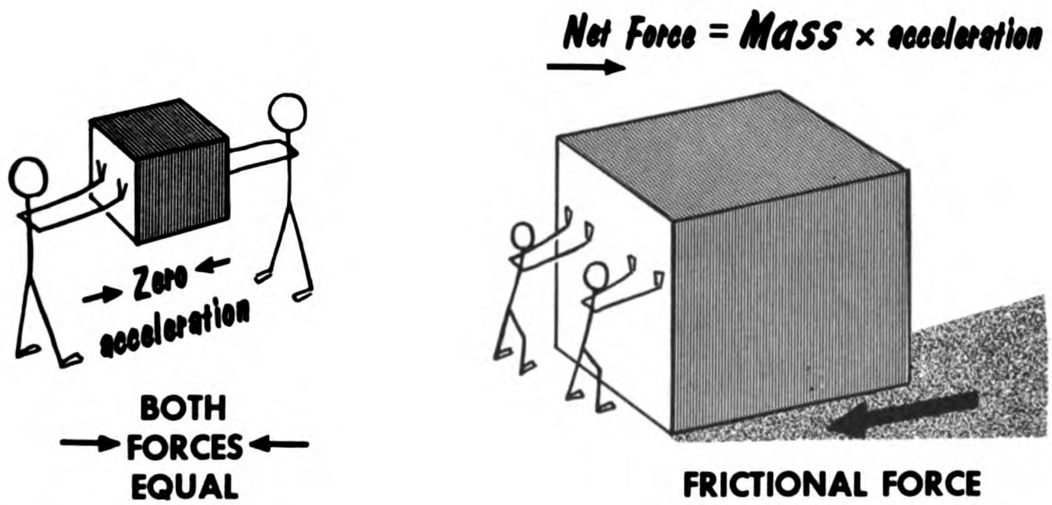


Fig. 4-18B

in the positive or negative *x direction*. If two applied forces are equal in magnitude but opposite in direction the net force is zero (Fig. 4-18B). On the other hand, forces in the same direction are added together to obtain the net force.

The mathematical shorthand for the algebraic summation of forces is illustrated (Fig. 4-19) by:

$$F_1 + F_2 + F_3 = Ma$$

$$M = \text{mass} = \text{weight} \div \text{gravitational acceleration} = \frac{w}{g}$$

$$a = \text{acceleration}$$

• ***Note***: F_3 is a negative number since it represents a force which acts in a direction opposite to the direction of the forces F_1 and F_2.

The equation may be written:

$$F_1 + F_2 + F_3 = M\ddot{x} \qquad (4-1)$$

where

x = distance traveled or displacement
$\dot{x}$ = velocity
$\ddot{x}$ = a, the acceleration

F_1 and F_2 are externally applied forces. F_3 is a frictional force which results from the object sliding in contact with another surface. $\ddot{x}$ is the acceleration of the mass.

This is an equation containing a second order derivative. Hence it is a differential equation of the second order. *The major use of general purpose analog computers is in the solution of differential equations.* This statement is not in conflict with earlier remarks concerning the simulation of analogous physical systems. The reason is that, for any physical system capable of at least momentary storage of energy (in the inertia of masses, compression of springs, magnetic fields of coils or electrostatic fields between plates of a capacitor), the dynamic behavior of that system is described by physical laws which in mathematical terms are differential equations.

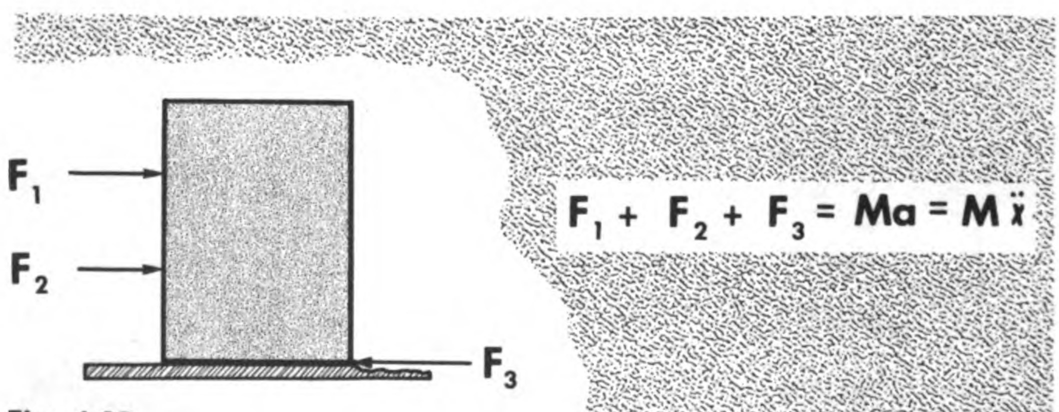


Fig. 4-19

Consider for example, Fig. 4-20A. Assume that only a small frictional force exists between the table and the metal block and that the magnitude of this force is proportional to the velocity of the block, and its direction opposite to the motion of the block. Next, we assume that the spring is

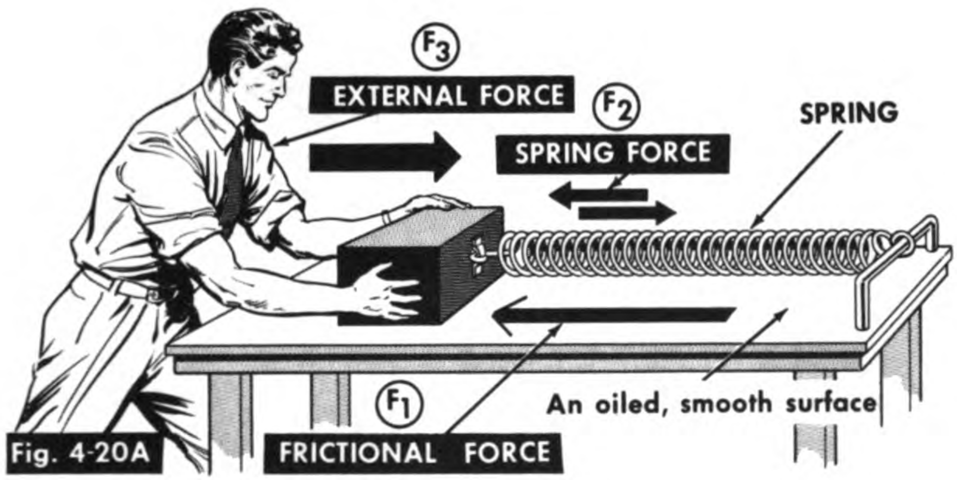


Fig. 4-20A

fixed at the right end and has the experimentally-determined property that the force required to compress or extend the spring is proportional to the amount of compression or extension: the compression or extension of the

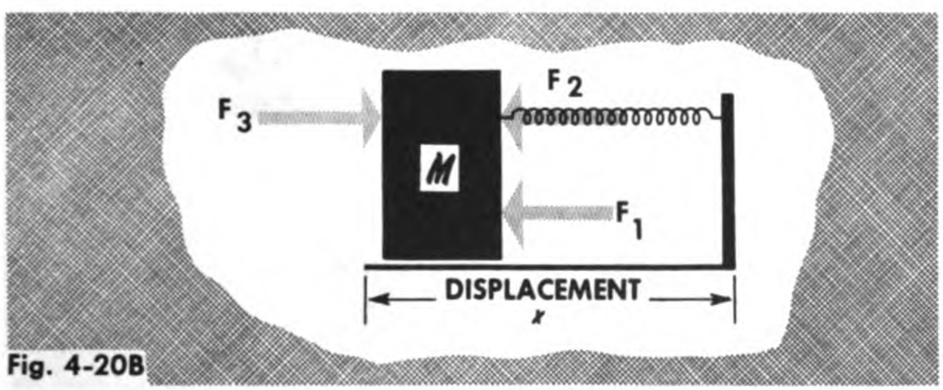


Fig. 4-20B

spring equals the movement of the block to right or left from the position for which the spring is relaxed (Fig. 4-20B).

Let x be the distance moved by the block to the right from its position in an initially relaxed situation. Then,

Frictional force is proportional to the velocity:

$$F_1 = -D\dot{x}$$

D is frictional constant of proportionality
Minus sign indicates F_1 opposes the motion (if motion is to right, force is to the left)

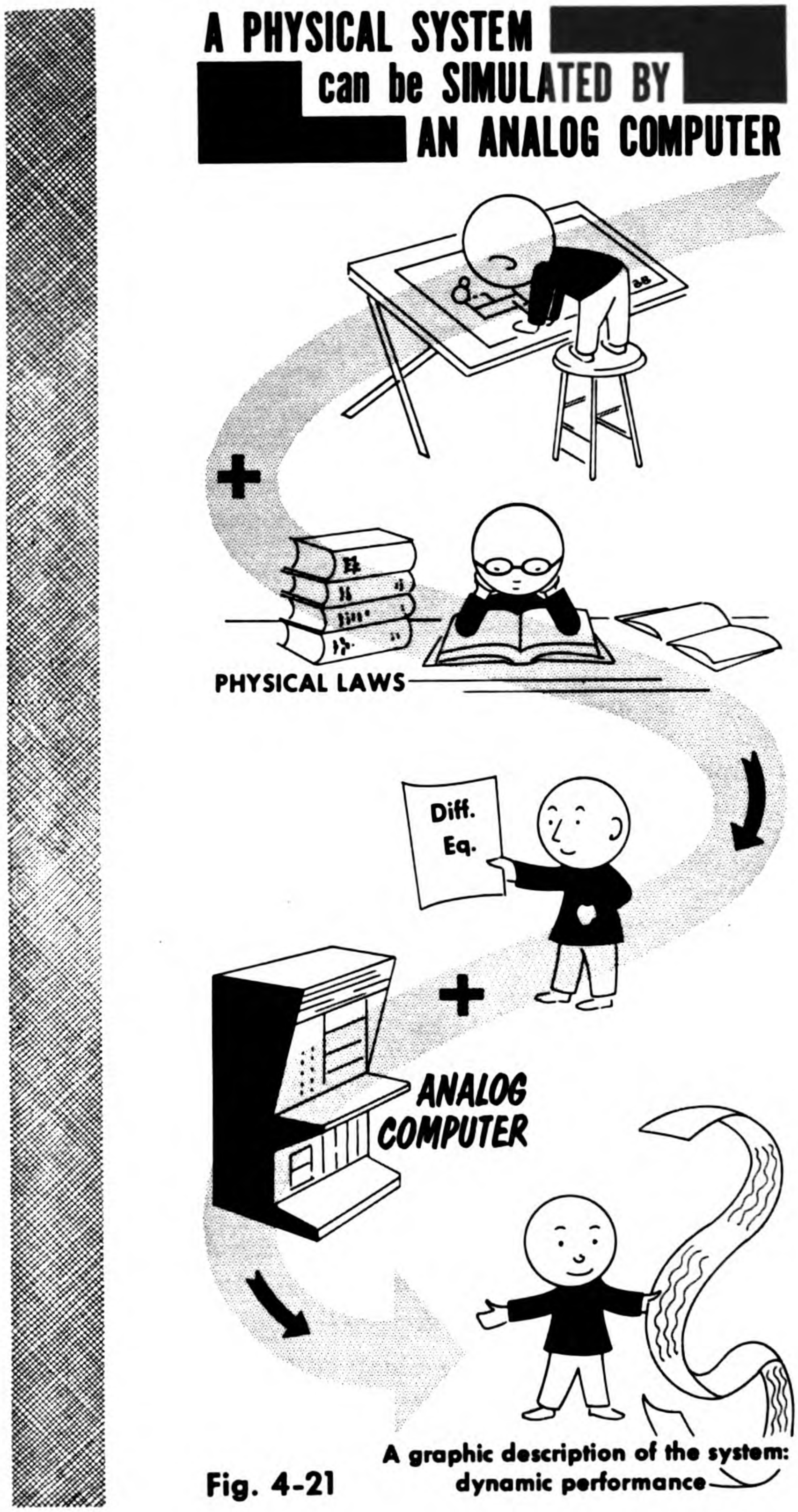

A PHYSICAL SYSTEM
can be SIMULATED BY
AN ANALOG COMPUTER
PHYSICAL LAWS
Diff.
Eq.
ANALOG
COMPUTER
A graphic description of the system:
dynamic performance

Fig. 4-21

Spring force is proportional to the compression or extension:

$$F_2 = -Kx$$

K is the spring constant of proportionality
Minus sign indicates:

if x is positive (compression) F_2 is to the left
if x is negative (extension) F_2 is to the right

Applied force = F_3 (determined by experimenter)

Now using Newton's law (summing the forces on the *block* only)

$$F_1 + F_2 + F_3 = M\ddot{x}$$

or

$$-D\dot{x} - Kx + F_3 = M\ddot{x}$$

or rearranging terms:

$$\begin{array}{rl} M\ddot{x} & \text{(inertial force)} \\ +D\dot{x} & \text{(frictional force)} \\ +Kx & \text{(spring force)} \\ =F_3 & \text{(applied force)} \end{array}$$

How to Simulate a Physical System with an Analog Computer

The simulation of even the most complicated physical systems proceeds in much the same way as above. That is:

1. Given a physical system, identify each element (M) and parameter (K, D) and assign coordinates (x) and positive directions.
2. Apply physical laws (Newton's, Kirchhoff's, conservation of energy, conservation of momentum, conservation of angular momentum, etc.), Write these laws in mathematical notation in terms of parameters and coordinates of the system (K, M, D, x and t) (Fig. 4-21).
3. This yields one or more differential equations. The solving of these equations (say, for x as a function time) represents, without a computer, probably the most difficult task of all, but with a computer, in most cases the easiest task. Indeed many such sets of differential equations can, for all practical purposes, *only* be solved on a computer, i.e., the labor required for hand solution would be prohibitive. The way a computer solves a differential equation will be shown later; essentially, it is simply a matter of interconnecting analog building blocks to form all the terms of the equations and to integrate each derivative. The solution is usually recorded as a time history of values plotted on graph paper.

Simulation of a Double Mass and Spring System

Earlier we wrote the equation for a single mass and spring system. That equation had the same form as the two equations given on p. 110 for the double mass and spring system (Fig. 4-22A).

Simulation of a Double Mass and Spring System

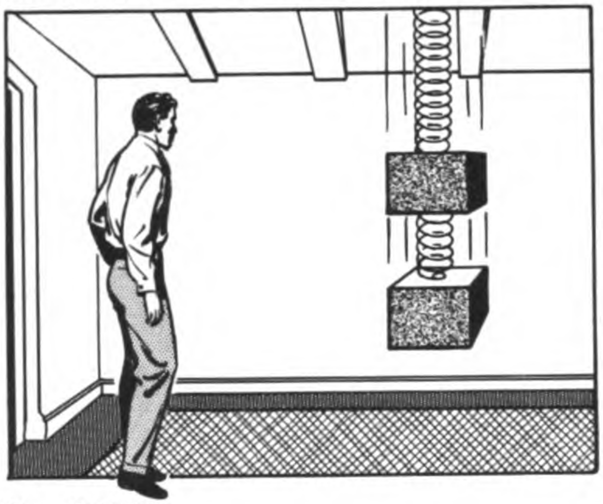


Fig. 4-22A

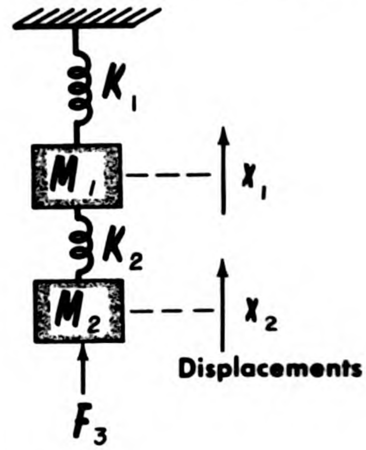


Fig. 4-22B

x_1 and x_2 are the displacements of the masses measured from their position when the system is at rest (Fig. 4-22B).

D_1 and D_2 are coefficients of viscous friction (air friction)

K_1 and K_2 are spring constants of proportionality

$$M_1 = \frac{w_1}{g}$$

$$M_2 = \frac{w_2}{g}$$

where w_1 and w_2 are weights, and g = gravitational acceleration.

Differential equations:

$M_1\ddot{x}_1$	$+$	$D_1\dot{x}_1$	$+$	$(K_1 + K_2)x_1$	$-$	K_2x_2	$=$	0
$M_2\ddot{x}_2$	$+$	$D_2\dot{x}_2$	$+$	K_2x_2	$-$	K_2x_1	$=$	F_3
(inertial forces)		(damping forces)		(spring forces)		(coupling forces)		(externally applied forces)

Notice that x_1 and x_2 appear in both equations. These are called *coupled equations* or a pair of *simultaneous equations*. They must be solved simultaneously. The analog computer can solve them simultaneously with ease.

There is a one-to-one correspondence between the terms of a differential equation and the elements of the physical system described, and between these and the building blocks of a computer program, which is shown on pp. 3-46 [see Fig. 2-7 (A)], and 3-47, for these particular equations.

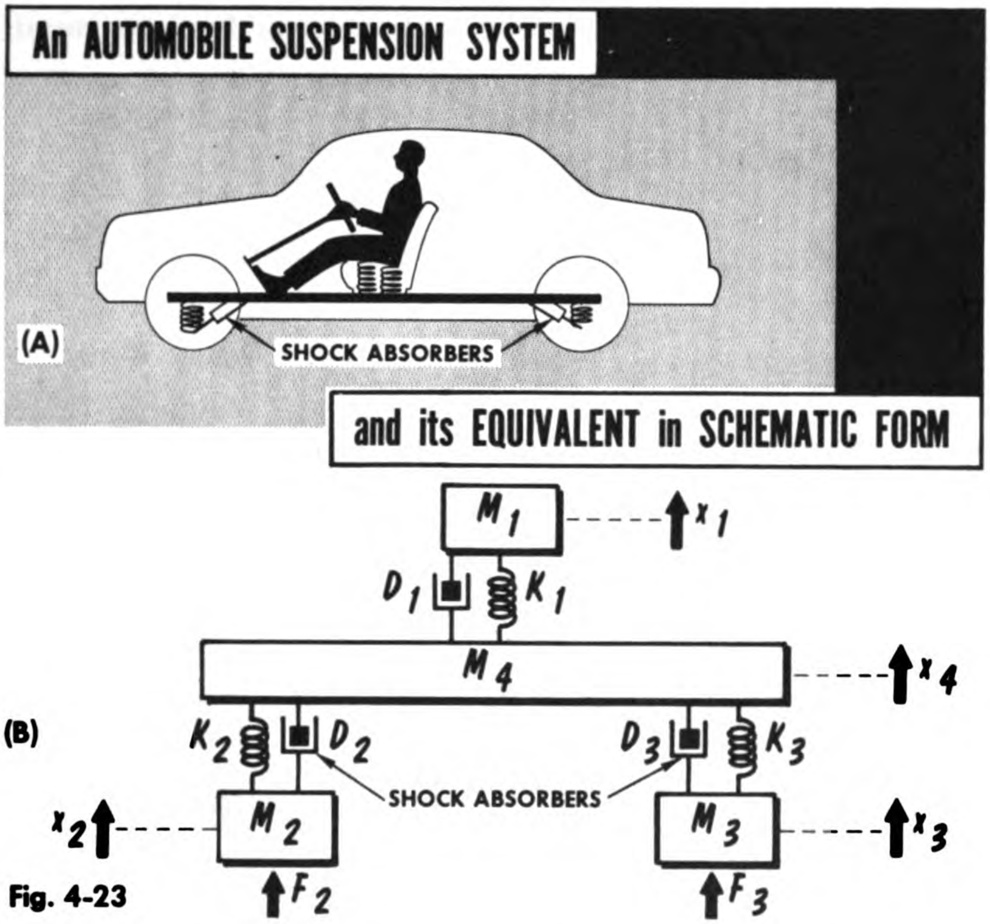


Fig. 4-23

Simulation of an Automobile Suspension System (Simplified)

[See Fig. 4-23 (A) and the equivalent diagram, Fig. 4-23 (B), of an automobile suspension system.]

M_1 = mass of passenger and seat
M_2, M_3 = mass of wheel assemblies
M_4 = mass of chassis

K_1 = seat spring constant
K_2, K_3 = front and back spring constants

D_1 = seat damping constant
D_2, D_3 = front and back damping constants (shock absorbers)

F_2, F_3 = road forces acting through wheels

x_1 = vertical displacement of passenger
x_2, x_3 = wheel displacements
x_4 = vertical displacement of chassis

If we assume that there is no pitching motion, then the differential equations are:

$$
\begin{array}{llllll}
M_1\ddot{x}_1 & + \; D_1\dot{x}_1 & + \; K_1x_1 & - D_1\dot{x}_4 - K_1x_4 & = & 0 \\
M_2\ddot{x}_2 & + \; D_2\dot{x}_2 & + \; K_2x_2 & - D_2\dot{x}_4 - K_2x_4 & = & F_2 \\
M_3\ddot{x}_3 & + \; D_3\dot{x}_3 & + \; K_3x_3 & - D_3\dot{x}_4 - K_3x_4 & = & F_3 \\
M_4\ddot{x}_4 & + \; (D_1 + D_2 + D_3)\dot{x}_4 & + \; (K_1 + K_2 + K_3)x_4 & - D_1\dot{x}_1 - K_1x_1 & & \\
 & & & - D_2\dot{x}_2 - K_2x_2 & & \\
 & & & - D_3\dot{x}_3 - K_3x_3 & = & 0 \\
\text{(inertial forces)} & \text{(damping forces)} & \text{(spring forces)} & \text{(coupling forces)} & & \text{(externally applied forces)}
\end{array}
$$

An analog computer study of such an automobile suspension system would most likely be used to determine values for the spring and shock absorber

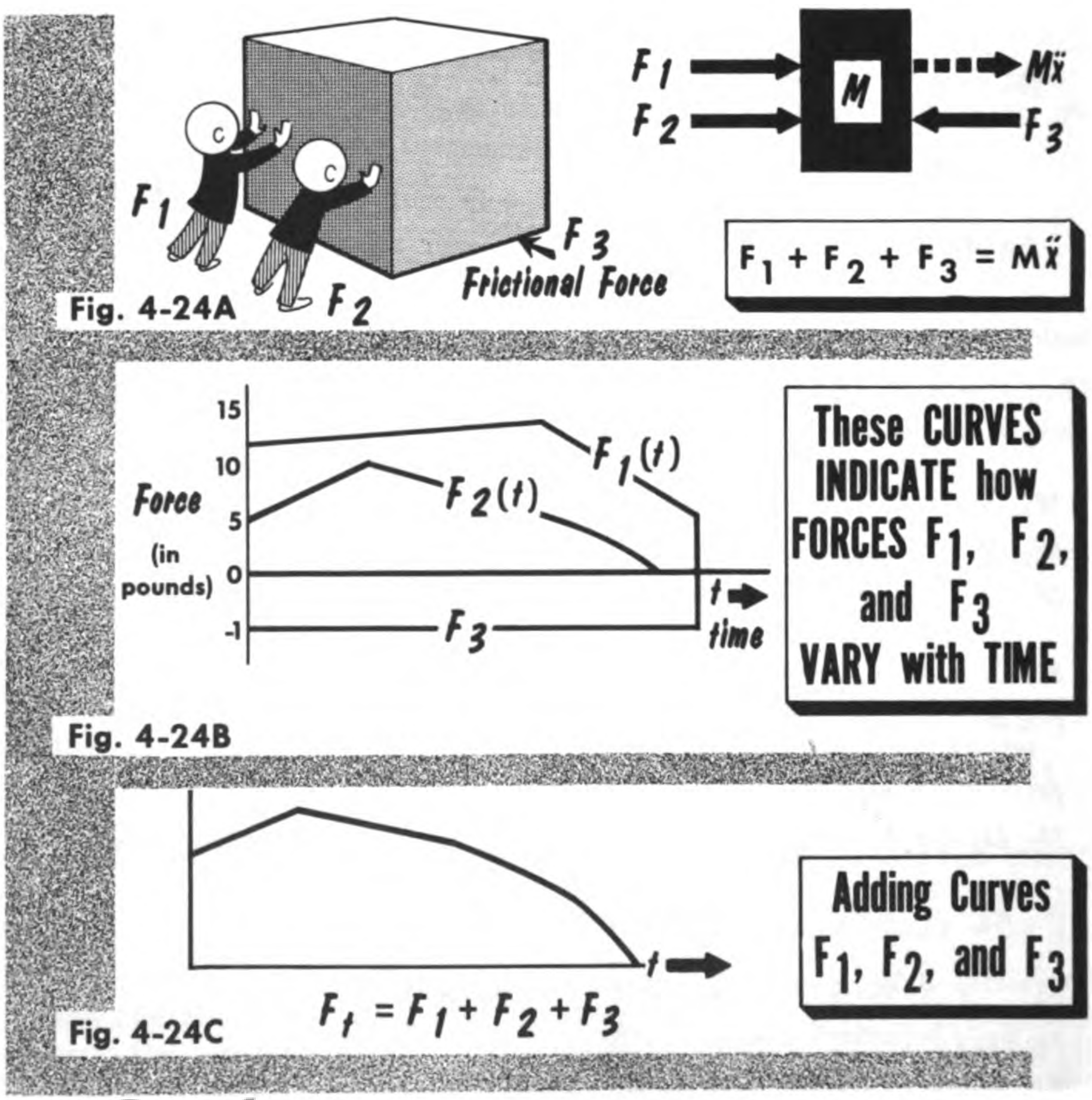


Fig. 4-24A

Fig. 4-24B

Fig. 4-24C

parameters. In particular it would be desired to find values which minimize the motion of the passengers and chassis ($\dot{x}_1$ and $\dot{x}_4$) for various road conditions (simulated by varying F_2 and F_3).

The Simplest Differential Equation: A Definite Integral

Consider next, differential equation (4-1) from p. 106 (Fig. 4-24A) repeated below:

$$F_1 + F_2 + F_3 = M\ddot{x} \qquad [4\text{-}1]$$

where

x = distance traveled or displacement
$\dot{x}$ = velocity
$\ddot{x}$ = a, the acceleration

Suppose we know precisely how F_1, F_2 and F_3 vary with time, that is, F_3 is a known negative constant (as long as the mass is moving to the right) and graphs of F_1 and F_2 are as given in Fig. 4-24B.

Adding these three curves graphically gives F_T (Fig. 4-24C).

Now $\ddot{x} = F_T/M$, and if we are interested in knowing how fast the block moves, as a function time, we can integrate the acceleration in order to obtain the velocity $\dot{x}$:

$$\dot{x}(t) = \int_o^t \ddot{x}dt = \int_o^t \frac{F_T}{M} dt$$

assuming that the initial velocity is zero.

Thus the solution of differential equation (4-1) for velocity requires only the integration of a definite integral. The position, x, of the block as a

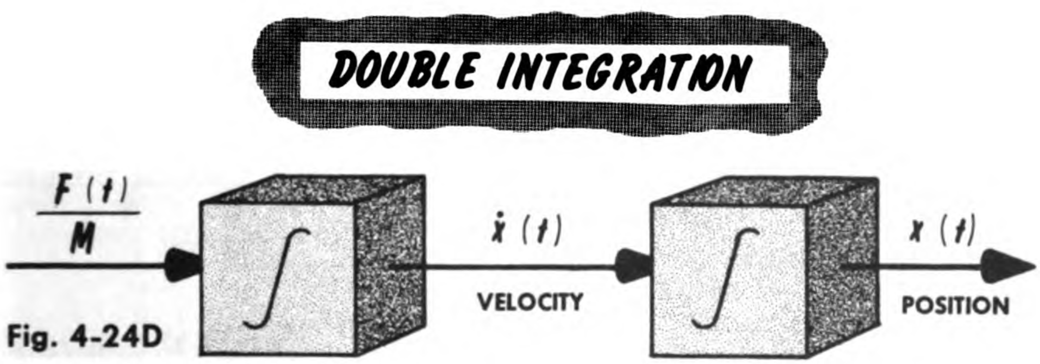


Fig. 4-24D

function of time is determined by further integration of the velocity function $\dot{x}$. Now assuming we could provide a shaft rotation or voltage which changed in a manner analogous to F_T, this computer variable could be fed into computer integrators as shown in Fig. 4-24D.

It is important that we distinguish between differential equations and the situation on p. 106 which simply requires the integration of two definite integrals. In the next example of a differential equation we will see that the analog diagram shows a connection from the output of both integrating devices back to the input of the first integrator. Such connections are characteristic of the flow diagrams for differential equations, and do not occur for definite integrals as above. Obviously this is because in the above case the derivatives are not functions of the variable itself.

A Second Order System

Suppose we rewrite the differential equation corresponding to the system shown in Fig. 4-25 (A):

$$M\ddot{x} + D\dot{x} + Kx = F_3 \qquad (4\text{-}2)$$

solving as before for $\ddot{x}$. That is:

$$\ddot{x} = -\frac{D\dot{x}}{M} - \frac{Kx}{M} + \frac{F_3}{M}$$

But now it appears that we cannot follow the previous procedures, adding up everything on the right-hand side and integrating it once to get the velocity, for we do not know the values of x and $\dot{x}$ until we integrate, and it appears we cannot integrate until we know what to integrate!

It turns out however, that both jobs can be done at the same time in such a way that the equation is solved for both $x(t)$ and $\dot{x}(t)$. This will be discussed in more detail when we consider implicit functions and feedback. But the solution by computer building blocks is best illustrated as follows. Assume you know $\ddot{x}$, and follow the path around the loop in Fig. 4-25 (B).

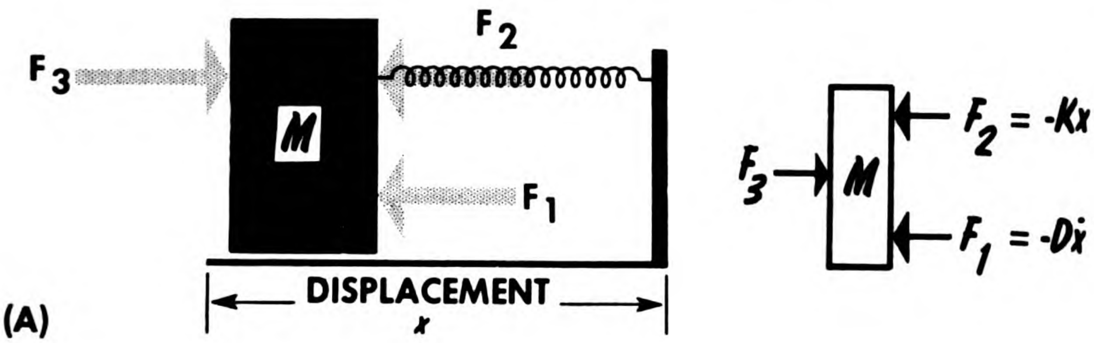
These two examples, equations (4-1) and (4-2), should illustrate the marked difference between *integration of a known function* and *integration of a differential equation.* In fact, we shall no longer consider equation (4-1) on p. 106 a differential equation, nor any equation that contains only one derivative of x, with all the other terms, known quantities. These are merely exercises in integration.

Solution of, or integration of a differential equation requires integration of terms which result from the integration itself!

Initial Values of the Integrated Function

We noted earlier that an automobile odometer registers the integral of the speed of the vehicle, or the distance traveled. Furthermore, we know that if we substract the initial from the final reading after a trip we have the integral of speed over that trip. This is expressed mathematically as

$$x(t) - x(o) = \int_o^t v(t)dt$$



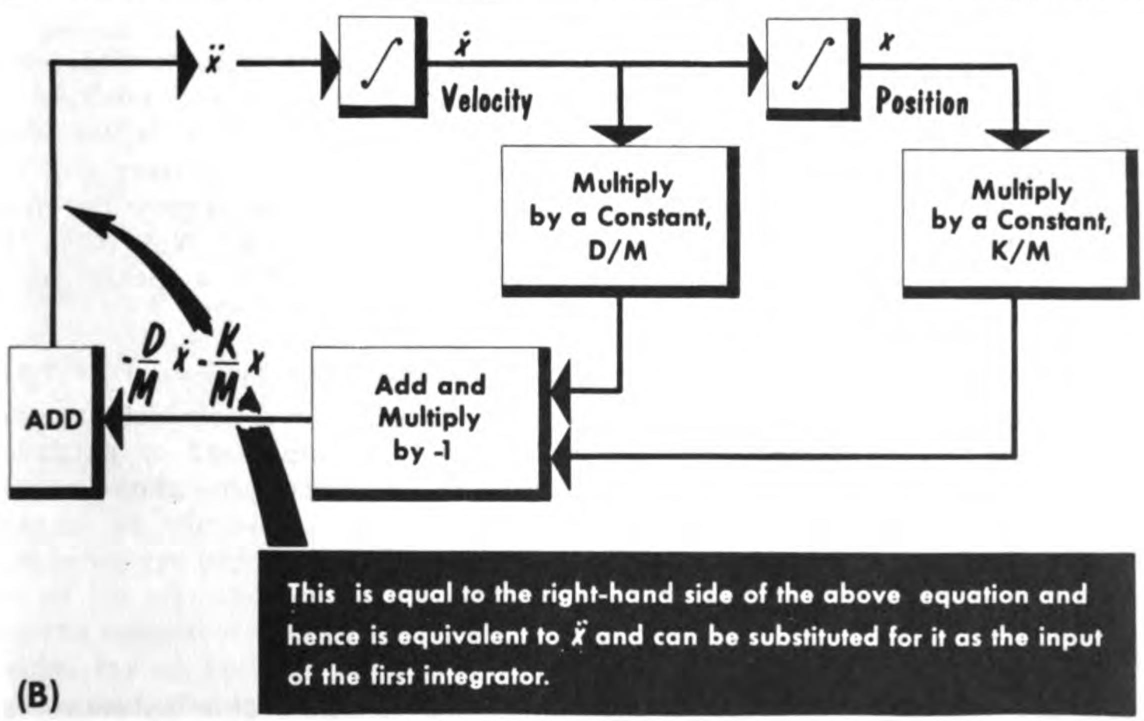
(A)

A Second Order System

Write the Equation:

$$\ddot{x} = -\frac{D}{M}\dot{x} - \frac{K}{M}x + F_3$$

ANALOG COMPUTER BUILDING BLOCKS



(B)

Fig. 4-25

where $v(t)$ is the velocity function and $x(o)$ is the initial reading of the mileage indicator (since the trip is assumed to begin at $t = 0$). The latter term is known as the *initial value* of the integral. Although sometimes the initial value is zero as when the car is new, it is always necessary to account for the initial value of an integral whether the integral occurs in a definite integral or in a differential equation.

For example, in the situation discussed on pp. 105–106 where the two men were pushing a box, the force functions and Newton's second law com-

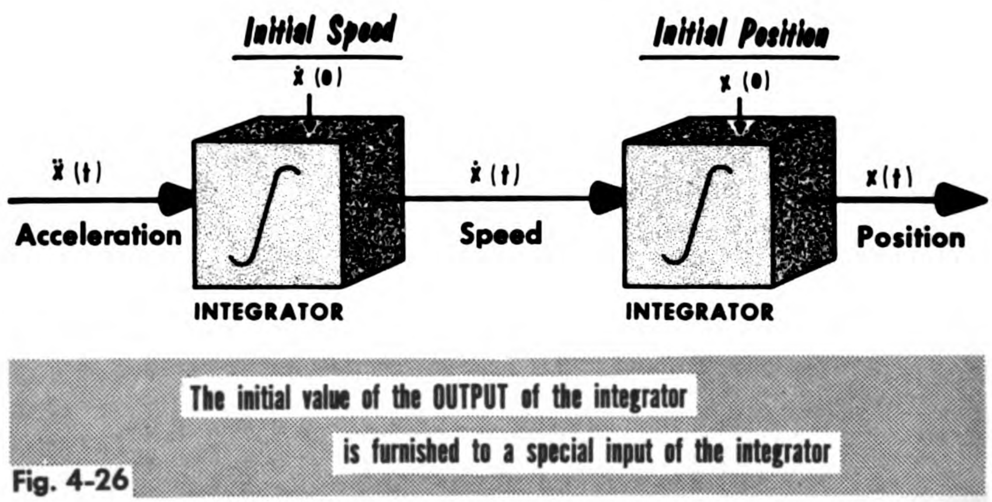


Fig. 4-26

pletely specify the *form* of the velocity and position functions. However, it is evident that we could not know the exact velocity and position at time, t, unless we know the *initial velocity and position*. That is, we must know the initial values of the two integrals, and our integrating devices must have provision for accepting the initial values and presenting them at the output at the beginning of the problem (Fig. 4-26). Note that the initial value of the *output* of the integrator is furnished to a special input of the integrator.

As a further illustration, consider briefly differential Eq. (4-2). Again the initial values $\dot{x}(o)$ and $x(o)$ are required. Clearly for a complete answer it is necessary to specify whether the spring is compressed or extended before the mass is released, and if the mass is released with a shove or not.

INTEGRATORS

Resistor-Capacitor Integration

We have noted that a mechanical integrator must be able to retain an accumulated count or an accumulated shaft or rod displacement. Similarly, a crude hydraulic integrator was suggested for which storage of liquid was required. An electronic integrator must also provide some storage capacity, and this is possible with inductors and capacitors. An in-

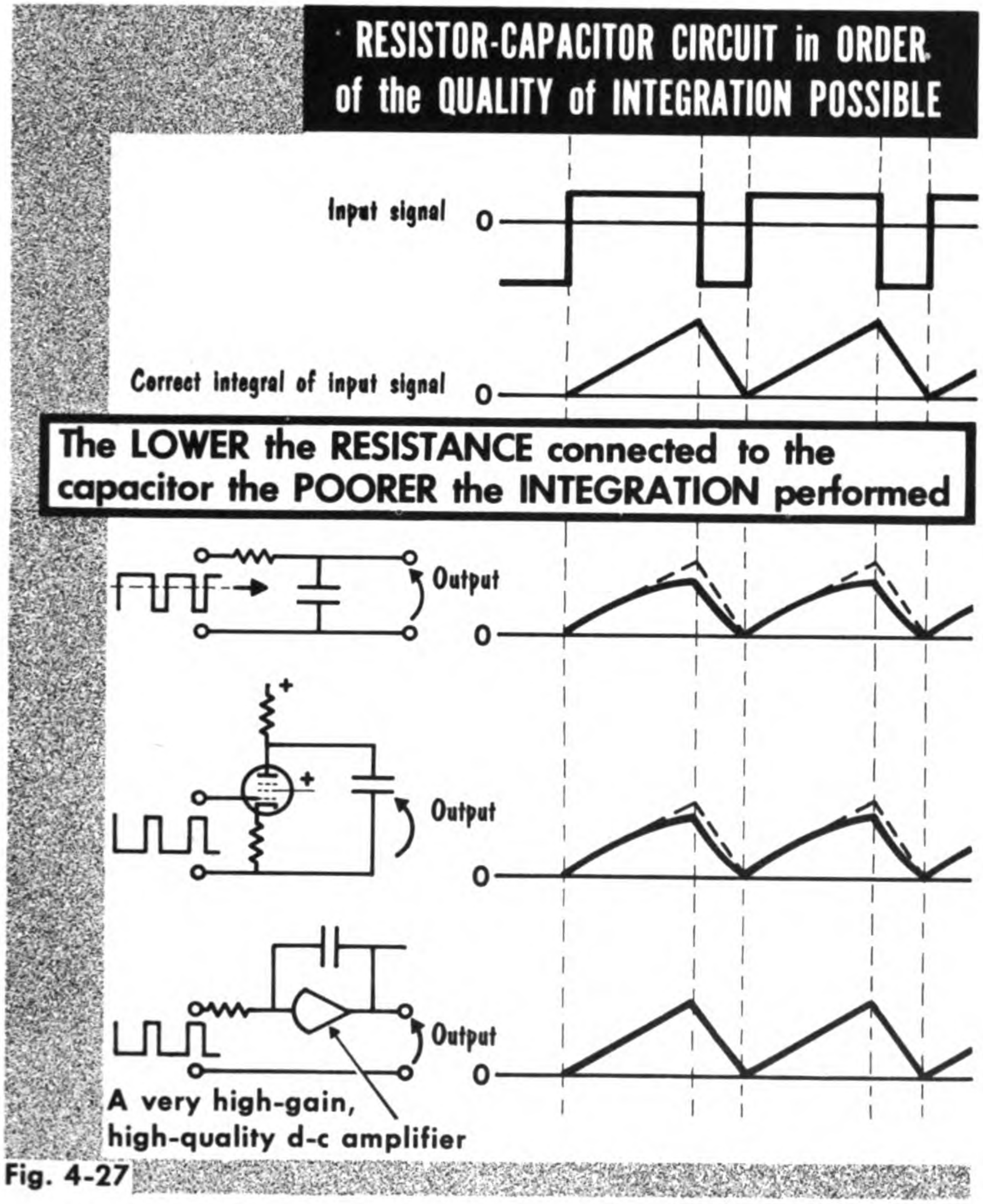


Fig. 4-27

ductor stores magnetic flux and passes a current proportional to the integral of the voltage across its terminals. The capacitor, however, is used more frequently. A capacitor stores electric charge and offers a voltage at its terminals proportional to the integral of the current flowing into the capacitor — like a storage tank for charge. The integration is quite precise for an *isolated* capacitor, but clearly if current is to be fed to the circuit and the voltage is to be detected as an output, the capacitor is necessarily connected to some associated circuitry. The lower the resistance connected to the capacitor the poorer is the integration performed. Figure 4-27 illustrates several resistance-capacitance circuits in order of the quality of integration possible.

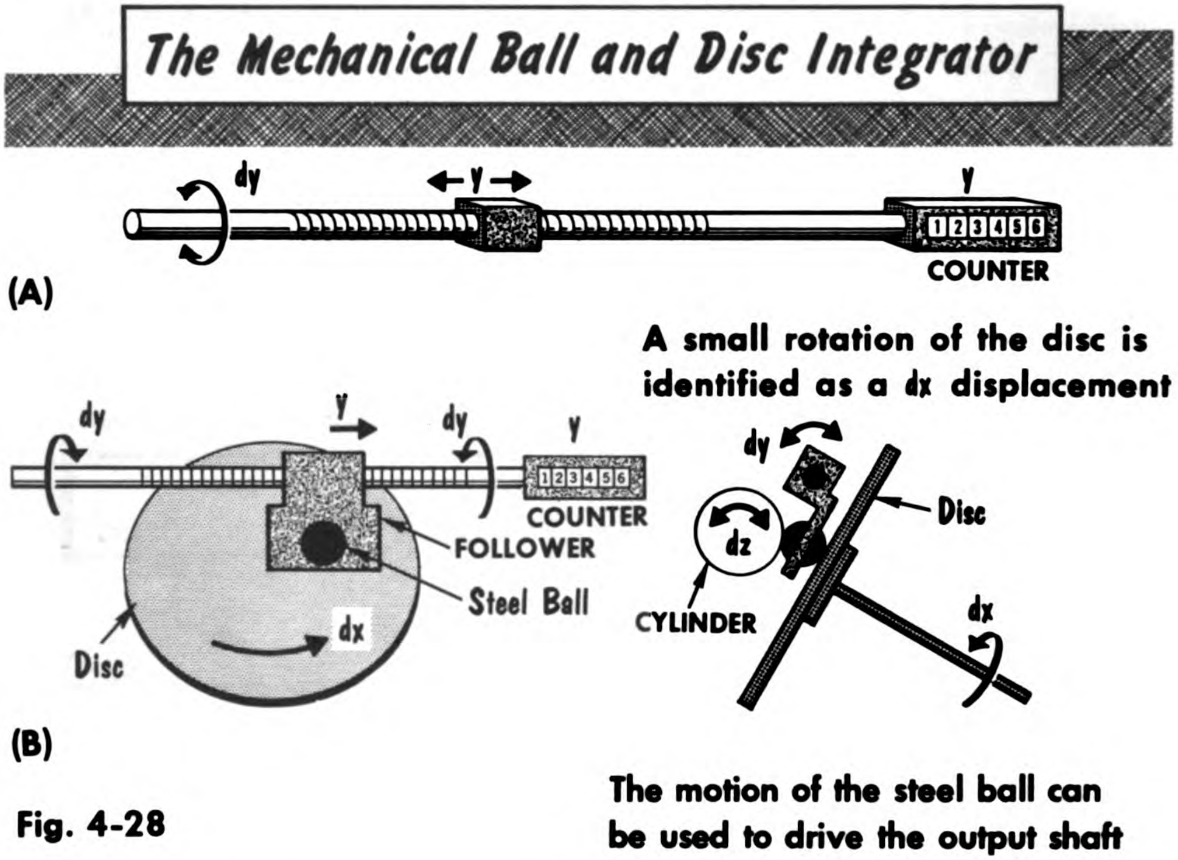


Fig. 4-28

The last device is precisely that used for integration in a general purpose d-c analog computer. In each case an initial value of the integral can be inserted by placing an initial charge on the capacitor.

The Mechanical Ball and Disc Integrator

In a mechanical computer where many or all computer variables have the form of shaft rotations, a small change of position of a shaft is termed an *incremental displacement* or *differential displacement*. If the revolutions counter on a particular shaft is identified with the variable y, then a differential change in y is called dy. For convenience, we may identify a single revolution of the shaft with a unit of dy. When the number y is very large, one revolution is, by comparison, a differential or very small quantity. Notice that this is consistent with our notation for integration, for y is the sum or the count of all revolutions and thus the integral of dy, that is:

$$y = \int dy$$

Now in addition to a revolution counter, let us equip the y shaft with a thread or lead screw, and a nut or follower that moves back and forth as the y shaft rotates [Fig. 4-28 (A)]. If the follower is started in the middle when the counter reads zero, then at any time later the distance between follower and the center is proportional to the count, and therefore to y.

Let the follower hold a small cage that retains a steel ball that is further supported by an inclined disc [Fig. 4-28 (B)]. The disc is driven by the x shaft. A small rotation of the disc is identified as a dx displacement. When the disc revolves the steel ball is free to roll. If the y follower is in the center, the ball is at the center of the disc and will not roll even when the disc revolves. When y is at the right end of its allowed travel, the ball rolls rapidly in one direction, and similarly in the other direction when the y follower is at the other side of the plate. The motion of the ball can be used to drive another shaft which is the output shaft, and is identified as the z shaft. The z shaft must not drive a very heavy load, and good traction is required to avoid slipping. The differential displacement dz is proportional to the product of the y position on the disc and the dx motion. That is $dz = Kydx$ (K is a constant of proportionality). For convenience, let us assume $K = 1$. Then the z revolutions counter will indicate the integrated quantity, $z = \int ydx$.

The Wheel and Disc Integrator

The ball and disc integrator uses a relatively heavy steel ball, rolling on a steel plate, to maintain adequate traction with the disc and the cylinder on the output shaft. However, the size of the ball makes it impossible to position the ball very accurately. To remedy this, it is possible to replace

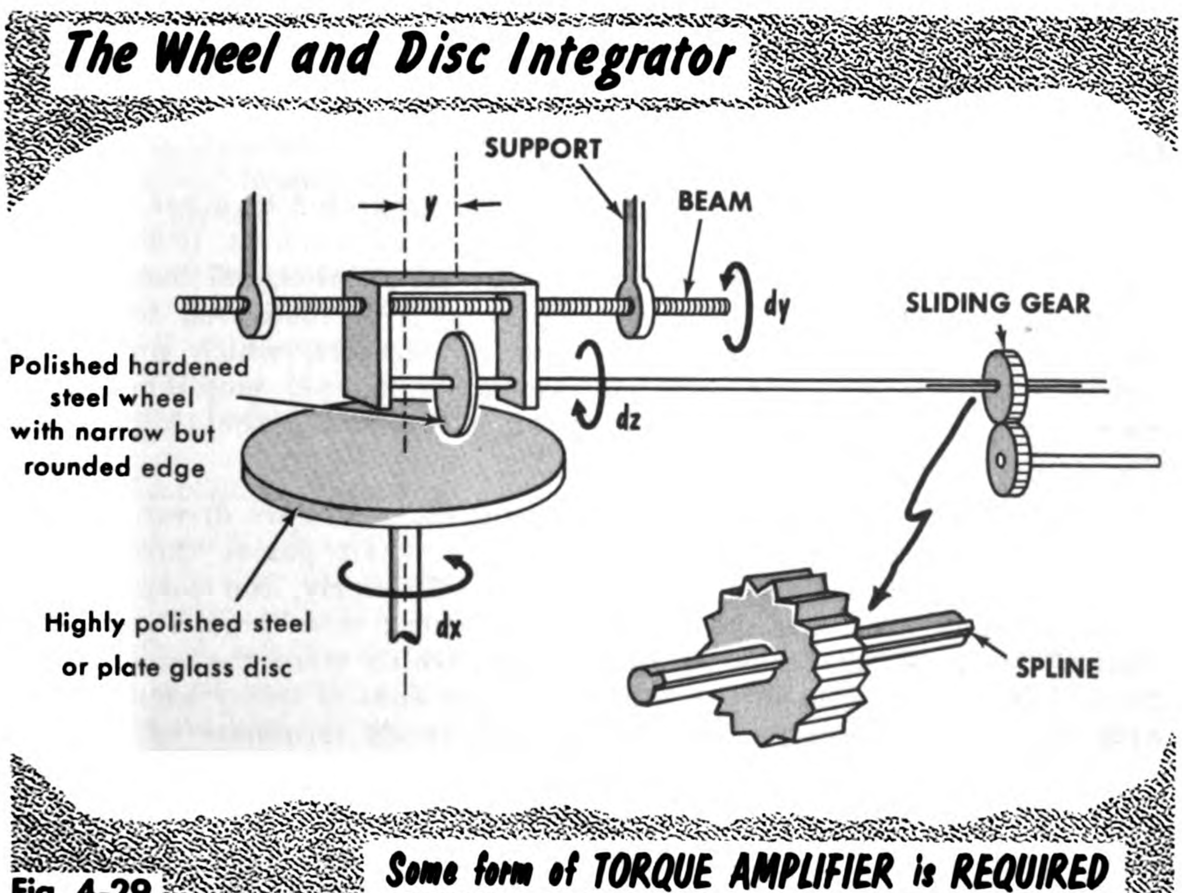


Fig. 4-29

the ball with a small, thin wheel mounted on the y follower. The axis of the wheel is the z or output shaft. The y follower moves the wheel and its shaft, therefore the output connection requires a gear which slides along the splined z shaft. Now if the wheel has a sharp edge it is evident that its position, or y, can be determined to a high degree of accuracy. However, it is also obvious that the wheel and disc device has the disadvantage of requiring *low* sliding friction as the y follower moves back and forth, while avoiding any slip in the other direction. The low sliding friction is accomplished by using a polished, hardened steel wheel with a narrow but slightly rounded edge, in conjunction with a highly polished steel or plate glass disc.

Any connection to the z shaft will cause a torque to be exerted on the small wheel. This loading torque may cause slipping, and thereby destroy the validity of the computation. Loading may be reduced by using step-up gear ratios on the output, but with the sharp edge wheel this is usually not adequate to prevent all slipping, and some form of a *torque amplifier* is required (Fig. 4-29).

Although the principle of the wheel and disc integrator was known to scientists in the earlier 1800's, the lack of an adequate torque amplifier precluded its use until about 1930.

Initial values for the integrals are established in both the ball and disc integrator and the wheel and disc integrator, firstly, by choosing a suitable initial position for the y follower (or the ball or wheel), and secondly, by inserting the desired count in the z shaft revolutions counter.

Torque Amplifiers

For a typical wheel and disc integrator to be connected to other mechanical building blocks, a torque amplification of as much as 10,000 is required. While the z shaft should have only an infinitesimal load placed upon it, the signal from the z shaft (dz) must be delivered to shafts, differential gears, lead screws and other integrators, which produce a considerable restraining torque. A mechanical torque amplifier which was used most often with the integrators built during the 1930's, is of the double-capstan type (Fig. 4-30).

Two identical drums each with two different diameters are driven at high speed in opposite directions. The z or input shaft passes through the center of one and is connected to a T bar. Similarly, the output shaft passes through the other drum, and is connected to a large T bar. The input T bar holds the ends of light strings which wrap the small diameters of the drums. The output T bar holds the ends of heavy bands which wrap the large diameters. The strings and bands terminate on a third center T bar. Tension is adjusted so that all bars are motionless when there is no change in the z shaft.

A displacement of the z shaft moves the input T bar which tightens one string and loosens the other, which in turn causes the center T bar to

PRINCIPLE of the CAPSTAN TORQUE AMPLIFIER

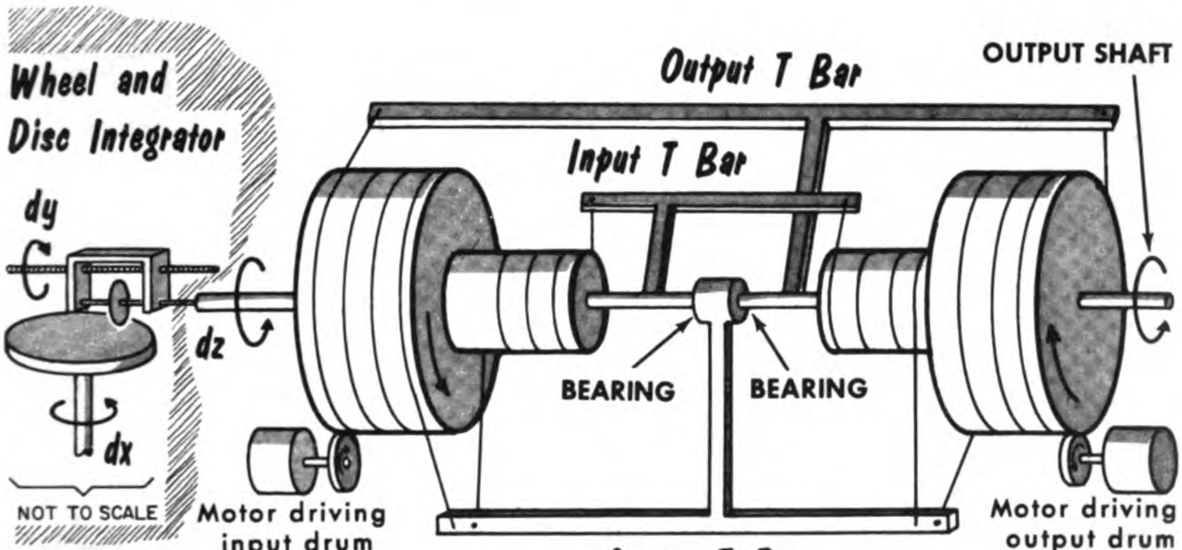


Fig. 4-30

move with the input T bar. Motion of the center T bar tightens one of the bands and loosens the other, resulting in motion of the output T bar and shaft identical to that of the input shaft, T bar, and center T bar.

Since the output shaft is driven by the drum motors through the friction of the heavy bands, the device is capable of delivering a very large torque for a small input torque. Hence it is called a torque amplifier. The amplifier illustrated is really a two-stage amplifier. The same principle applies when using drums having only one diameter.

Torque Amplifiers: Electric

Instead of using drums, belts, and T bars, adequate torque could be obtained by driving the output shaft directly with a high-torque electric motor, if only we had an electric signal proportional to the changes in the z shaft. A number of techniques have been used to obtain such a voltage. The most ingenious and popular method utilizes an optical system, and places no load whatsoever on the z shaft (Fig. 4-31A).

An extra wheel is placed on the z shaft. It is made of strongly polarized glass and is used to pass a light beam which, due to the direction of its polarization, contains all the information about the z shaft position necessary to drive the torque motor. The beam passes through two fixed pola-

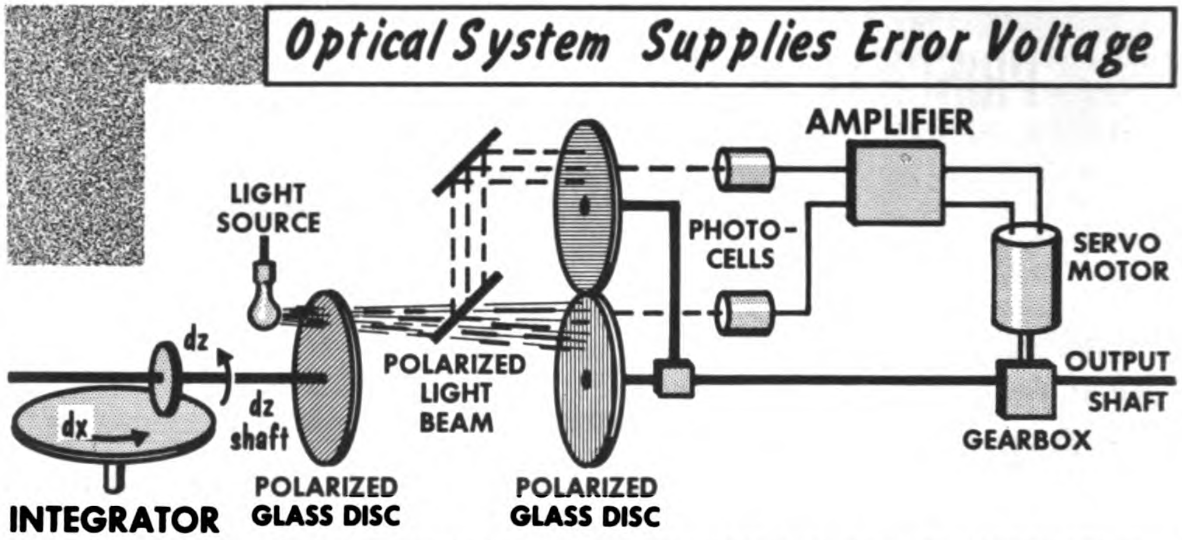


Fig. 4-31A

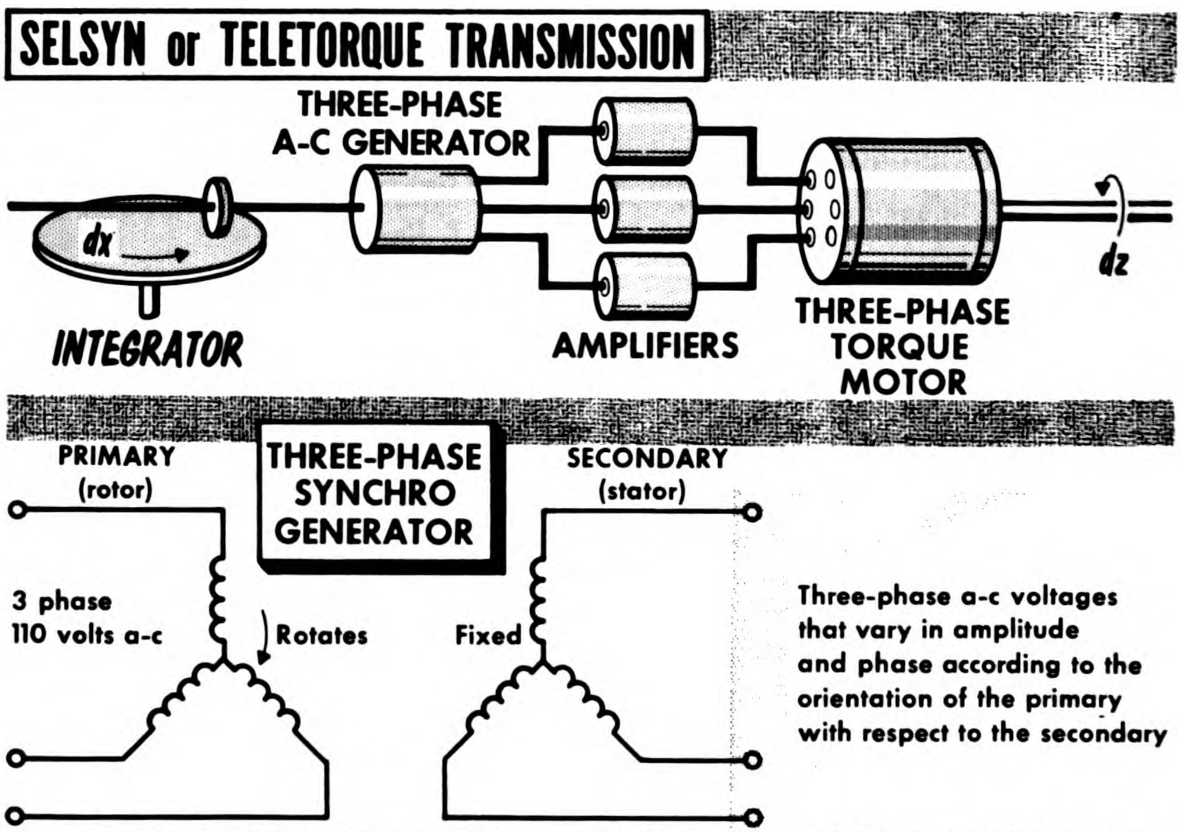


Fig. 4-31B

rized filters and is detected by two photocells. The electrical signals from the photocells are amplified and combined to form the correct input to the torque motor. The motor rotates the two polarized filters until one passes a maximum signal and the other a minimum signal.

Other techniques use:

1. Capacitive coupling between plates on the z shaft and fixed plates.
2. Rotation of a dielectric material on the z shaft between two fixed plates to cause a change in capacitance proportional to dz.
3. Inductive coupling from the rotating primary of a three-phase transformer mounted on the z shaft, to a fixed transformer secondary. The signals from the three secondary windings are amplified and sent to the control winding of a three-phase torque motor. In such a system the motor shaft is synchronized to the z shaft and the transmission system is known as *synchro*. Other popular names are selsyn and Teletorque transmission (Fig. 4-31B).

Velocity Servomechanism

In some systems (ac or dc) a motor is used as an integrator. The motor speed is proportional to the voltage applied to its terminals and hence

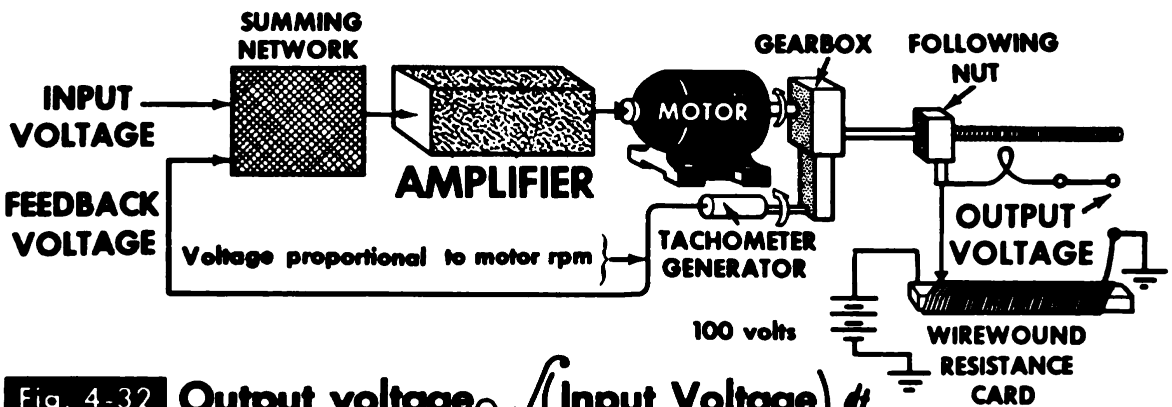


Fig. 4-32 **Output voltage $\sim \int$ (Input Voltage) dt**

the number of revolutions turned is proportional to the integral of the voltage input. A shaft or rod or revolutions counter may be the output of the integrator, if a mechanical output is desired. On the other hand, if the system is primarily electrical, a d-c or a-c voltage output may be obtained by driving a wiper arm across a potentiometer card or the arm of an autotransformer (for ac).

To make the motor speed coincide as closely as possible with the input voltage, thus avoiding overshooting or oscillating about the correct speed due to the motor inertia, it is usually necessary to design an electronic amplifier and feedback circuits for use with the motor. Such a system falls in the class of devices called *servomechanisms*, or just *servos*. Hence this integrator is often called a *velocity servo* (Fig. 4-32).

The initial value of the integral is placed in this integrator by adjusting the initial position of the following nut on the output lead screw so that the output voltage is the desired initial voltage.

QUESTIONS

1. What is integration? — differentiation?
2. Describe the operations of integration and differentiation of a time dependent variable in physical terms.
3. Distinguish between physical, mathematical, and computer variables. In what way are they alike?
4. Name three continuous integrating devices with which you are familiar. How do they operate?
5. Why is differentiation not a standard operation in analog computers?
6. Of the following equations which is a differential equation? Explain.

$$ax(t) + bx^2(t) = f(t)$$
$$ax(t) + b\dot{x}(t) = f(t)$$
$$a\ddot{x}(t) + b(t) = f(t)$$

7. What constitutes a differential equation? What determines the order of a differential equation? Explain the term "simultaneous differential equation".
8. What are the major steps in a computer simulation of a physical system?
9. What do the individual terms represent in the equations for the double mass and spring system?
10. What do you understand by "initial values" and why are they important in the mathematical investigation of physical systems?
11. Under what conditions could a simple resistor and capacitor be used for integration?
12. How are capacitors used for integration in the electronic analog computer?
13. What are the differences and relative merits of the ball and disc, and wheel and disc integrators?
14. What is the major difficulty with such (Q. 13) mechanical integrators?
15. Describe two means for driving large loads from a mechanical integrator?

Volume 2

GENERAL PURPOSE ANALOG COMPUTERS

Chapter *1*

GENERAL PURPOSE ANALOG COMPUTER TYPES

Introduction

The intent of this book so far has been to introduce the basic ideas of the scientific and engineering uses of physical analogies, and to describe some of the components used to put those ideas to work. No attempt has been made to classify computers themselves or to distinguish between building blocks which are and are not used in general purpose computers. A number of the interesting devices discussed are found only in special purpose computers, due to the special nature of the device itself or to the lack of compatibility with other general purpose devices. For example, no general

Passive-Element Computers are used primarily for ***static problems***	***Active-Element Computers*** are used for solving ***time-dependent dynamic problems***
1. Conductive sheets	1. Mechanical differential analyzer
2. Electrolytic tanks	2. Electromechanical differential analyzer
3. Resistor networks	3. A-c electronic-analog computer
4. Rubber sheets	4. D-c electronic-analog computer
5. Pin and rod systems	5. Digital or incremental differential analyzer
6. Resistor and capacitor networks	
7. Network analyzers	

Fig. 1-1

purpose hydraulic computers are known to exist, though special purpose hydraulic devices are not uncommon.

Now that the ideas and building blocks have been developed, this chapter is devoted to classifying and describing general purpose computing systems, for while the d-c electronic analog computer predominates today, there are other systems in regular use. Although some of these other systems are limited to the solution of a certain class of problems and might be considered special purpose computers, those described here have sufficient flexibility in their programming and apply to a large enough class of problems for us to retain them in the general purpose category (Fig. 1-1). The computer types to be considered are:

• *Passive-Element Computers:* Computers with only passive building block components and signal generators. These are used primarily for static problems, and for studying the regular sinusoidal behavior of systems driven by sinusoidal excitations, which result after any transient behavior has decreased to zero.

1. Conductive sheets
2. Electrolytic tanks
3. Resistor networks
4. Rubber sheets
5. Pin and rod systems
6. Resistor and capacitor networks
7. Network analyzers.

• *Active-Element Computers:* Computers with at least one kind of building block containing an amplifier; used for solving dynamic time-dependent problems.

1. Mechanical differential analyzers
2. Electromechanical differential analyzers
3. A-c electronic analog computers
4. Digital or incremental differential analyzers
5. D-c electronic analog computers*

In the foregoing list of computer types there are a few terms that require further explanation.

Passive-Element Computers: Frequently, this group of computers solves equations in which the space dimensions, length, breadth, and height, (x, y, z,) are the independent variables, rather than time. Such equations describe physical problems in which temperature, or potential, or pressure distributions, heat or fluid flow patterns (steady flow), or stresses on metal plates and rods are important. To solve these equations use is made of the physical dimensions of the computer components themselves. Conductive sheets,

* Because of its importance, the d-c electronic analog computer is dealt with separately at the end of this chapter, after the digital differential analyzer, and is then presented in considerable detail in the following chapters.

tanks of electrolytic fluid, or resistor networks enable static or "steady-state" voltages and currents to be developed throughout the networks. No dynamic losses are incurred and no amplification is needed. Only external driving functions (steady forces or currents or voltages) need be applied to the

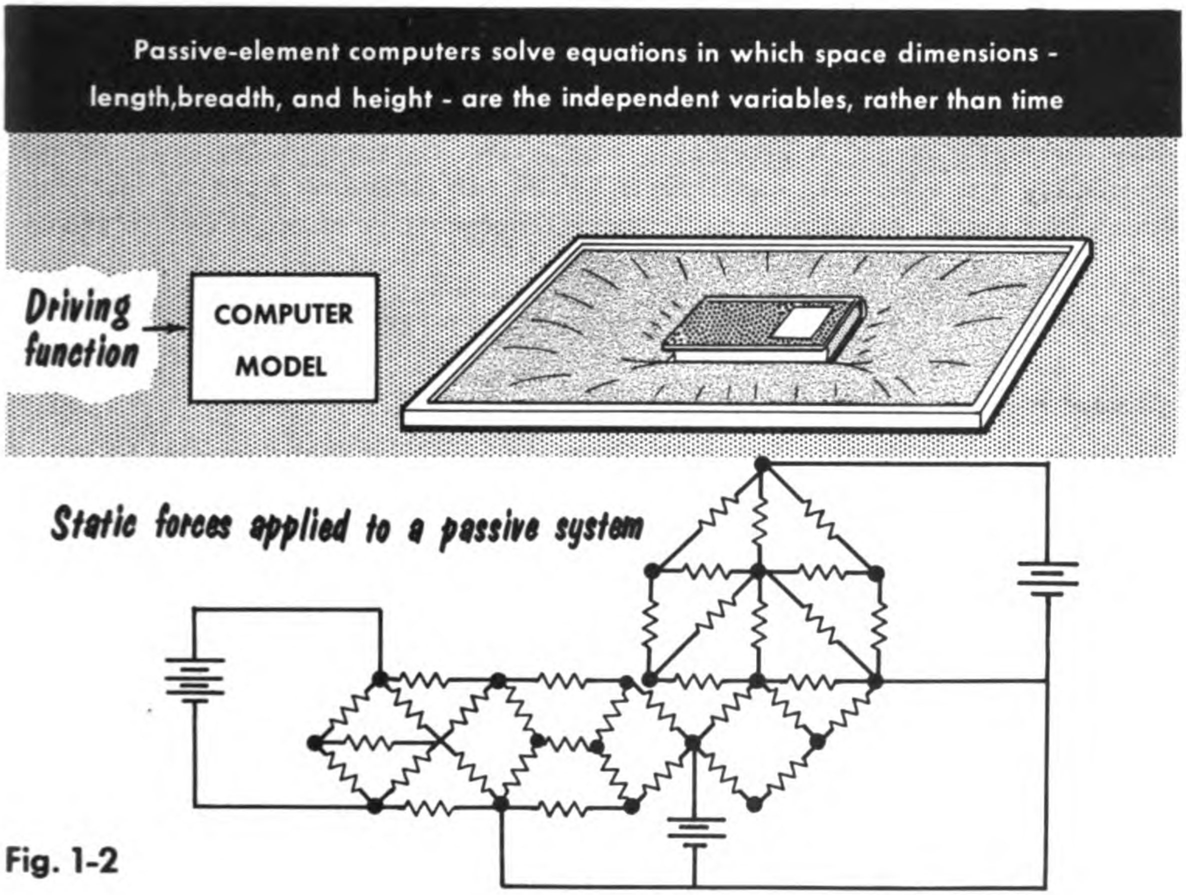


Fig. 1-2

passive elements and then the long-term displacements or voltages and currents observed (Fig. 1-2).

As an exception to the above, passive-element network analyzers using capacitors and inductors can compute the time variations of a physical system in addition to the space variation. That is, one can simulate the "transient" behavior of a physical system with these computers. This can be accomplished without the use of amplifiers provided the time variations are very rapid, compared with, for instance, the time for a capacitor to lose a measurable amount of charge through leakage.

Computers for purely static, spatial problems are sometimes called *field plotters* and *potential analyzers*. Computers for dynamic (time varying) spatial problems are usually termed R-C or R-L-C network analyzers.

Differential Analyzer and Active-Element Computers: Notice that all the second group (active-element) computers are called differential analyzers save the a-c and d-c electronic analog computers. Commonly, these two also

are called *electronic differential analyzers.* Thus it appears that the term *differential analyzer* in some way implies the active nature (see below) of the computer elements. This is because differential analyzers are analog systems for solving time-dependent differential equations, computing the second-by-second small (differential) changes in the system variables. To accomplish this to a high degree of accuracy over relatively long periods of time, some amplification is required to overcome the losses in the computer components and to isolate one component from another. Usually the amount of amplification required is very large. In mechanical computers torque amplifiers are used. In electronic computers high amplification feedback amplifiers are used. Any computer having at least one kind of building block containing an amplifier is considered an *active-element computer*.

PASSIVE-ELEMENT COMPUTERS

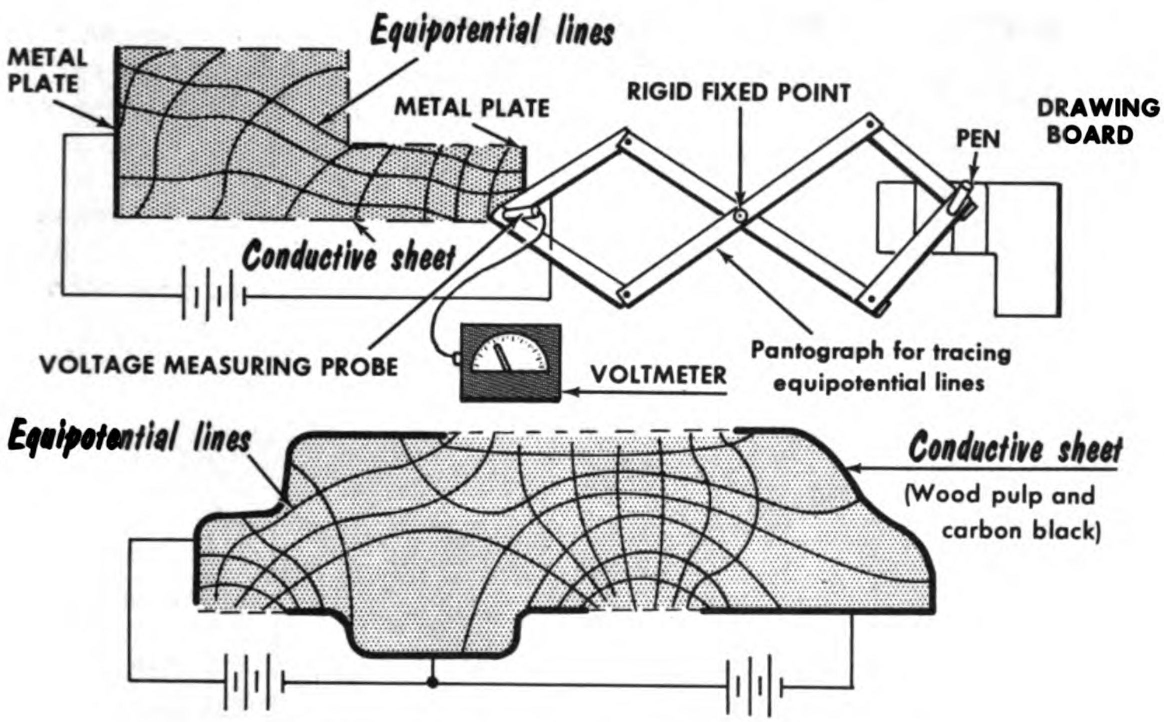
Conductive Sheets

One large class of engineering problems requires the determination of static distribution of physical fields, subject to certain boundary conditions. These fields may be the electric and magnetic fields about the electrodes of a vacuum tube or a transmitting antenna, or they may be the reception pattern or sensitivity field of a TV receiving antenna, or again, the pressure and streamline patterns for the *steady* flow of a fluid through a nozzle. As a final example, the fields may be the stress and strain patterns of a stressed elastic object. In each case the boundary conditions are the forces, pressures, voltages or currents which create and sustain the fields as well as the physical boundaries of the space containing the fields. Since these fields are frequently static, not changing with time, the simulation of such systems is much simpler than the simulation of dynamic systems. No time integrators, or multipliers, are required. It is only necessary to find an analog with the same field distributive properties as the primary system. A number of such analogs are available:

1. Conductive sheets of paper, metal, compositions
2. Electrolytic tanks
3. Resistance networks
4. Rubber sheets
5. Metal rods and pins

Two of these were illustrated earlier. Each is called a passive-element computer or device.

Although the analog may relate an electric system to a mechanical system there is usually a direct correspondence between the boundaries of the analog device and the shape of the space containing the fields in the primary system. Thus the analog or model is built in the shape of the primary system. If the problem can be described in two dimensions the analog device used might be a conductive or rubber sheet. Three-dimensional problems must be solved with three-dimensional analogs, for example, the elec-



The TELEDELTOS paper used by Western Union is a good CONDUCTIVE SHEET

Fig. 1-3

trolytic tank. It is possible to build resistor networks and pin-and-rod models in two or three dimensions.

A useful conductive sheet material is the Teledeltos paper developed and used by Western Union Telegraph Co., for telegrams, telegraphic reproduction of graphic data (such as weather maps), and instrument recordings. This paper has carbon black mixed with the wood pulp and will conduct electricity (Fig. 1-3). If the boundaries, which are to be held at fixed voltages, are painted on with a highly-conductive (silver) ink and the free boundaries are cut to the correct corresponding "physical" shape, then when the appropriate voltages are applied, patterns of current paths and equipotential lines will be established within the paper, simulating the field patterns in the primary system. These patterns are detected by sensitive voltage measuring instruments.

Electrolytic Tanks

For the many three-dimensional field problems the electrolytic tank is probably the best technique for analog simulation. Though probably not the neatest since it requires the handling of acid solutions, it is much simpler to set up than the adjustment of thousands of resistors in a three-dimensional resistor network.

As seen before, a scale model of the primary system is placed in an electro-

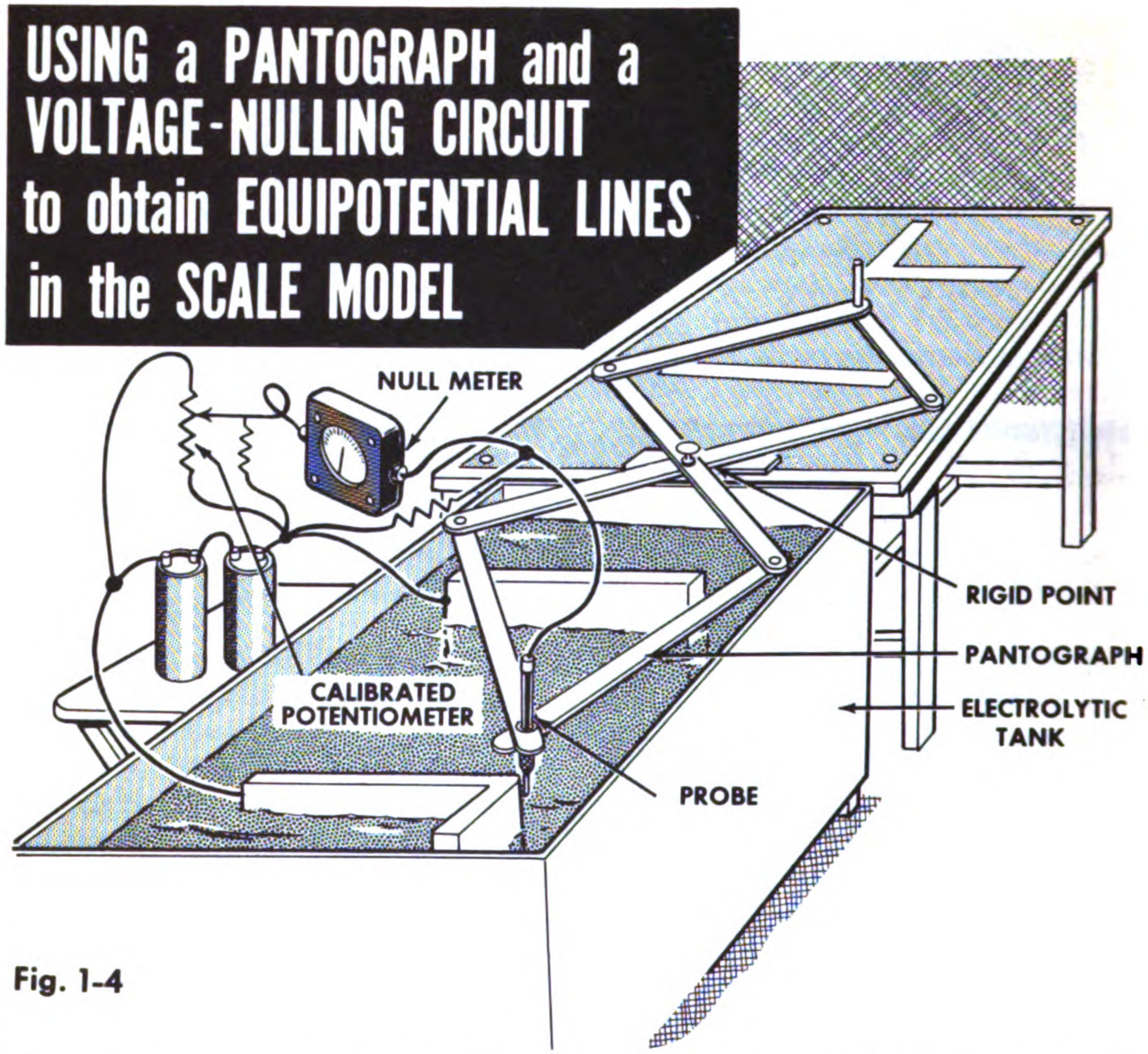


Fig. 1-4

lytic solution, voltages are applied to parts of the model, and a probe is used to detect the location of the equipotential lines in the space between the conducting portions of the model. In a two-dimensional model, a simple pantograph and a voltage-nulling circuit (Fig. 1-4) greatly simplifies the recording of equipotential lines. A particular voltage is selected with the potentiometer, and the operator moves the probe to make the meter read zero. Since the meter is connected in a bridge circuit it reads zero when the probe voltage is equal to the selected voltage. Maintaining this zero, or null condition while moving the probe, amounts to following an equipotential line corresponding to the selected voltage. The pantograph scribe draws an up-side-down picture of the path taken by the probe.

Resistor Networks

Upon first thought it would appear that a resistor network could not be an analog for an electric field or a pressure field. A resistor network consists of many discrete branches, while the fields we have been discussing are continuous phenomena. The resistor network analogy is, of course, only an approximation to a true analog. If a sufficiently fine mesh of resistors is

used, a very satisfactory simulation can be performed. In fact, if the mesh is made infinitely fine the network approaches the conductive sheet. Indeed, were we to start with a conductive sheet and cut many small slits in the paper *parallel to the flow of current*, we could probably remove enough strips, without seriously disturbing the current and potential distributions, to make the sheet look like a network of small resistive strips of paper! (Fig. 1-5A).

Usually, a regular array of resistors is used, either a rectangular, triangular or diagonal array. The boundary of the network is adjusted to approximate

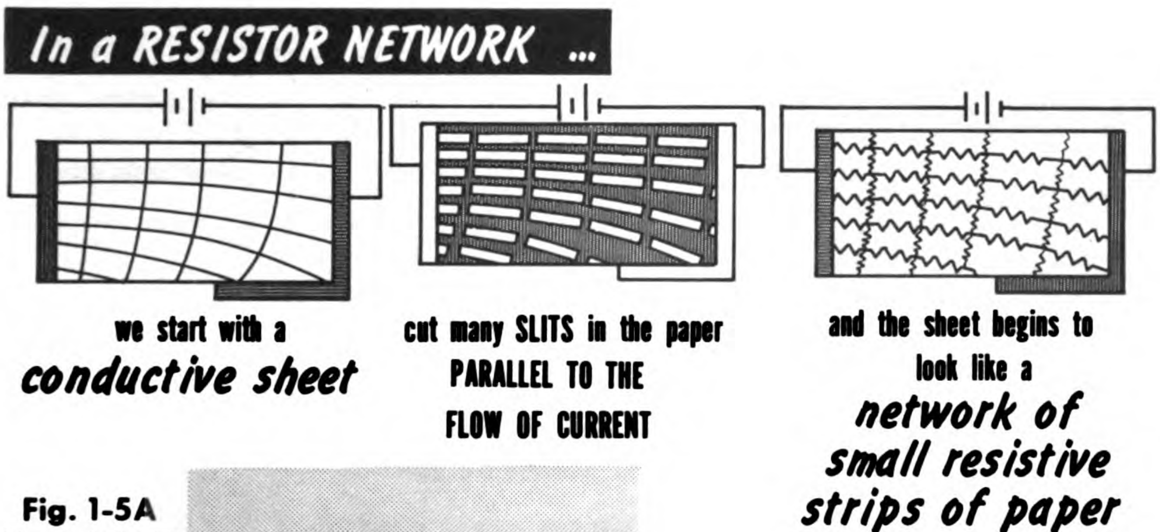


Fig. 1-5A

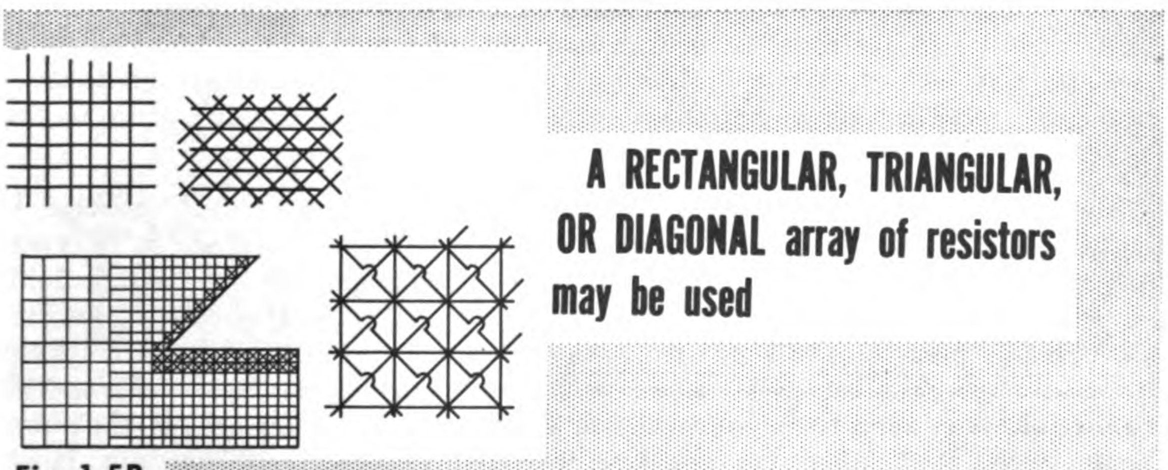


Fig. 1-5B

the real boundary. At very irregular boundaries extra resistors and a finer mesh may be required to obtain the desired accuracy in the simulation (Fig. 1-5B).

Voltage or current generators are used for simulating the applied forces, or pressures etc. Nodes, or intersections, which are at the same potential

CUTAWAY of a VACUUM PENTODE

TWO-DIMENSIONAL VIEW of a PENTODE CUTAWAY

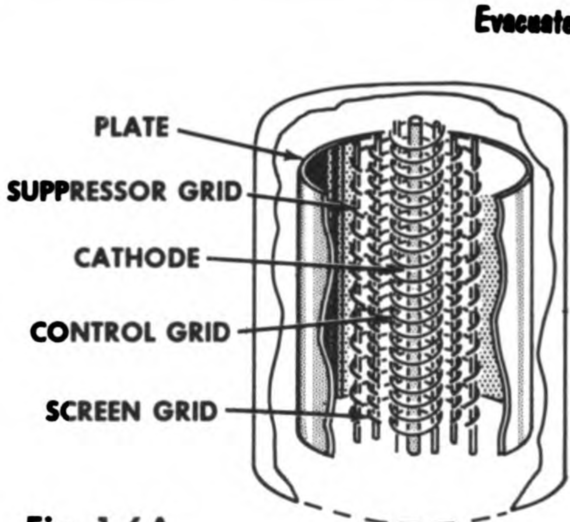


Fig. 1-6A

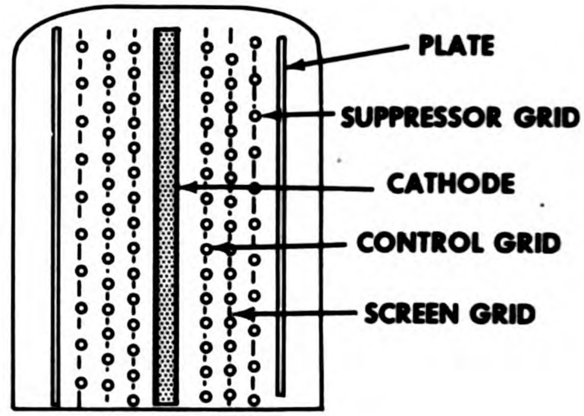


Fig. 1-6B

are simply measured and recorded in order to plot points on the equipotential lines, although the null meter and pantograph, shown earlier with the continuous electrolytic tank, are of no use here.

Rubber Sheets

Rubber-sheet computers have been used extensively to study the trajectories of electrons and gas ions in vacuum and gas tubes. A precise knowledge of the effect of each electrode (control grid, screen grid, suppressor grid, etc.,) in a tube upon the motion of the electrons and gas ions is imperative to the development of tubes with specific electrical properties.

Although electrons are usually emitted radially from a central cathode, it is often adequate to study simply a vertical cross section, as a two-dimensional problem, making appropriate correction for the radial field. Now the reason a passive-element computer can be used to study the trajectories of electrons is that electrons will always follow the electric field lines, always crossing the equipotential lines at right angles. Thus the rubber sheet is used to simulate the field distributions throughout the tube. Depressions in the sheet represent regions of *high* positive potentials. High positive potentials attract electrons; in a similar manner small steel balls will roll toward the lower regions of a horizontal rubber sheet as if "attracted". Negative potential regions which repel electrons are simulated by elevated portions of the rubber sheet and, of course, the small steel balls will be "repelled" from such regions (Figs. 1-6A and 1-6B).

With aluminum rods and plates in place under and above the sheet to simulate the electrode potentials of a multi-element tube, multiple exposure

photographs are taken of the steel balls rolling across the sheet. These simulated electron trajectories can be very accurate for cases where the space-charge effects are unimportant, that is, when the number of steel balls (electrons) is sufficiently small that any one is not affected by the presence of the others. When a large number of projectiles are present, the electrons mutually repel each other, while the steel balls come together because of the additional depression of the rubber sheet. Thus the rubber-sheet and steel-ball model fails as a useful analog when the electron-beam density is high (Fig. 1-6C).

Pin-and-Rod Systems

One characteristic common to all the fields considered here is that the equipotential lines (pressure, electric voltage) are always perpendicular to the flow lines or stream lines (or electric field lines). This does not mean the fields always form a rectangular grid but simply that the two sets of lines always cross at right angles [Fig. 1-7(A)].

A clever way to simulate such a field was first used in the last century. A

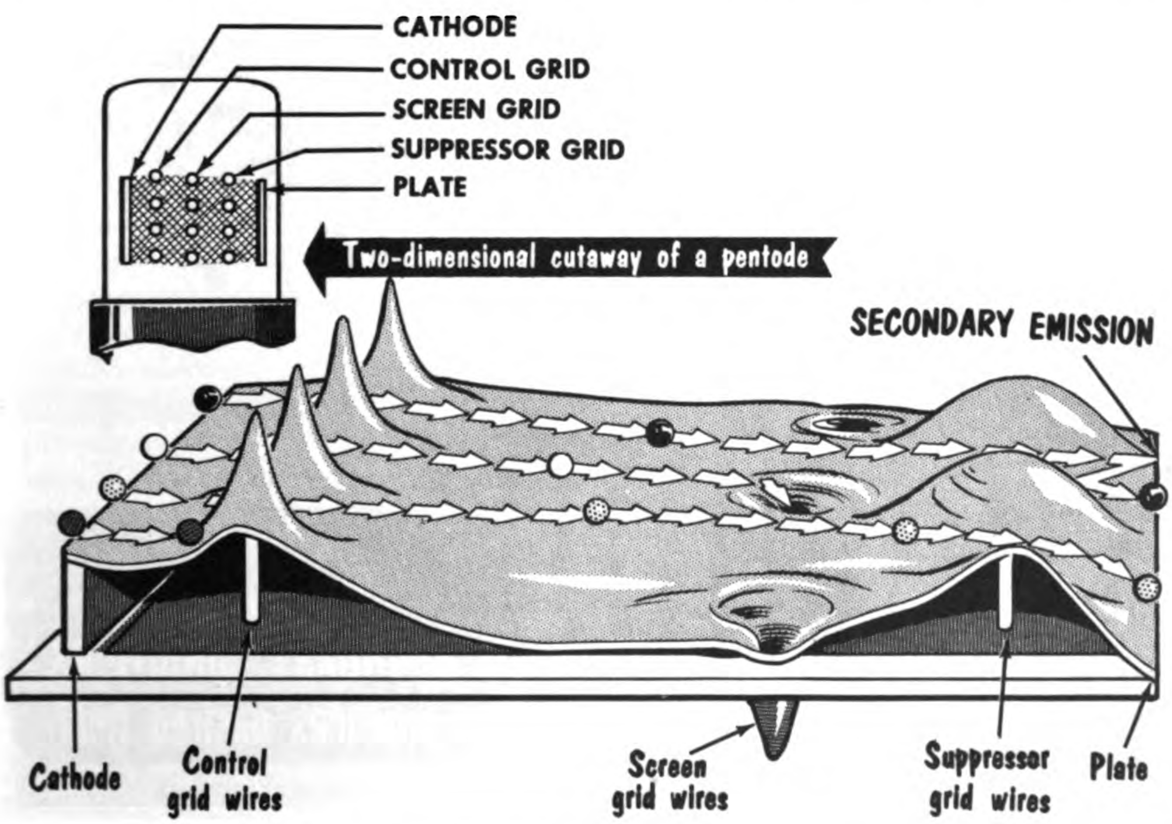


Fig. 1-6C RUBBER SHEET ANALOGY of POTENTIAL DISTRIBUTION in a PENTODE

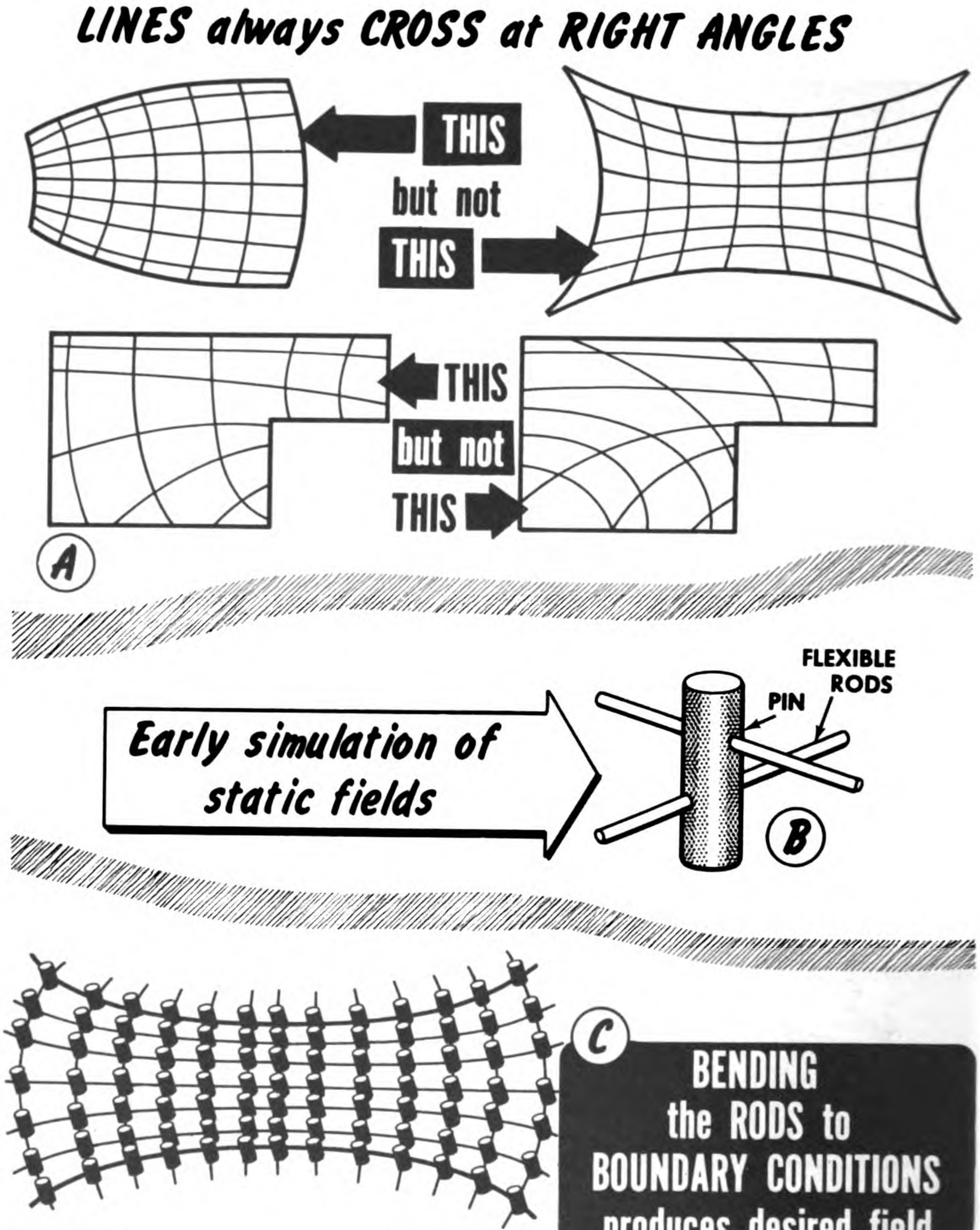


Fig. 1-7

system of many small pins with long stiff wires passing through them is assembled. Two wires pass through each pin at right angles as shown. The pins are free to move back and forth on each wire [Fig. 1-7(B)].

If the edges of this array of pins and wires are bent to fit certain boundary conditions specified by a problem, the pins will adjust their position permitting the wires to correctly describe the desired fields. Figure 1-7(C),

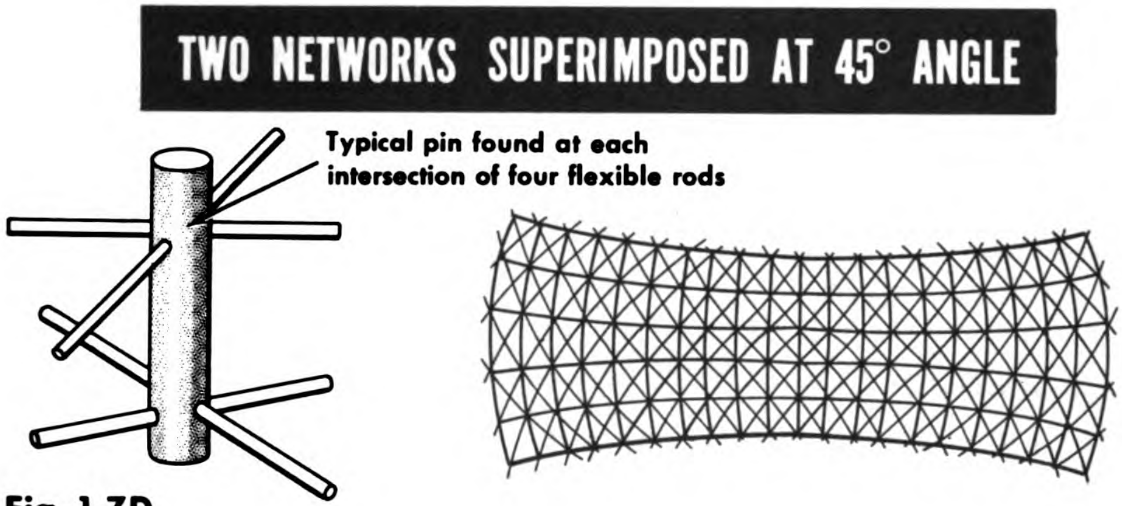


Fig. 1-7D

for instance, might represent the pressure potentials and the streamlines for nonturbulent flow through a nozzle. For greater accuracy two networks may be superimposed at 45°, that is, each pin has four wires through it, one at each 45° angle (Fig. 1-7D).

Resistor and Capacitor Networks

It is not true that passive-element computers are used only for static problems. Any of the preceding devices could be used in a simulation with time-varying boundary conditions. For instance, to simulate varying grid and plate voltages in a vacuum tube one need only move the aluminum supports representing them in the model up and down very slowly. Moreover, passive-element computers are often used to solve very important classes of problems in the propagation of energy across fields. One kind of such propagation is called *diffusion*. Diffusion is a very common phenomenon: heat diffuses through a solid; liquids diffuse through sand; radioactive particles diffuse through almost anything; and transistor junctions are often made by diffusing one material into another. Diffusion is distinct from other transport phenomena such as radiation, convection, or wave propagation.

Simulation of diffusion requires only the addition of a capacitor to each node of a resistance network (Fig. 1-8). If we were to simulate the change in temperature throughout a slab of material due to a sudden application of heat at one boundary, we would first discharge all the capacitors then suddenly apply a voltage (proportional to the temperature at the input side) to one side of the network. Due to the presence of the capacitors there would be a delay before the voltage would be detected across the network corresponding to the diffusion of the heat across the slab. Typical voltage recordings at several points in the slab are shown. The speed at which the heat waves propagate depends upon the size of the capacitors. Some R-C network analyzers use small capacitors, operate rapidly and re-

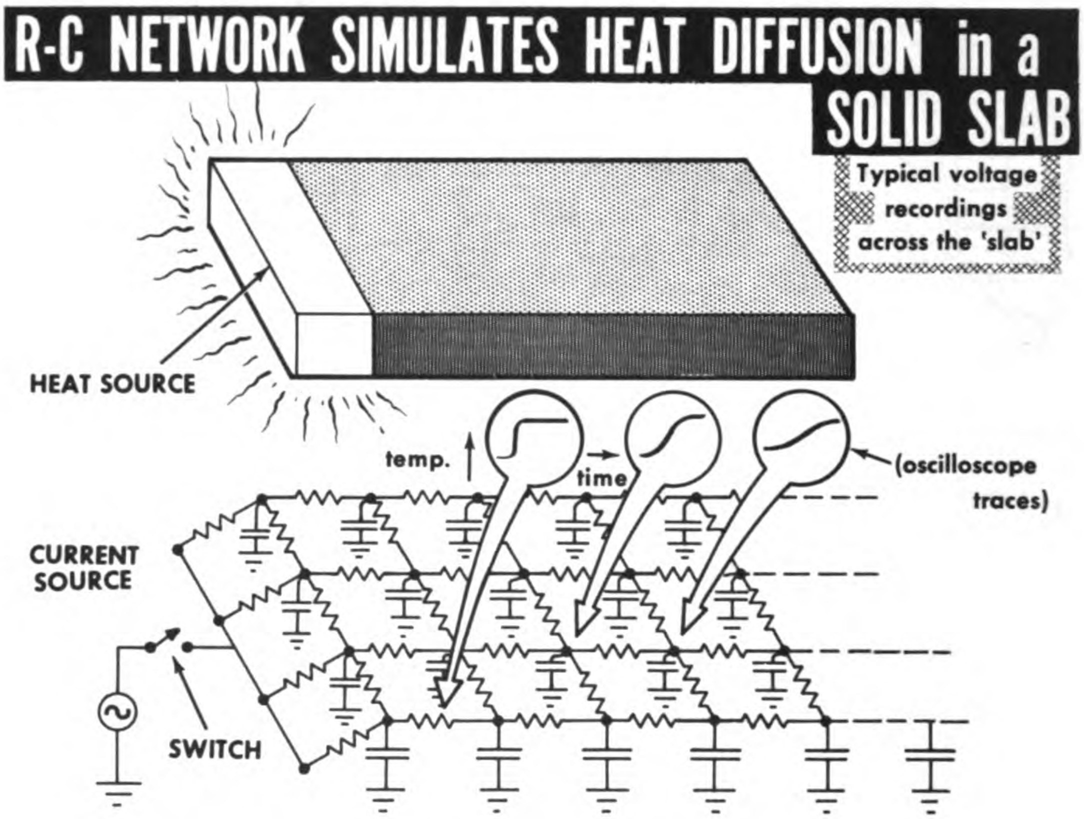


Fig. 1-8 Due to the presence of capacitors there is a delay before the voltage can be detected across the network, corresponding to the diffusion of heat across the slab

petitively, and provide a display of the results on an oscilloscope. Other computers operate very slowly.

It can be shown mathematically that if the mesh size in the network analyzer is relatively small, the behavior of this kind of circuit is a very good approximation to the diffusion of heat through a continuous medium.

Resistor, Capacitor, and Inductor Network Analyzers

In the R-C network analyzer just referred to, electrostatic energy is stored by the capacitors in a manner analogous to the storage of heat throughout a slab. This computer can solve the "diffusion equation" (a mathematical statement of the diffusion process). The R-*L*-C network analyzer (Fig. 1-9) has two kinds of energy storage elements: *inductors* which store *magnetic* energy and *capacitors* which store *electric* energy. In some analogies these two kinds of energies are considered to be the analogs of kinetic and potential energy in a mechanical system. The presence of these two energy stores makes possible the simulation of wave-like propagation. In fact, the R-*L*-C network analyzer will solve the wave equation, in the sense that it

simulates wave motion in a continuous primary system with voltage and current waves in a discrete electric circuit. The degree of approximation in such a simulation depends primarily upon the mesh size or number of circuit branches used to represent a unit length or unit area of the primary system.

The Electric Network Analyzer

The most common large-scale use of the electric network analyzer is in the simulation of electric power distribution systems. Many electric power producers have installed these computers to help their power dispatchers make important decisions more rapidly. The building blocks of the computer are simply inductors, capacitors, resistors and a-c voltage generators. These units are connected to form an electrical model of the large electric network served by the company and of its power generating stations. When in operation the model network delivers a-c power to its several branches in the same manner as the primary system. Meters in the model indicate small currents which are proportional to the large currents in the primary system. The model voltage generators are adjusted to produce voltages proportional to the voltages of the generating stations. The resistors, capacitors, and inductors are chosen to yield the electrical characteristics of large sections of the primary system network. Since most industrial and residential power requirements are for motors, (which are inductive) more

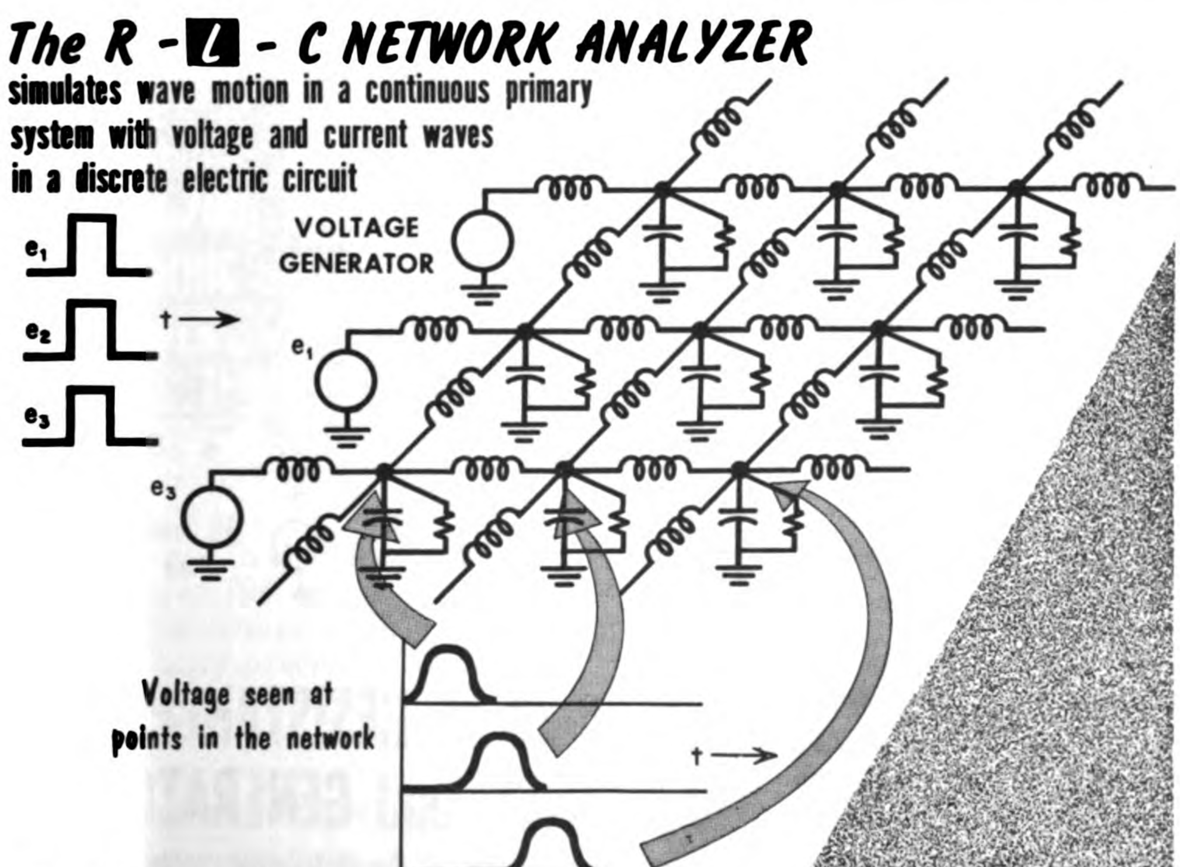
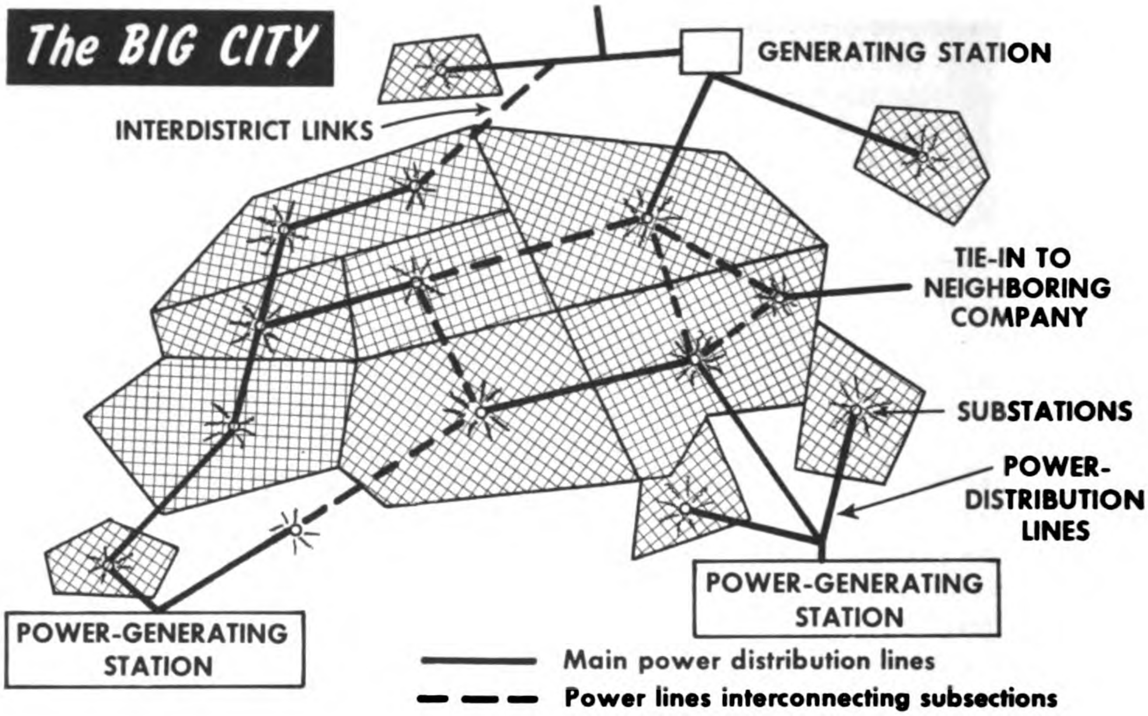
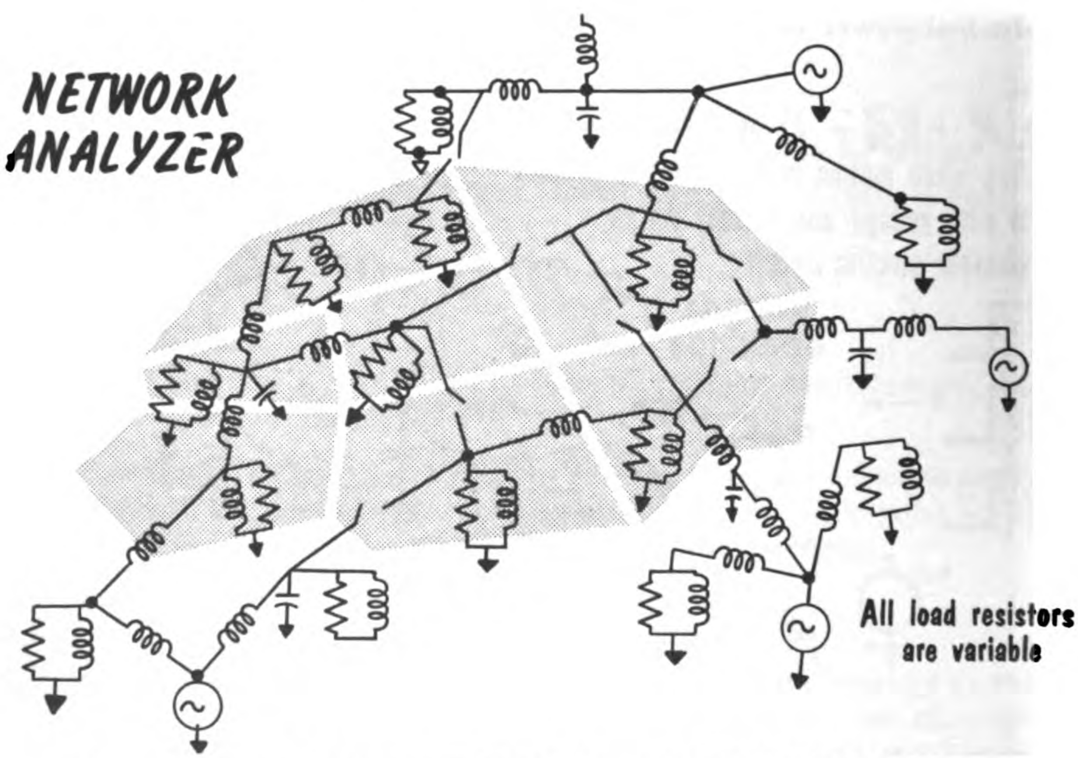


Fig. 1-9 Propagation of a square wave





The ELECTRIC NETWORK of RESISTANCE, CAPACITANCE, INDUCTANCE, and GENERATORS

is a time-scaled and magnitude-scaled model of the distribution system

Fig. 1-10A.

inductors are used than capacitors. Resistors simulate all power-absorbing devices.

It is costly to send power a long distance to a user because of line losses. Furthermore, it is costly to operate a generating station at less than rated power output, and of course, it is undesirable to exceed the power capacity of a generating station. Since electric power cannot be stored in huge quantities very conveniently, the power company dispatchers have an enormous task in keeping all their power stations at full capacity, while at the same time supplying power to a constantly changing load (customer demand), keeping line losses at a minimum by delivering power from the nearest generator, and avoiding overloading of any generating station. To complicate the dispatcher's problems further, neighboring companies continually buy and sell power between themselves to meet the above and other desired operating conditions. Faced with a myriad of choices, the power dispatcher uses the network analyzer to determine the alternatives resulting in the lowest losses and maximum profits (Fig. 1-10A).

Because the power involved in the primary system is large, and in the model it is relatively small, any small error in a current in the network analyzer

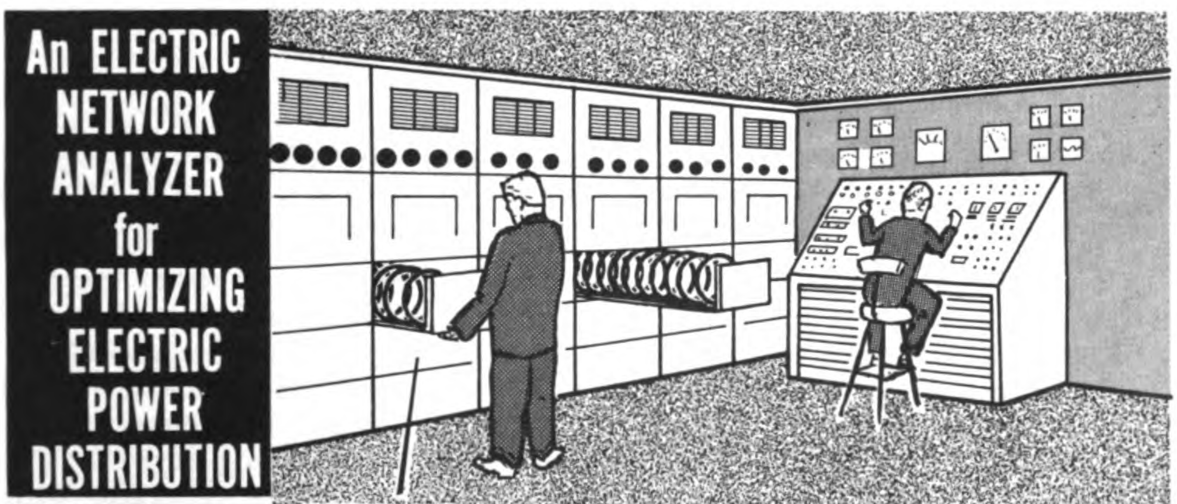


Fig. 1-10B

would result in a correspondingly large error in the calculation of the performance of the primary system (Fig. 1-10B). Such errors could be very costly. To reduce them it is necessary to build the network analyzer of exceptionally precise components. Consequently the resistors, capacitors, and inductors of network analyzers for 60-cycle electric power distribution are commonly quite large and expensive.

High-Frequency Network Analyzers

The power distribution network analyzers just described form an electric network which is almost a scale model of another electric network (the dis-

tribution system). Neglecting errors in that the many small branches of the primary system are lumped together in the model, one might say that the network had been scaled down in size or in magnitude but that the *time scale* had not been changed. That is, the network size and power require-

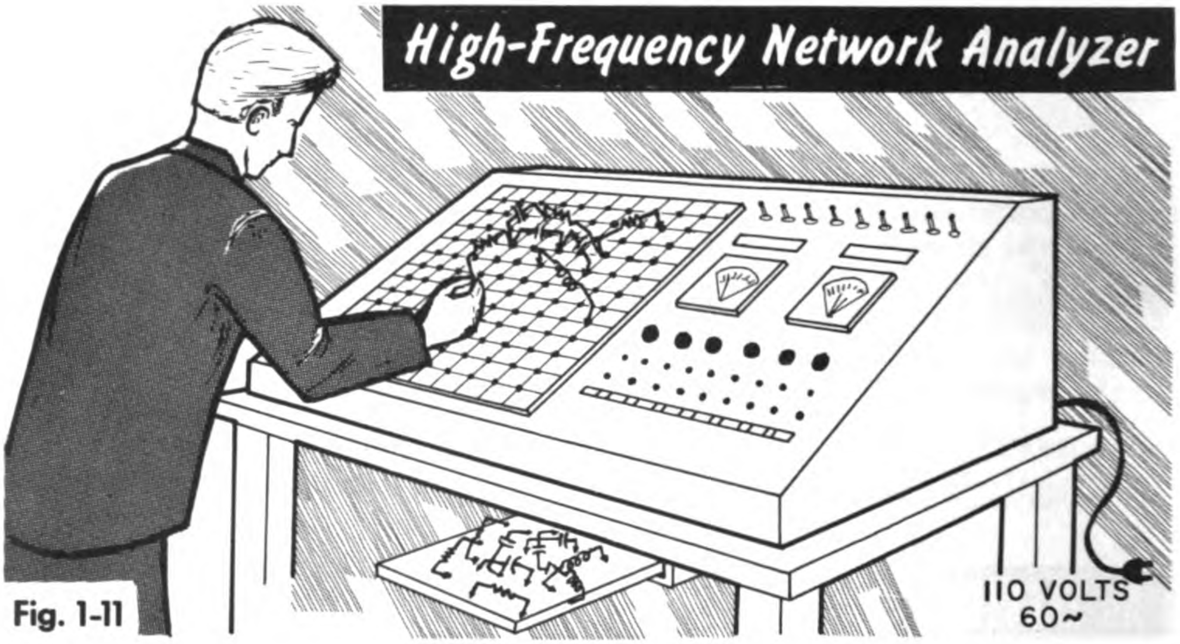


Fig. 1-11

ments are much smaller than the primary system, but the time scale or the frequency of the a-c voltage is the same, i.e., 60 cps. The fact that the frequency is low in the model is another reason the building blocks are so large.

It is quite possible to construct an electric network of resistance, capacitance, inductance, and generators, which is both a "time-scaled" and "magnitude-scaled" model of another electric system. By merely increasing the basic frequency of the a-c generators one can have a "faster than-real-time" computer that, moreover, consists of inexpensive small components. Such a computer (Fig. 1-11) can simulate more than just a simple network; for instance, the primary system might be an electric motor or generator with many complicated windings and intercoil capacitances. Instead of 60 cycles, typical frequencies would be 50 to 100 kilocycles per second, in the radio-frequency range.

ACTIVE-ELEMENT COMPUTERS

MECHANICAL

The Bush Differential Analyzer

It is interesting to note that the idea of using mechanical devices to perform integration of functions and of differential equations was first suggested by Lord Kelvin, in 1876, and that a practical embodiment of this idea was

not achieved until 1925. In the latter year Dr. Vannevar Bush and his associates at M.I.T. developed a device for multiplying two functions together and integrating the product. The functions were formed into shaft rotations by hand-operated function generating tables. The shafts operated wipers on potentiometers, thereby forming voltage functions proportional to the desired input functions. The two voltage signals were then connected to a multiplying integrator of the watt-hour meter type. To avoid loading the sensitive meter to drive any output equipment, the revolutions of the

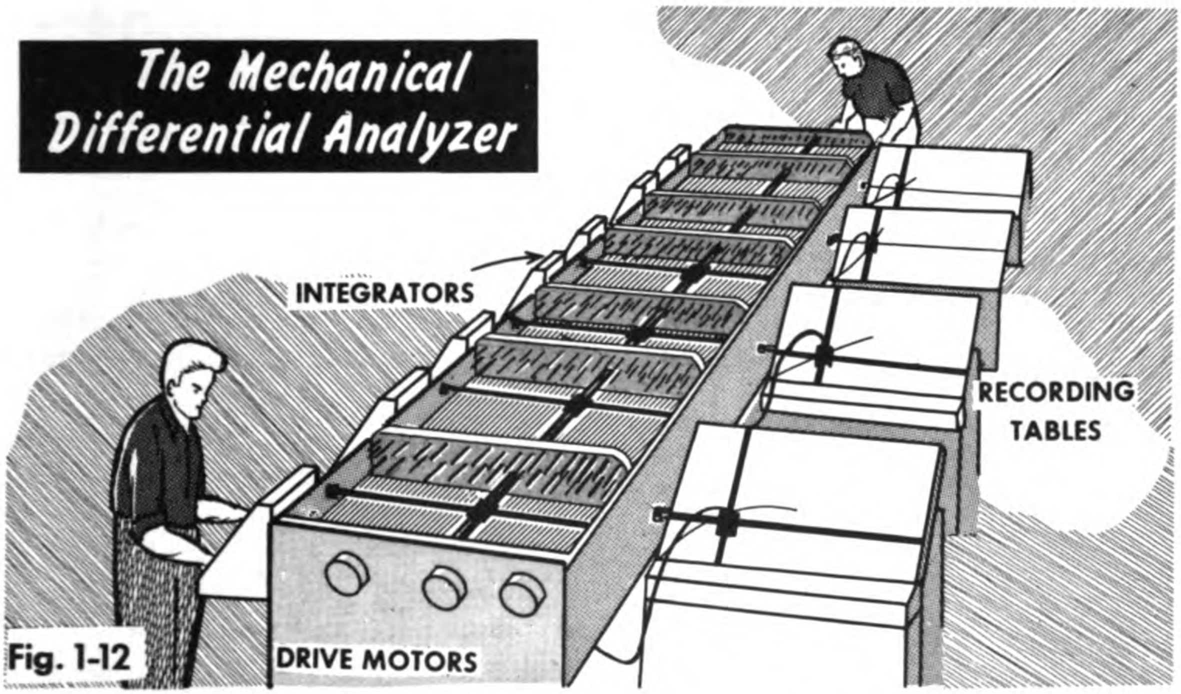


Fig. 1-12

meter output shaft were detected electrically and applied to the signal amplifier. This was a simple form of torque amplification.

The use of this continuous "integraph" led in 1930 to the design and construction of what is perhaps the most famous of all early forms of analog computers: the Bush mechanical differential analyzer. The computer was a general purpose machine consisting of six wheel and disc integrators with capstan-type mechanical torque amplifiers, several hand-operated function generating tables, and many interconnecting shafts and gear boxes. The function generating tables were also used for recording the results of computations in graphic form.

Many other mechanical differential analyzers were constructed during the 1930's, with a varying number of integrators, and improvements in accuracy and size. An early model of a computer was built by D. R. Hartree in England, with components of a Meccano toy construction set. Several replicas of the Bush computer were built by the Moore School of Electrical Engineering, University of Pennsylvania. One improved version with fourteen integrators is *still in use*. Although wear has reduced the figure, it

once was capable of very high accuracy: a precision of one part in 10,000 to 100,000, for some units.

During World War II these computers were invaluable assets to scientific laboratories. The design and study of many military projects (from aircraft and tank design to automatic control systems and ballistic problems) were effected on the numerous computers throughout the country. These mechanical computers suffer by comparison with electronic analog computers of today (Fig. 1-12) primarily because of their size, slow speed, and the large amount of time and effort required to prepare a machine for a problem solution.

Due to the bulkiness of the components and necessity for very accurate positioning of many gears, shafts, and other mechanical units, a typical setup time for a problem on a computer with 10 to 15 integrators would be from 10 days to two weeks.

The Electromechanical (Nordsieck) Computer

Improvements in the design of the mechanical differential analyzer during the 1930's and 1940's included the electrical and electrical-optical torque

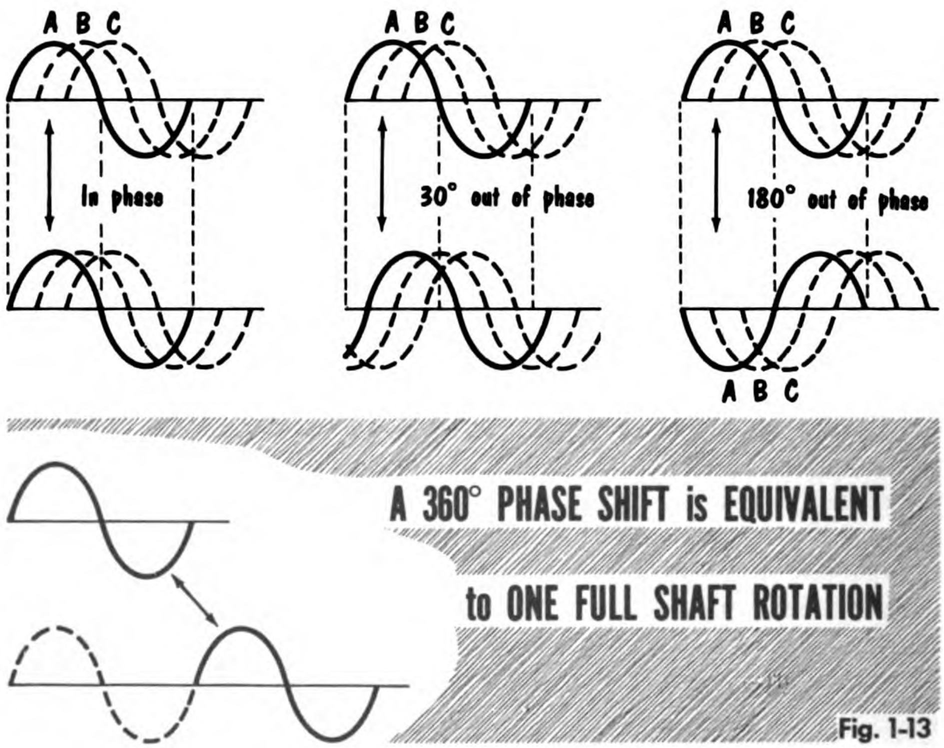
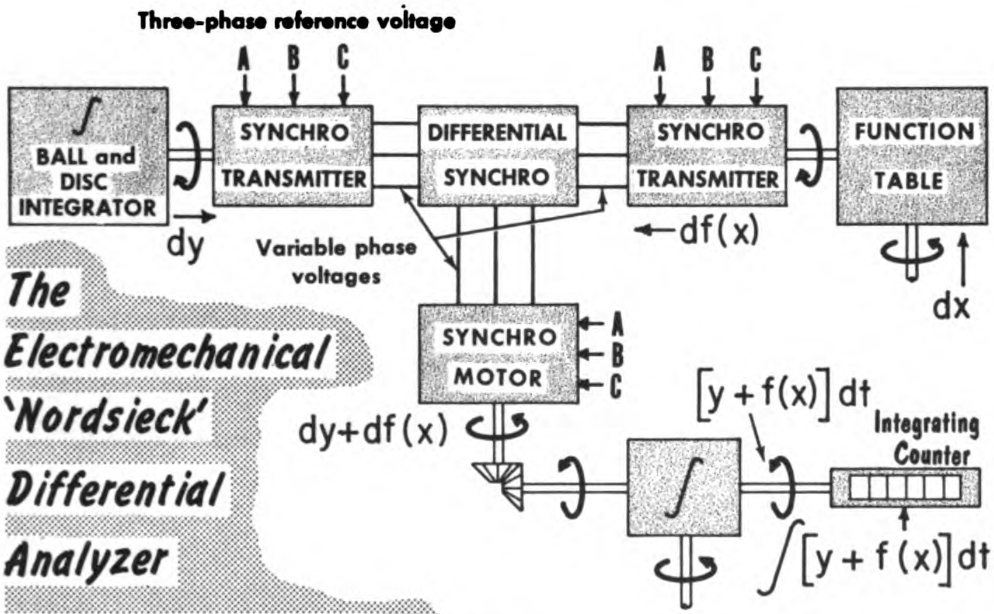


Fig. 1-13

amplifiers described earlier, and the use of very narrow wheels and plate glass discs. Replacement of the hand-operated crank on the function generator table with an automatic servo system significantly reduced the manpower required for operation of the computer, as well as improving the accuracy. However, the most significant change in the differential analyzer

(without considering all electronic systems) was the replacement of all the interconnecting shafts and gears with *synchro servomechanisms.*

A servomechanism is a mechanical device which slaves (servo) one variable to another (usually a shaft position to a voltage). A servomechanism which



The Electromechanical 'Nordsieck' Differential Analyzer

incorporates a synchro transmission system with mechanical computing elements

Fig. 1-14

uses as the controlling variable the phase difference between a variable a-c voltage and a fixed reference voltage (usually balanced three-phase voltages) is known as a *synchro*, *selsyn*, *telesyn*, *microsyn* or *teletorque* system. With synchro transmitters the position of the output shaft of a mechanical integrator or a function generator table is translated into a variable phase of a three-phase a-c voltage. A synchro receiver or motor accepts such a voltage variable and translates the phase into a shaft position at the input to an integrator or function table. A differential synchro performs the same task as a differential gear, that is, it adds (with proper attention to signs) two variables. In this case the two variables are voltages of fixed magnitude with phase angles proportional to shaft positions. The sum of the phase angles is proportional to the sum of the shaft positions. A phase change of 360° is equivalent to one full shaft revolution (Fig. 1-13).

The first computer to incorporate a synchro transmission system with mechanical computing elements is the Nordsieck differential analyzer (Fig. 1-14). The synchro system evidently reduced the bulk and weight of the mechanical differential analyzer significantly. Furthermore, the set-up time was also reduced enormously, for only simple electrical connectors and flexible cables were required for interconnection of integrators, function tables and recording devices.

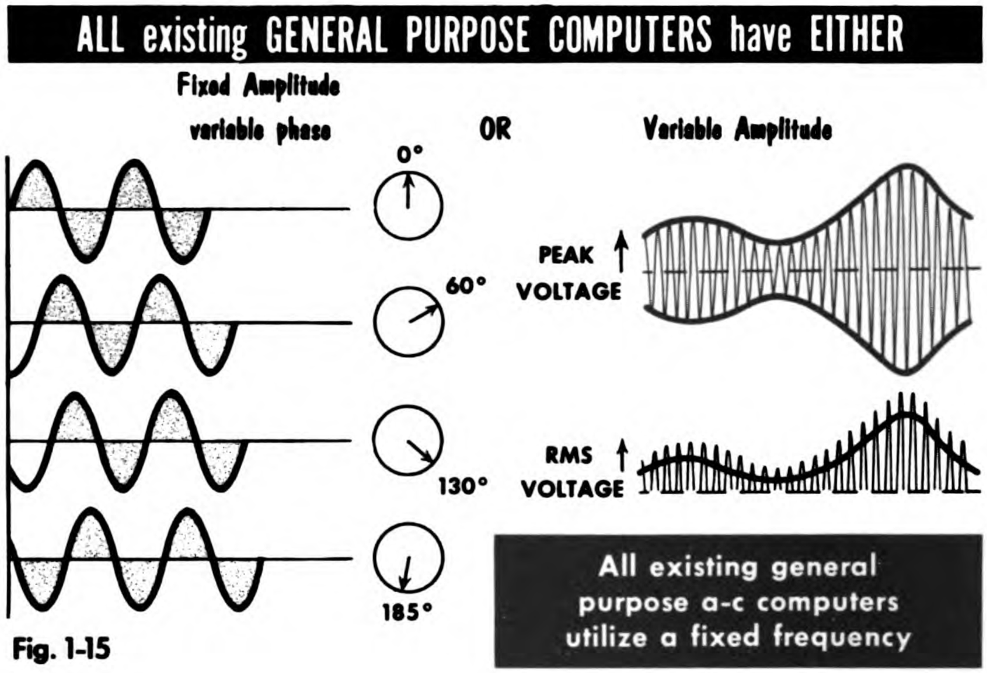


Fig. 1-15

A-C ANALOG COMPUTERS

A-C Variables

Some early general purpose computers and some present day aircraft simulators utilize alternating voltages as the primary computer variable rather than d-c voltages. Alternating voltage systems afford certain economies due to the use of simple circuits for high-gain amplifiers, and their freedom from any serious drift voltages eliminates the need for elaborate drift-stabilizing networks. When servomechanisms are required for turning shafts, as in a servomultiplier (used extensively in both a-c and d-c analog systems), the a-c servo amplifier and servo motor are generally less expensive for equivalent performance. In fact, the a-c servos are so much more desirable that most d-c servomultipliers convert the d-c input voltage to ac and use an a-c servo amplifier and motor! On the other hand, a-c computers are generally less accurate, and are more limited in computing speed because more electromechanical units are usually required. We will see below that even the integrators are electromechanical units. The primary computing speed limitation in analog computers, ac and dc, is in the slow dynamic performance of mechanical building blocks. The development of high precision, all-electronic building blocks for d-c computers is well ahead of that for a-c computers. Nevertheless, it is of value to take a close look at the a-c computing techniques.

An a-c voltage has three controllable properties: frequency, magnitude (or amplitude) and phase. All existing general purpose a-c computers utilize

a fixed frequency. Both the magnitude and the phase of an a-c voltage are used as computer variables (Fig. 1-15). One technique, described previously under the subject of synchros, uses fixed amplitude voltages with variable phase relationships to drive synchro motors — the motor shaft positions vary with the phases of the driving voltages. The technique to be discussed here modulates (varies) the amplitude of an a-c carrier wave. That is, the a-c voltage of fixed frequency (60 or 400 cps) acts as a carrier of the computer variable signal. All signal frequencies must be very much less than the carrier frequency. The signal is said to be contained in the *envelope* of the modulated carrier signal. The magnitude of the signal is described by either the peak values of the carrier signal or by the rms (root mean square) value of the carrier signal (the so-called "effective value" of the a-c voltage).

A-C Variables: Phase Detection

Note that in the description just given, the peak and the rms value of the carrier signal and hence the magnitude of the computer variable signal is

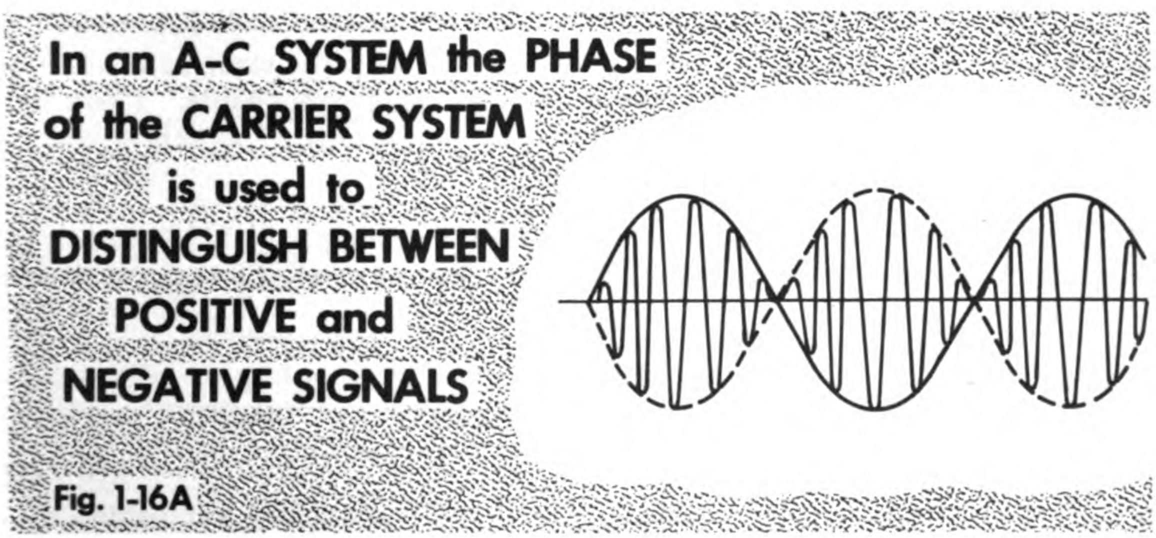


Fig. 1-16A

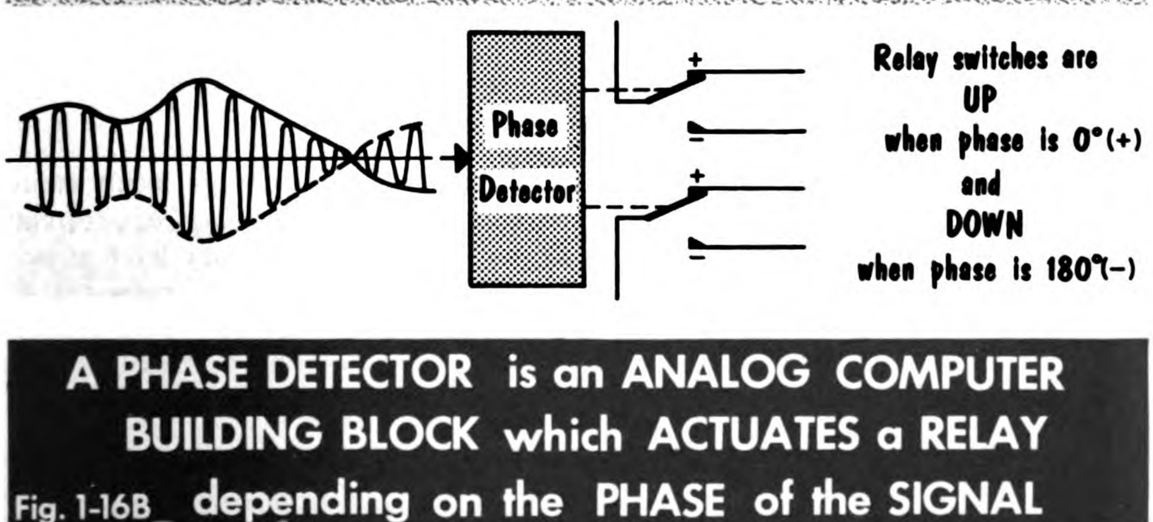


Fig. 1-16B

always positive. It is meaningless to speak of a negative rms value of an a-c voltage. It is generally desired however, that computer variables be able to take on both positive and negative values. In an a-c system of this sort the *phase* of the carrier signal is used to distinguish between positive and negative signals. The phase of the carrier signal is always arranged to be in phase with the reference phase (positive variable), or out of phase with the latter by 180° (negative variable) (Fig. 1-16A). No other phase positions are permitted. Small phase shifts occurring in computing elements are carefully balanced out by phase-correcting circuits so that only zero and 180° phase positions are possible.

In a d-c computer it is an easy matter to determine when a variable is positive or negative simply by observing the polarity of the d-c voltage. However, in our a-c system, if it is desired, for instance, to actuate a relay with the sign of a variable, it is necessary first to detect the phase of the carrier. For such purposes there is an a-c analog computer building block called a *phase detector* which compares the signal with a reference voltage to actuate a relay according to the sign of the signal (Fig. 1-16B).

A-C Building Blocks

Many a-c computer building blocks are substantially the same as d-c analog computer building blocks and some are even interchangeable. There are, however, some notable exceptions, the most important of which is the a-c integrator, discussed a little further on.

- *Multiplication by a constant* (Fig. 1-17A) is accomplished by resistor (or reactance) voltage dividers, potentiometers, transformers (fixed, adjustable, and autotransformers).

- *Addition and subtraction (addition of out-of-phase voltages)* (Fig. 1-17B) is handled by resistor networks with high-gain a-c feedback amplifiers (summing amplifiers) and sometimes by transformers.

- *Multiplication of two variables* (Fig. 1-17C) is usually performed by a servomultiplier (position servo), which is discussed in detail later for the d-c case.

- *Sign detection* requires a phase detector and relay.

It is often necessary to convert the a-c signal to a d-c voltage, with amplitude proportional to the envelope of the carrier signal, and sign appropriate to the carrier phase. This conversion might be required for use with special building blocks or recording and indicating instruments. It requires the use of a *phase-sensitive demodulator*, sometimes called a linear-phase detector. For details, the reader is referred to a later section on the d-c amplifier drift stabilizer which utilizes just such a demodulator.

- *Function generators* are usually servo driven electromechanical or even electro-hydraulic-mechanical devices with cams and potentiometers.

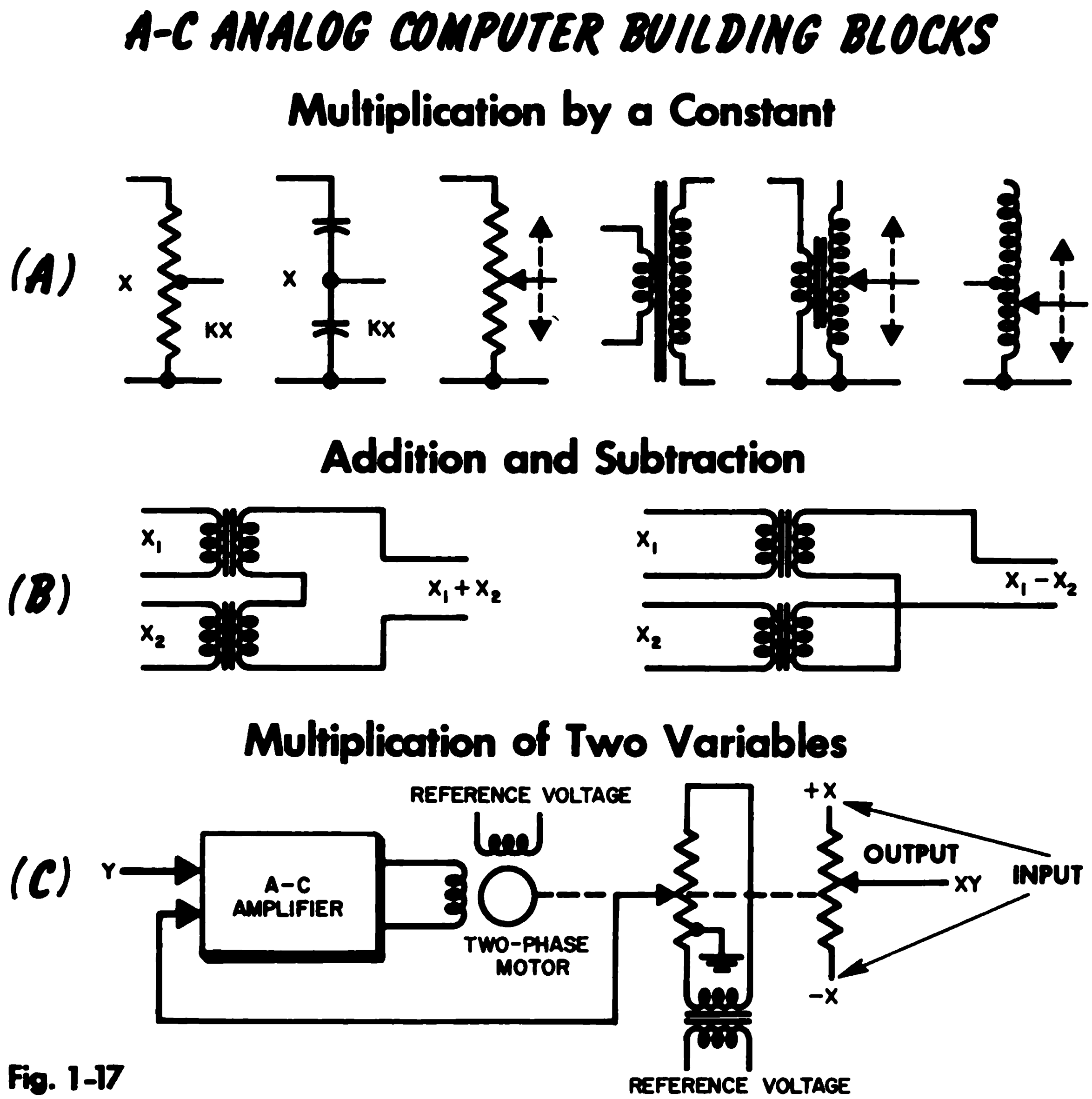


Fig. 1-17

The Integrator

The a-c modulated carrier type signal is described mathematically by the expression

$$v(t)\ K \sin (2\pi f_c t)$$

where v(t) is the computer variable (the envelope), K is some constant, and f_c is the carrier frequency (Fig. 1-18A).

In the case illustrated v(t) is also sinusoidal, hence the expression for the complete voltage signal is

$$E \sin (\omega t + \phi) \times K \sin (2\pi f_c t)$$

Now if we wish to perform the operation of integration we must be careful and integrate the correct variable. Clearly if we integrate the above func-

tion we will have the integral of the product of v(t) with the carrier K sin $(2\pi f_0 t)$. Of course we wish the integral of v(t) alone, that is

$$\int_0^t v(t)dt$$

In principle it is possible to dissociate v(t) from the total function and integrate it by use of a carefully tuned, high quality, resonant circuit. How-

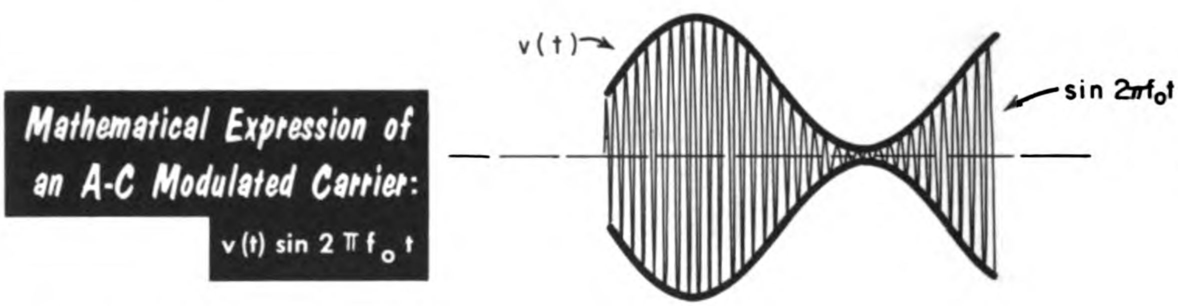


Fig. 1-18A

ever, component tolerances, temperature variations, and the difficulty of tuning a parallel L-C circuit to precisely the carrier frequency, f_0, prevents this technique from being practical.

One direct approach is to change from ac to dc with a phase-sensitive de-

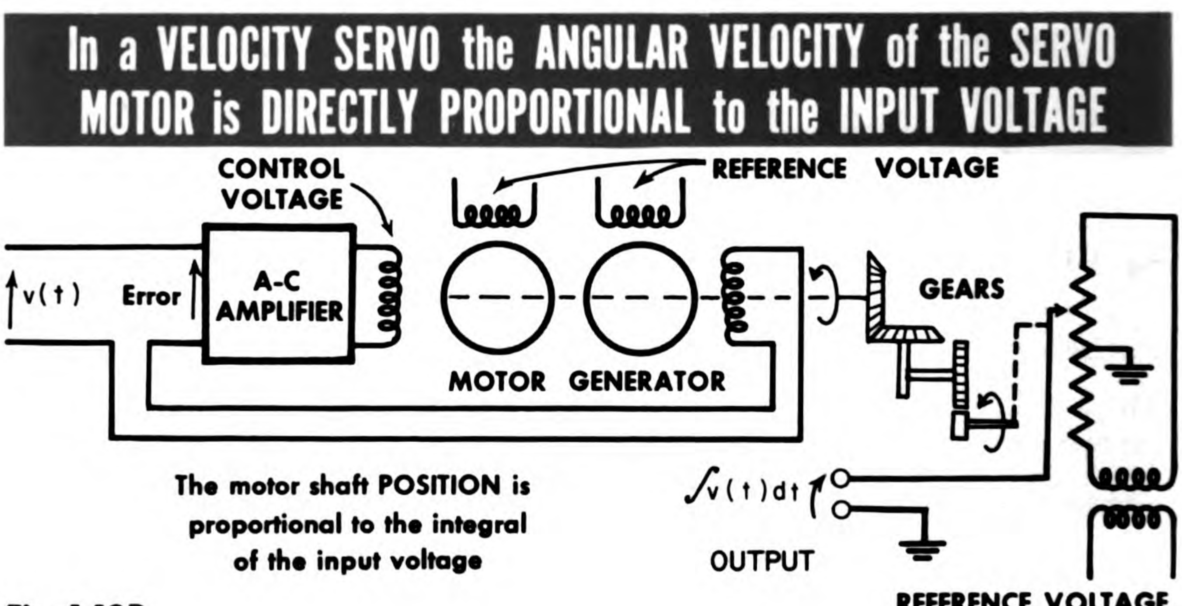


Fig. 1-18B

modulator, perform a d-c integration, and reconstruct the a-c by a synchronous modulator. This technique, however, has all the *dis*advantages of both a-c and d-c systems.

By far the best and most common a-c integrator is an a-c "velocity servo"

(Fig. 1-18B). In a velocity servo the angular velocity of the servo motor is made directly proportional to the input voltage. This is done by using as a feedback voltage the output of a small tachometer which generates a voltage proportional to the shaft angular velocity. The series addition of the input voltage and the feedback generator voltage creates an error voltage (since the generator voltage is of opposite phase). The error voltage is amplified, and applied to the control winding of the two-phase servo motor. The servo motor speed will change and the tachometer voltage will change to reduce the size of the error voltage. Since motor speed is proportional to input voltage, the motor shaft position (number of revolutions) must be directly proportional to the integral of the input voltage. Thus the a-c voltage on the wiper of a geared-down potentiometer is also proportional to the desired integral.

Trigonometric Building Blocks

Trigonometric functions are obtained for varying magnitude a-c signals by resistance-card resolvers. The alternating reference voltage is applied to a square wirewound resistance card, which has two wipers driven by a shaft through the center of the card (Fig. 1-19A). The shaft may be driven by a

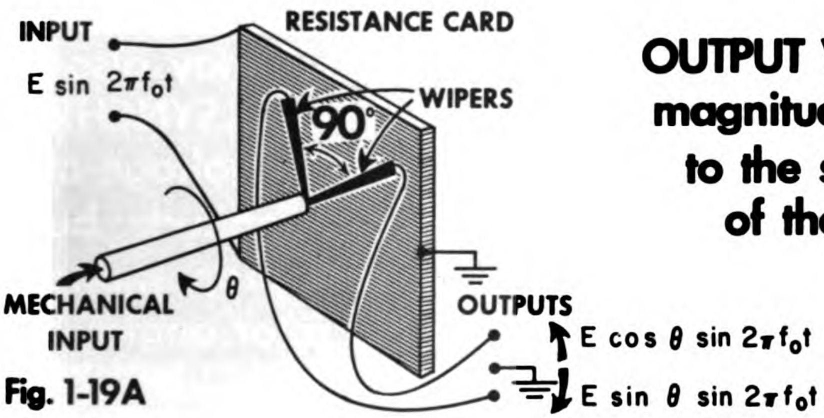


Fig. 1-19A

OUTPUT VOLTAGES have magnitudes proportional to the sine and cosine of the shaft angle

See Fig. 1-18 (B)

synchro or a position or velocity servo. The output voltages from the wipers have magnitudes proportional to the sine and cosine of the shaft angle.

Another method uses the transformer action of a two-phase induction machine where the two stator windings (physically displaced by 90°) provide the sine and cosine function of the rotor angle when the single rotor is excited by some a-c voltage (Fig. 1-19B). This is equivalent to resolving a vector $\vec{E}$ into its components, E cos Θ and E sin Θ.

This same device may also be used in a manner similar to a synchro generator to provide a voltage of constant magnitude but with phase proportional to the rotor angle, by exciting the stator windings with voltages 90° out of phase and taking the output from the rotor. This may represent the rotation of a vector of magnitude E(t) through an angle Θ.

TRIGONOMETRIC FUNCTIONS are obtained due to the TRANSFORMER ACTION of a TWO-PHASE INDUCTION MACHINE

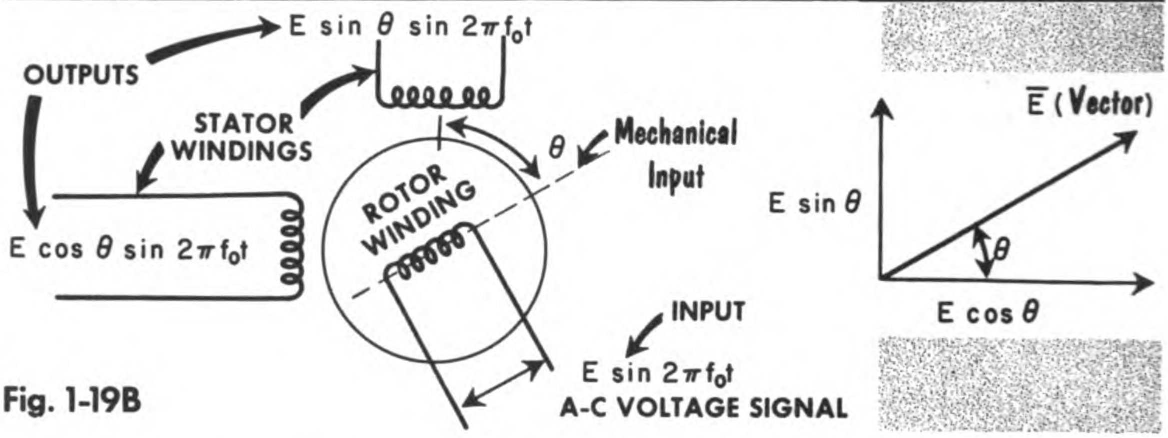


Fig. 1-19B

This device may also be used to obtain an output voltage whose *magnitude* varies according to the magnitude of the applied stator voltage and whose *phase* varies proportional to the rotor angle (Fig. 1-19C). The stator windings are excited by some input voltage, $E(t) \sin 2\pi f_0 t$, and the same volt-

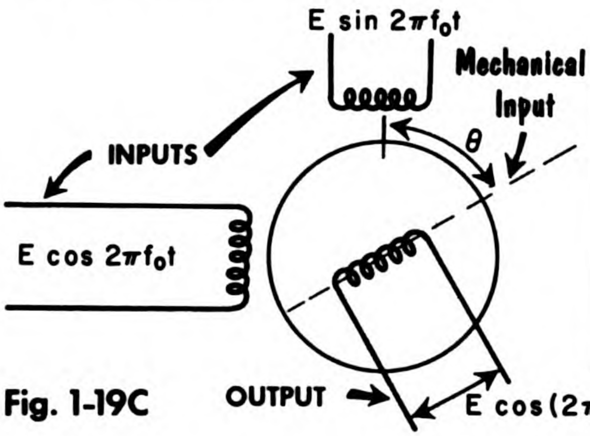


Fig. 1-19C

This DEVICE may also BE USED like a SYNCHRO to provide a voltage of constant magnitude, but with phase proportional to the rotor angle

age shifted in phase by 90°, $E(t) \cos 2\pi f_0 t$. Then the rotor voltage is $E(t) (\sin 2\pi f_0 t) (\cos \Theta) + E(t) (\cos 2\pi f_0 t) (\sin \Theta)$; but this is the same as $E(t) \sin (2\pi f_0 t + \Theta)$. This says the magnitude varies as $E(t)$ and the phase as Θ. The operation amounts to forming a single voltage signal which contains both the magnitude and angle of a vector. Such a voltage might be called a *vector voltage.*

Vector Operations

Vector rotation can be accomplished by a device such as that shown in Fig. 1-19C, but with a second rotor winding at right angles to the other rotor winding (Fig. 1-20A). Let the stator voltages be the components of a given vector, that is $\vec{E}(t) = [E_x(t) + E_y(t)] \sin 2\pi f_0 t$.

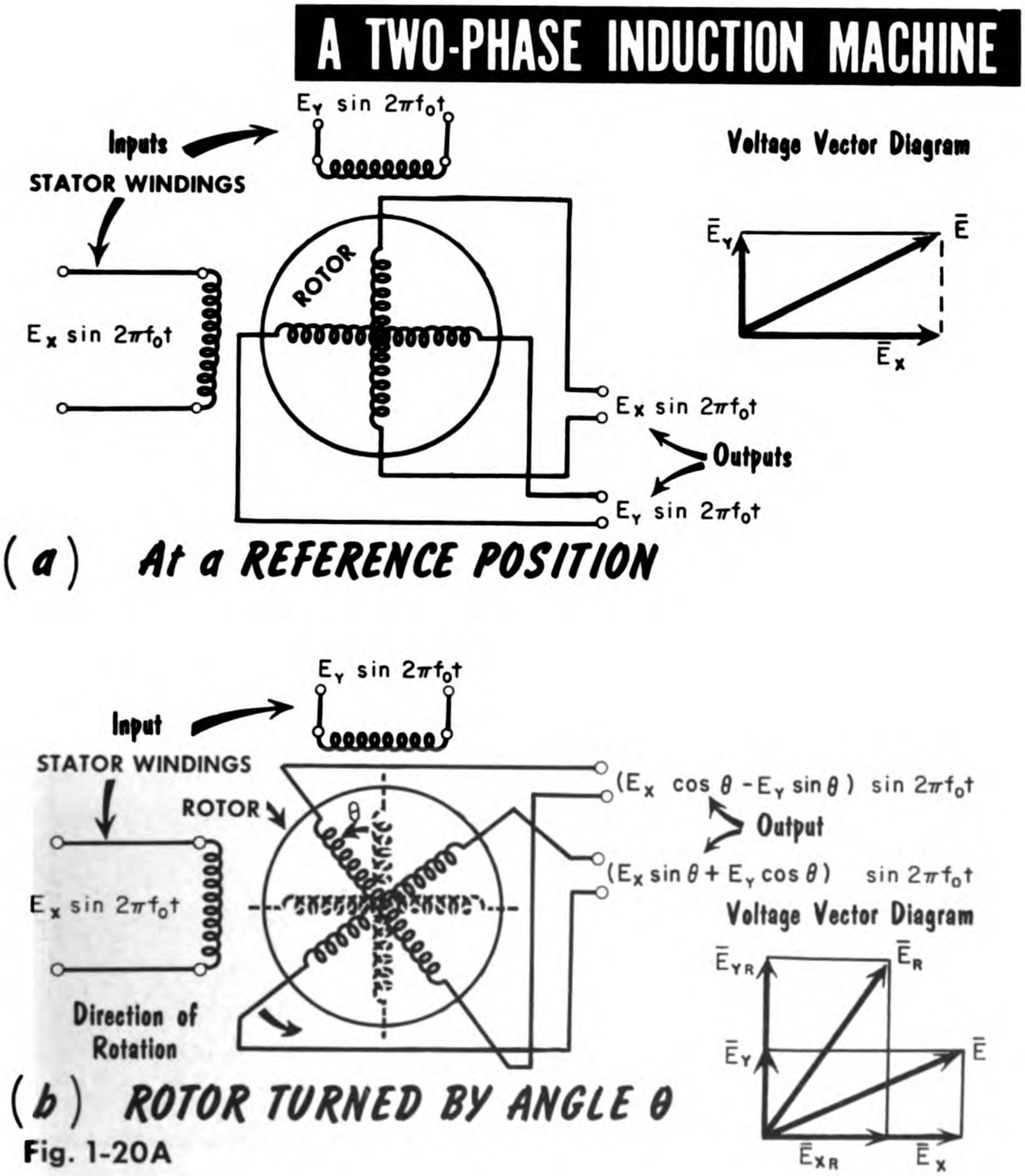


Fig. 1-20A

Each stator (input) winding induces a voltage in each rotor (output) winding proportional to the voltage applied to the stator and reduced by the sine or cosine, as appropriate, of the rotor angle. If a particular stator-rotor winding pair are parallel (or "lined up"), full input voltage is induced, but if perpendicular, no voltage is induced. Thus, except when $\Theta = 0$, 90°, 180°, and 270°, there are two components of voltage induced in each rotor winding. These rotor voltages are described by the equations

$$E_{xR} = E_x \cos\Theta \sin 2\pi f_o t - E_y \sin\Theta \sin 2\pi f_o t$$

and

$$E_{yR} = E_x \sin\Theta \sin 2\pi f_o t + E_y \cos\Theta \sin 2\pi f_o t$$

Since each voltage is an a-c sinusoidal voltage with the same frequency, f_o, the sums are sinusoidal voltages with the same frequency and with magnitudes indicated by

$$\text{magnitude of } E_{xR} = E_x \cos \Theta - E_y \sin \Theta$$

$$\text{magnitude of } E_{yR} = E_x \sin \Theta + E_y \cos \Theta$$

If E_x and E_y are components of a vector $\vec{E}$, then E_{xR} and E_{yR} are components of $\vec{E}$ rotated through an angle Θ.

A simple device for vector addition of two vector voltages $[\vec{E}_1(t) = E_1(t) \sin (2\pi f_o t + \Theta_1)$ plus $\vec{E}_2(t) = E_2(t) \sin (2\pi f_o t + \Theta_2)]$ is the common-cathode summing amplifier (Fig. 1-20B). The voltage across the resistor has a magnitude proportional to $E_3(t)$ and phase angle Θ_3, which are defined by the vector addition in the illustration.

Vector addition can also be accomplished by adding the components of two vectors by pairs, resulting in the components of the vector sum

$$E_{3x} = E_{1x} + E_{2x}, \quad E_{3y} = E_{1y} + E_{2y}$$

Thus the form of the adding device depends upon the available forms of the vector information (that is, magnitude and angle, or *x* and *y* components).

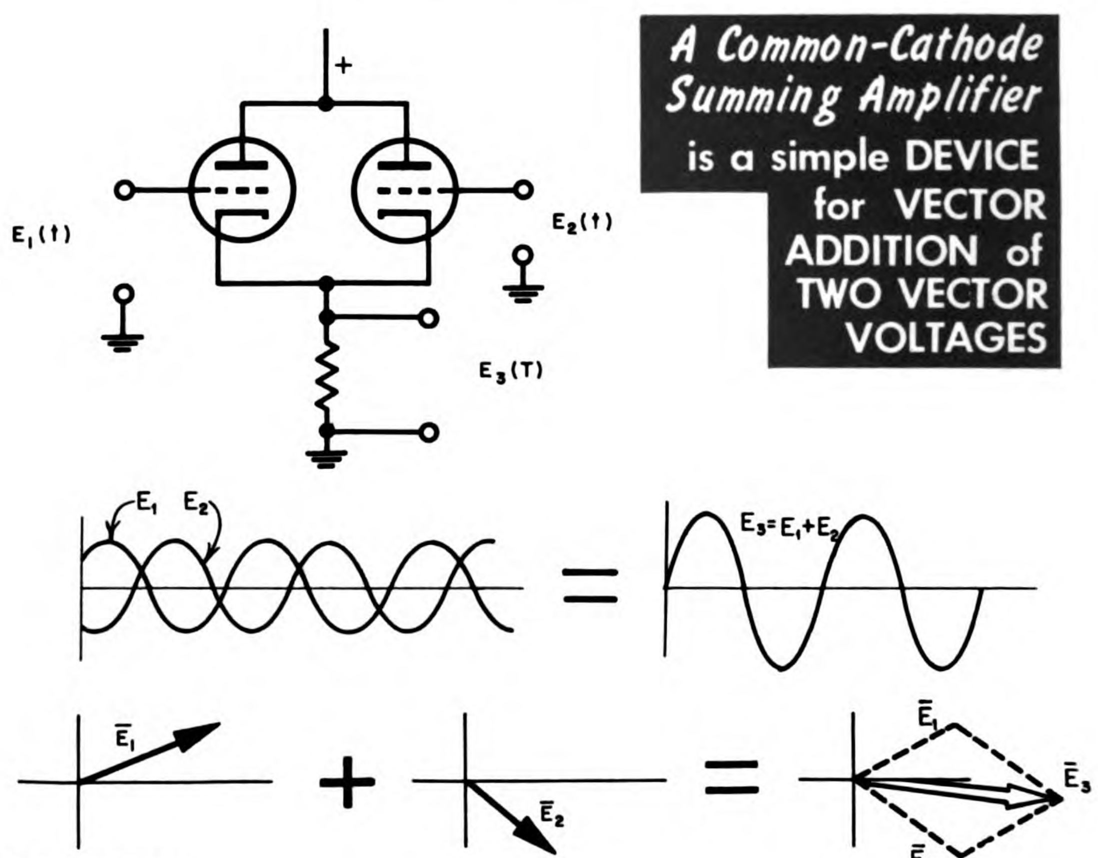


Fig. 1-20B

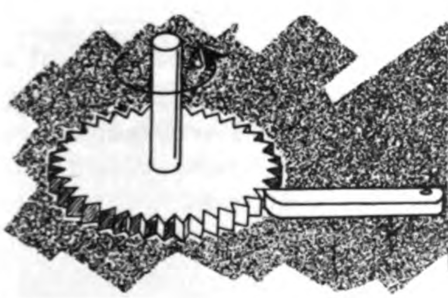
INCREMENTAL ANALOG COMPUTERS

The Digital Differential Analyzer (DDA)

In Chapter 4 of Volume 1 it was explained how the mathematical process of integration could be approximated by a continuous summation of numbers. The numbers represented the areas of small segments of equal width under the curve of the function being integrated, and the cumulative sum approximated the total area under the curve. The method is simple, and the result can be as accurate as one desires by using very small segments. Such a method is termed *numerical integration*, and is employed by general purpose digital computers as well as by persons performing hand calculations.

A similar, approximate means of integration is accomplished by a continuous counting of increments that represent segments of equal area. As above, functions are approximated by stair-case-type functions, and in this case the "stair-steps" are of equal size, and occur at unequal spacings. The accuracy of this method depends on the weight or value assigned to an increment relative to the values of the functions. For accurate results, an increment must represent only a very small change in a variable or function. A digital differential analyzer (DDA) is a general-purpose computer consisting primarily of incremental integrators. The discrete, incremental changes in computer variables are represented by sequences of unevenly spaced electronic pulses of equal weight. The integrators continually count pulses, some positive, some negative. Electronic circuits are employed to count the pulses, and to hold the total count as a number.

To understand better the operation of a DDA, consider a hypothetical mechanical equivalent. Envision the Bush mechanical differential analyzer with all the shafts of the wheel and disc integrators equipped with ratchets, i.e., the 2 input shafts and the 1 output shaft having toothed wheels and pawls as shown in Fig. 1-21A. Such an arrangement would restrict the



A FICTITIOUS MECHANICAL WHEEL and DISC INTEGRATOR with shafts equipped with ratchets, helps clarify the idea of a differential analyzer operating with discrete or digital variables

Fig. 1-21A

shaft positions to a number of discrete positions determined by the teeth of the wheel. One increment of shaft rotation would correspond to a count of 1 in a shaft-revolution counter. This fictitious integrating device is suggested solely to clarify the idea of a differential analyzer operating with discrete or digital variables. The device might be an integrator in a *mechanical digi-*

tal differential analyzer or *mechanical DDA*. A real DDA is an electronic device which operates with discrete or digital variables also, and performs the same mathematical operations as the mechanical DDA. Instead of discs on a ratchet the electronic variables are voltage *pulses*. Instead of a shaft revolution counter, an electronic *pulse counter* is used. The electronic circuits in a DDA are very much the same as those in a general purpose digital computer. In spite of this and the digital nature of the DDA variables, the DDA is more accurately classified as an *analog computer*, for the computer integrates differential quantities (pulses) rather than a numerical representation of such. The integration is performed in a manner approximately analogous to the behavior of some physical variables. If each pulse represents only a minute change the DDA can approximate very closely the dynamic behavior of a physical system.

One input to the DDA integrator is the differential change in the variable, y, called dy. dy is not a true mathematical differential, but serves to identify the train of pulses counted by the y pulse counter. The count of the y counter (that is, the quantity y) is then multiplied by the other input, dx,

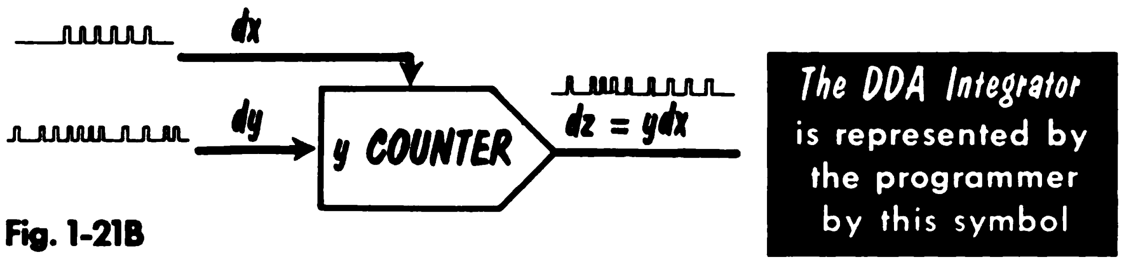


Fig. 1-21B

to form the product ydx which is identified as dz, the output pulse train. The DDA integrator is represented by the programmer with the symbol in Fig. 1-21B.

In a manner very similar to the programming of any other kind of analog computer, the DDA is programmed by:

1. Drawing a flow diagram showing the interconnection of building blocks to solve the desired equation [Fig. 1-22 (A)]. For example:

$$\ddot{x} = -\dot{x} - x$$

or

$$d\dot{x} = -\dot{x}dt - xdt$$

2. Actually interconnecting the building blocks according to the flow diagram [Fig. 1-22 (B)].
3. Providing the initial values of the variables held by the integrator counters.
4. Turning on the clock-pulse generator and recording the results.

Every time a pulse enters the left-hand end of the DDA integrator it is

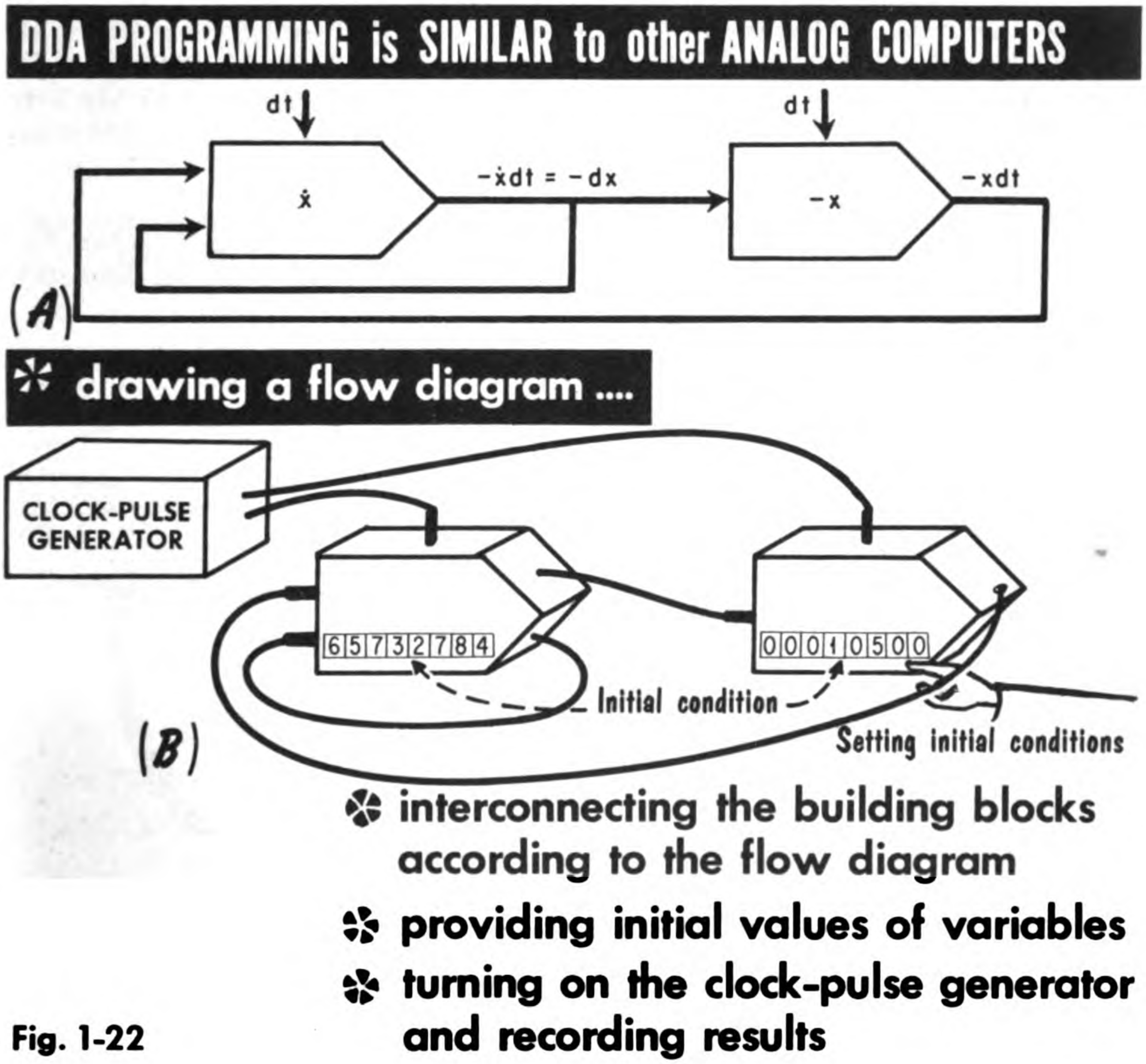


Fig. 1-22

added (or subtracted if the pulse is negative) to the y counter. Every time a pulse enters the top of the device a multiplication takes place in a manner which produces pulses at the output that occur at a rate proportional to the product ydx. The number in the y counter is taken to be a fraction; the maximum value of y must be less than unity. Thus the output pulse rate, dz, is the input rate, dx, attenuated or reduced by the fraction, y. The multiplier is a *rate attenuator*.

The Serial DDA

We have described briefly, a DDA of a most advanced type, in which many, separate integrators are interconnected to solve problems. Such DDA's are only recently on the market at the time of publishing this book. The potential in speed and accuracy of these computers is very great.

DDA's developed between 1949 and 1958 are all of a single type known as the *serial DDA* or *sequential DDA*. In these machines a very clever technique

is used to design a computer with up to 100 integrators, but with the circuitry for only *1* integrator. A great saving in complexity and equipment is achieved. In fact, the integrators do not exist as such, but only in the programmer's notation. However, the integrations are executed as if there were

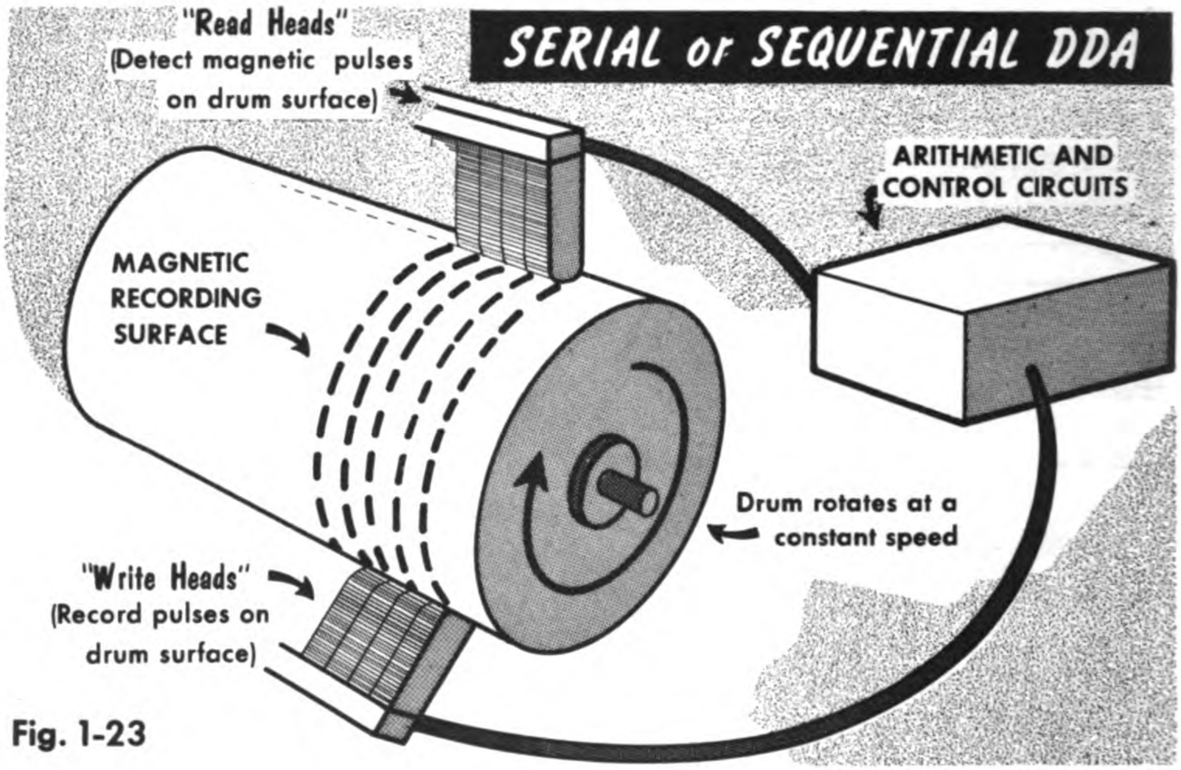


Fig. 1-23

many integrator building blocks. The circuitry is shared by all integrations, the calculations for each being performed in sequence. The intermediate results of each calculation are stored on a magnetic drum (Fig. 1-23), while the other integrators are being "serviced" by the circuitry. In this manner all the integrations are executed in one drum revolution. Typical speeds for drums run from 30 to 60 revolutions per second. Thus it is possible to service 100 integrators in 1/60 of a second. However, while such speeds may seem fast, they can hardly compare with a computer in which all integrators are operating all the time. Hence all serial DDA's are, relatively speaking, very slow computers.

THE D-C ANALOG COMPUTER

Introduction

The overall purpose of this book is two-fold: first, to present and illustrate the philosophy and underlying principles of analog computing, and second, to describe in detail the construction and operation of one specific kind of computer — the d-c analog computer. The brief discussions of other computing devices and computers have been given to fulfill the first purpose as

well as to serve as an introduction to the d-c analog computer. The choice was made to concentrate on this particular kind of computer because it is the only analog computer in really widespread use. Moreover, excluding transistor versions of this system and the digital differential analyzer, it would seem that the general purpose d-c analog computer described in later chapters of this book will continue to dominate the analog computing field for many years to come. Notwithstanding the contrary opinions of some engineers experienced in the ways of digital computers, there are many, many computer applications that are and always will be handled best by analog computers.

The general purpose d-c analog computers to be described at length are appropriately divided into several categories. The principles of operation of all of them are essentially the same and in some cases the building blocks are identical. The division into the following categories (Fig. 1-24, A-D) is primarily along lines of performance capabilities (speed and accuracy) and thus indirectly along lines of different application.

General Purpose D-C Analog Computers

- *Desk size*, 1%–5% precision, low cost computers with 10 to 20 unstabilized amplifiers. Primarily for educational purposes and small experimental design problems.
- *Desk size*, high-precision (0.1%), transistorized computers with 10 to 40 chopper-stabilized amplifiers, plus multipliers, squaring units, and function generators. For serious scientific investigations not requiring a large number of amplifiers.
- *Laboratory size*, high-speed, low-precision (5%–10%) repetitive-type computers with 10 to 200 high-frequency, medium-gain amplifiers. For experimental design of small to large linear and certain nonlinear systems.
- *Laboratory size*, very high-precision (0.01%–0.1%), computers with 20 to 500 wideband, very high d-c gain (3×10^8), chopper-stabilized amplifiers, and a full complement of nonlinear building blocks (multipliers and function generators). Superior performance for all applications; the most common (and most expensive) of all types.

Computer Types

Low cost *desk size computers* come in a variety of forms from "do-it-yourself" kits, to "assemble-it-yourself" computers made of many small black boxes interconnected by phone cords, and to professional-like units with removable "patch panels" [Fig. 1-24 (A)].

The primary disadvantage of these computers is the lack of automatic drift stabilization for the amplifiers, with the result that the unavoidable drift in the d-c amplifiers must be balanced out by manual adjustment every so often. Very few, if any, automatic features are built in, such as initial condition RESET relays, or HOLD relays; these must be wired up by the

GENERAL PURPOSE ANALOG COMPUTERS are DIVIDED INTO SEVERAL CATEGORIES

(a) Heathkit desk-size, kit-type computer

(b) Donner desk-type small computer with removable program board

Fig. 1-24 (A)

programmer with the relays and connectors provided. While such a lack is a hindrance in large-scale computing it makes these computers ideal for instructional purposes.

While great care and considerable expense is required in the design and construction of a high-precision computing system, large size is not necessarily a factor. Advances in transistor technology have resulted in computers which are compact, lightweight, and use very little electric power. The Electronic Associates TR-10 computer illustrated in Fig. 1-24 (B) is an example of a *high-precision, transistorized, desk-size computer*. One or more such computers might be utilized by an engineering firm not requiring a large computing laboratory. For example, when there are several small problems to be investigated over and over again for different conditions, small computers, tailored to the particular problems, might be preferred, to the use of one large computer at a central facility.

The *high-speed, repetitive-type computer* illustrated in Fig. 1-24 (C) differs sufficiently from the others in principle to warrant a brief description here. The significant point is that these computers produce a complete solution to the programmed equations in as short a period as several milliseconds. The solution is produced not just once, but hundreds of times per second, that is, repetitively. The solution is presented to the programmer upon the screen of an oscilloscope. The object is that the programmer be able to manually change selected parameters in his equations and *without delay* observe the

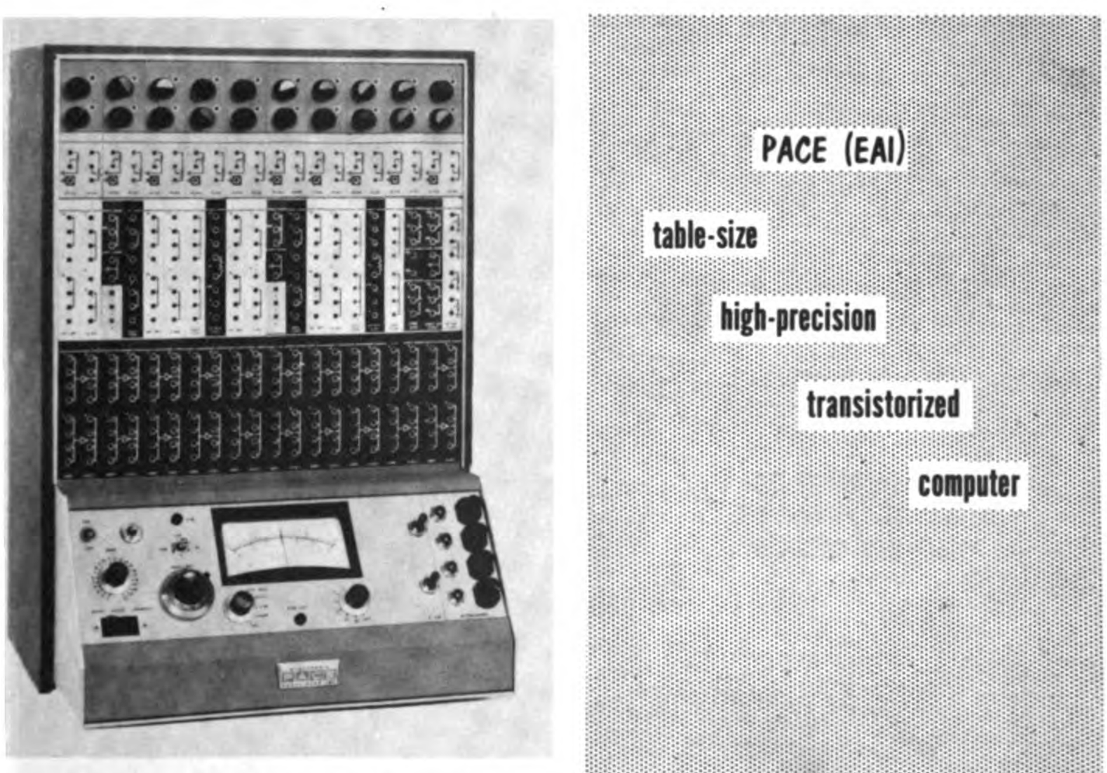
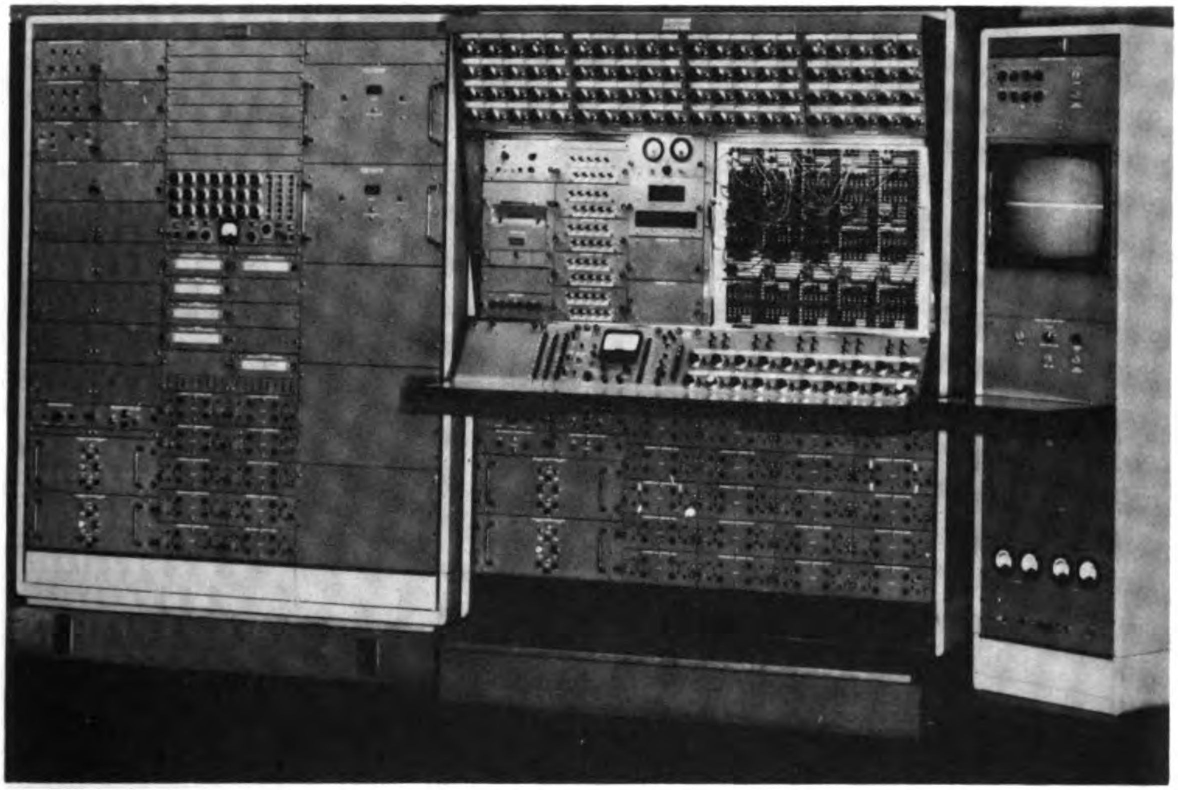


Fig. 1-24 (B)


Computing amplifiers for summing and integrating
Scope display for repetitive operation
Philbrick
high-speed, repetitive-type
laboratory-size computer
MULTIPLIER
DIVIDER

Fig. 1-24 (C)



PACE (EAI)
a fully-expanded, laboratory-size,
high-precision, high-speed analog computer
Fig. 1-24 (D)

effect of the changes on a sustained display of the solution. Since the solutions consist of rapidly changing variables and the total solution repeats hundreds of times per second, the amplifiers and all computer circuits must be capable of handling very high frequencies. This capability is accomplished at the expense of high gain (amplification) or, what is comparable, the flexibility and rapid presentation of these computers is obtained at the expense of accuracy. While very good for investigations of a qualitative nature, the high-speed repetitive computer has limitations in studies requiring precise quantitative results.

The ***high-precision, laboratory size, d-c electronic analog computer*** illustrated in Fig. 1-24 (D) is a most important engineering and scientific tool. The remainder of this Volume, and Volume 3 is devoted to its study, but a rapid survey of the characteristics and components of the computer is given first in the following several pages. A detailed description of each component and control feature is given in the rest of this Volume, and in Chapter 1 of Volume 3.

Typical of solution times on high-precision computers is 5 to 120 seconds. Longer times are possible for special purposes but the majority of problem solutions require less than one minute. This should be compared with typical digital computer solution times for comparable problems of one minute to several hours. Absolute comparisons of this sort are meaningless, since, for example, digital computers are capable of precisions beyond those possible with analog computers. However, since digital computers have only one "computing center" and must perform all mathematical operations in a sequential manner, while analog computers perform all computations simultaneously and continuously, the latter are inherently much faster devices [Fig. 1-25 (D)].

A word of caution must also be offered regarding the use of the terms "high precision" and "accuracy" (Fig. 1-26). These two terms are often used interchangeably, indiscriminately, and without regard for the reader's understanding of what is meant. The authors wish to restrict the meaning of the word *accuracy* to its use as *a measure of the difference between a computed solution and a true solution to a problem.* Accuracy should not be used to describe physical devices or computer building blocks. The word *precision* is then assigned to the duty of describing *the quality of* such *physical components.* Precision may be indicated by a percentage (0.01%) or by a ratio (10^4 : 1 or one part in 10,000). In both cases a reference scale must be indicated, for surely one must ask, "0.01% of *what*?" When referring to analog computers the reference scale is understood to be the *full-scale* variation of the voltage variable. In the large majority of cases this is 200 volts (from

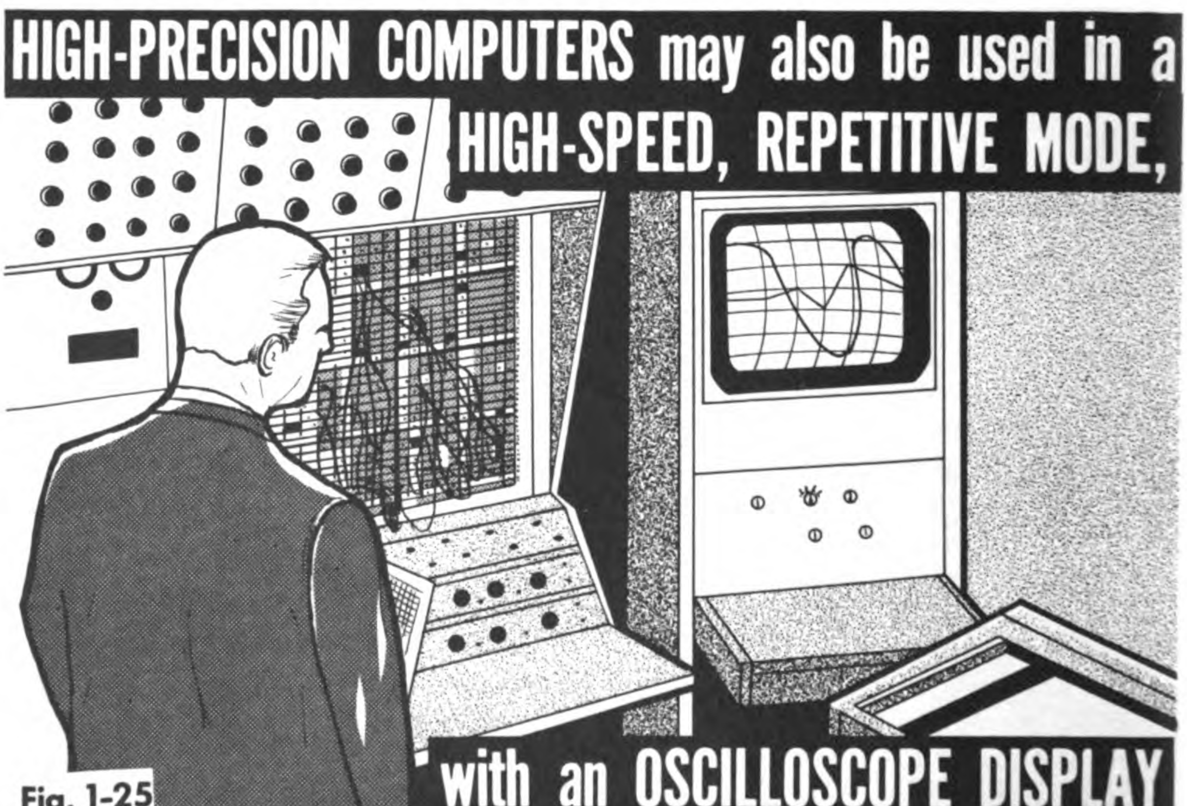


Fig. 1-25

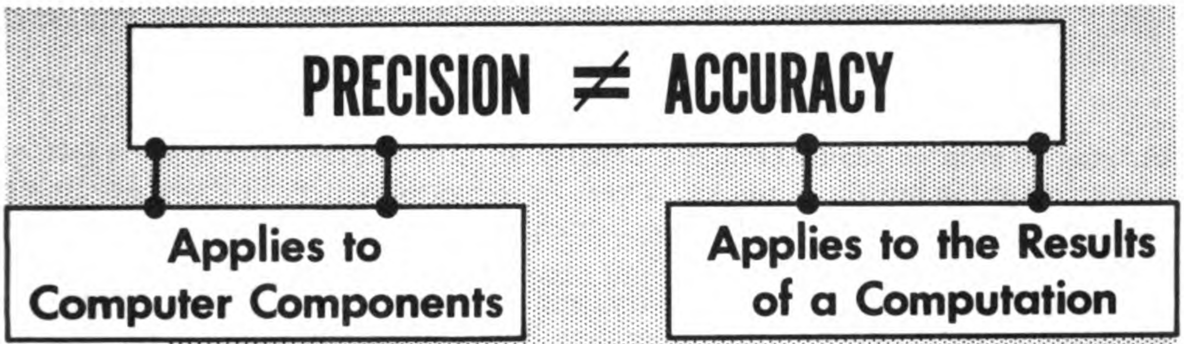


Fig. 1-26

−100 volts to 100 volts. To determine the smallest detectable signal variation it is necessary to know the reference scale.

Computer Precision

In the high-precision analog computer one speaks of component precision of 0.01% of full scale, meaning that the quality of the components is such that the smallest discernible computer variable change is 0.02 volt or 20 mv (which is 0.01% of 200 volts) (Fig. 1-27). Another way of saying it is that the precision is the ratio of the maximum or full-scale range to the smallest

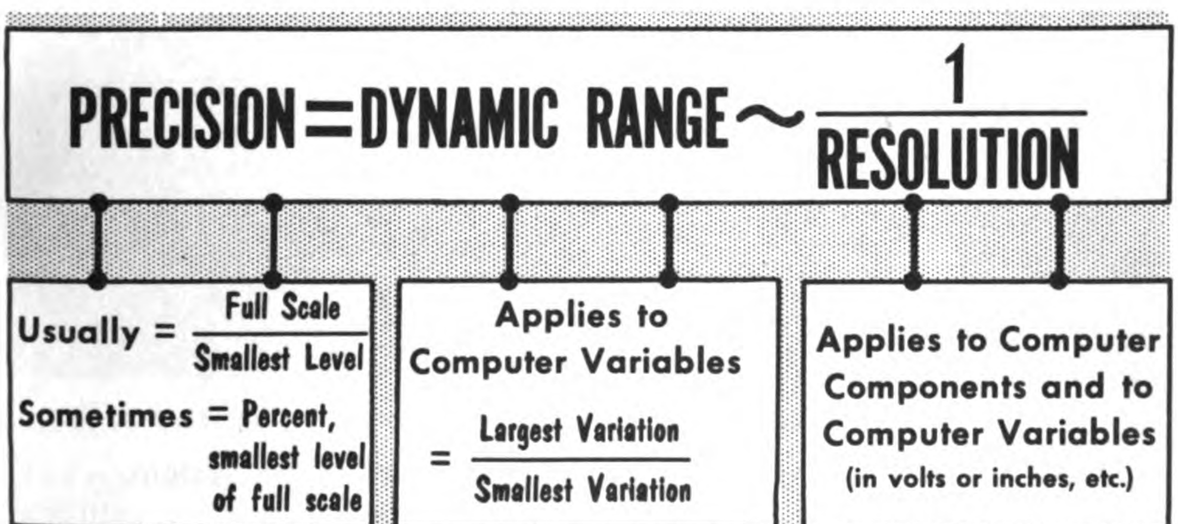


Fig. 1-27

discernible change of the computer variable; or, 200 volts : 0.02 volt = $10^4 : 1$. Now one might suggest that voltages smaller than 20 mv can be easily measured with good quality instruments. And so they can, but 20 mv is still considered the minimum resolution (the reciprocal of precision) of a good analog system, for this is just above the voltage level of many extraneous, random, uncontrollable voltage variations (generally called electronic noise) in a typical computer system.

The term *dynamic range* is the ratio of largest possible to smallest variation and hence is the same as precision, although it refers more to computer variables than to computer components.

Analog computer manufacturers usually supply specifications for each of their computer building blocks. These specifications describe the precision of the units, range of variables over which the precision figures are valid, the phase shift at some high frequency, d-c drift characteristics, percent of linearity, resolution and granularity, and other characteristics as appropriate. Such specifications place tolerance limits upon the errors that may be expected in each computing unit. It cannot be overemphasized, however, that no matter how good the specifications of the individual units are, one must not give in to the temptation to draw conclusions about the precision of a complete computer or a particular computer program, based upon the component precisions. As nice as it would be to have a precision figure for an entire computer or program from which to predict the *accuracy* of the solutions, it is pure folly to attempt such a thing. There exists no practical way to obtain a good estimate of the "precision" of a computer program even

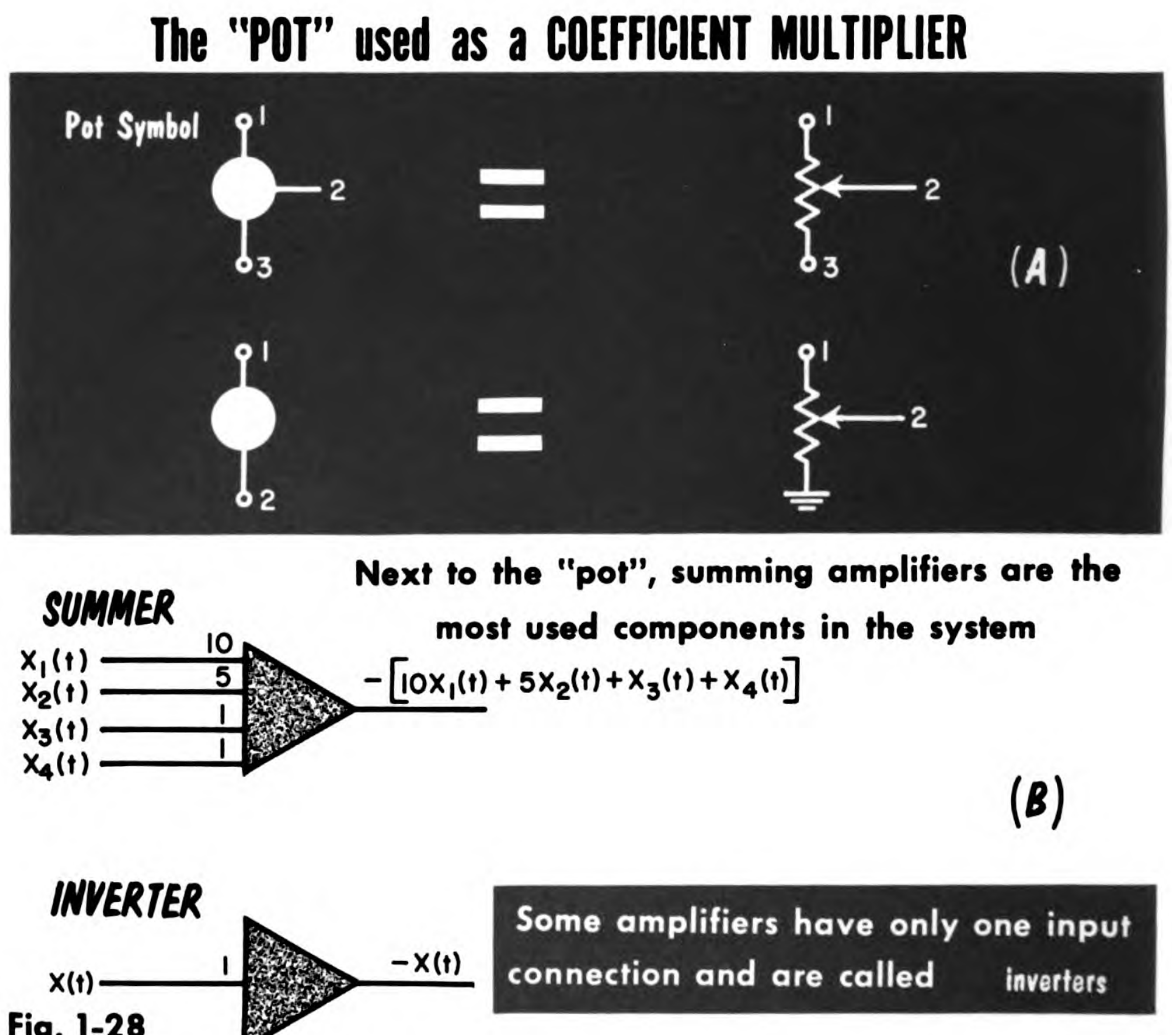


Fig. 1-28

if one knew what was meant by the expression! The only way to estimate the accuracy of solutions is to calculate the accuracy for special cases for which the true solution is known.

Building Blocks and Schematic Symbols

At this point it is appropriate to describe, very briefly, each of the building blocks commonly found in the general purpose d-c analog computer. Not all technical terms used here will be explained, for considerable attention is devoted to each item later. It is intended throughout the next several sections that the reader learn something about the computer as a system and about its operation.

For convenience in programming each building block is assigned a particular schematic symbol which is used to represent the unit in the programmer's schematic or flow diagram of every computer program. A "program" is an arrangement of interconnections of building blocks forming a computer circuit which will compute a solution for a given set of equations. This arrangement of interconnections is most easily displayed by a schematic diagram using the computer symbols for the following building blocks.

The *potentiometer* is used for coefficient multiplication. The symbol is a small circle with two or three connections. When only two connections are shown it is understood that the bottom of the "pot" (short for potentiometer) is grounded [Fig. 1-28 (A)]. Arrows are not required, for the direction of travel of a signal is adequately indicated by the rest of the diagram. The signal always flows into the top of a pot.

The pot is the most used component and the *summing amplifier* is next on the list. *Summers* have from two to as many as seven input connections. In case of many inputs, some of the inputs multiply the variable by a constant such as 5 or 10. This must be indicated on the symbol. The output voltage is always the negative of the sum of the inputs; hence the amplifier multiplies the input sum by *minus one* [Fig. 1-28 (B)]. However, *all* amplifiers multiply by -1, so this sign inversion is not shown in the symbol but must be remembered by the programmer. Some amplifiers may have only one input connection and are called *inverters*.

The next most used component is the *integrating amplifier*. The integrating amplifier has the same input connections as the summing amplifier, with the exception that a special kind of input connection is provided for inserting the initial value of the integral at the beginning of the problem. Such initial conditions are referred to as "IC's" and the input connection is the I-C input [Fig. 1-29 (A)].

Both the amplifiers just referred to are *negative* feedback amplifiers, for a specific feedback impedance is provided for the summer and for the integrator. It is necessary sometimes to have an amplifier with no feedback built in so that the programmer can build special circuits. Such an amplifier is called a *high-gain amplifier* (hga) [Fig. 1-29 (B)]. It has the same sign in-

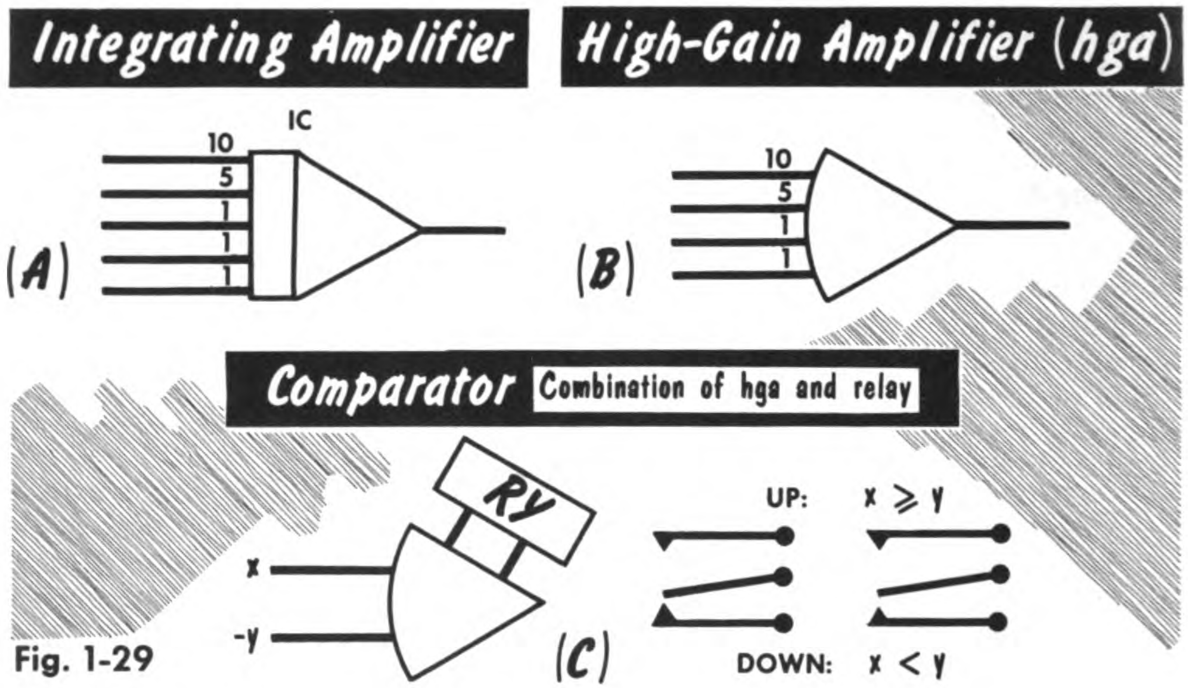


Fig. 1-29

version properties and input connection properties as other amplifiers but has an extremely high gain. The hga cannot be used without *some* form of feedback; it is up to the programmer to provide it. Since the overall gain is not determined until the feedback is provided, the input factors of 1, 5, and 10 mentioned, indicate only relative amplification. In the summer and integrator the 1, 5, and 10 are referred to as "gains of 1, 5, or 10" for those inputs.

A high-gain amplifier is often used to drive a mechanical relay. The high gain makes the relay throw faster. The relay coil and circuits to limit the output of the amplifier are shown by a box, and the double-pole-double-throw (dpdt) contacts of the relay are shown separately. If the sum of the input voltages is positive the relay will throw to one side, if negative to the other side. The combination of hga and relay forms a comparator, for if x and $-y$ are inputs, the relay will be in one position or the other accordingly as x or y is the larger [Fig. 1-29 (C)].

At this point however, we are discussing and introducing standard programming symbols and the *functions* for which they stand; the actual operation of the relay need not concern us here.

More Building Blocks and Symbols

Double-pole-double-throw switches for manual control at the console are very useful for making quick changes of parameters or circuits. These are called *function switches* [Fig. 1-30 (A)].

Diodes are common elements for use in the feedback path of high-gain amplifiers. Both thermionic (1) and solid-state (2) diodes are used; current flows in the direction of the arrows [Fig. 1-30 (B)].

Electronic multipliers form products between two or three input variables. These units are particularly useful when the input variables are rapidly varying, at frequencies from 10 to 100 cps [Fig. 1-30 (C)].

All multipliers introduce a *scale factor* of 1/100, the output being one hundredth of the product of the two inputs.

Servo multipliers [Fig. 1-30 (D)] provide up to five channels of multiplication, that is, five products with the input to the servo. The speed of the servo input variable *x(t)* is restricted to less than 10 cps, but these multipliers are less complex than electronic multipliers, and they have the additional feature of function generation described below. The square-box symbol includes the servo amplifier, motor, and "follow-up" or feedback potentiometer. The circles are the five servo-driven potentiometers, with the mechanical connection indicated by the dashed line. Top, bottom and the wiper arm (arrow head) of the potentiometer are shown.

Because of their physical appearance these potentiometers are called multi-

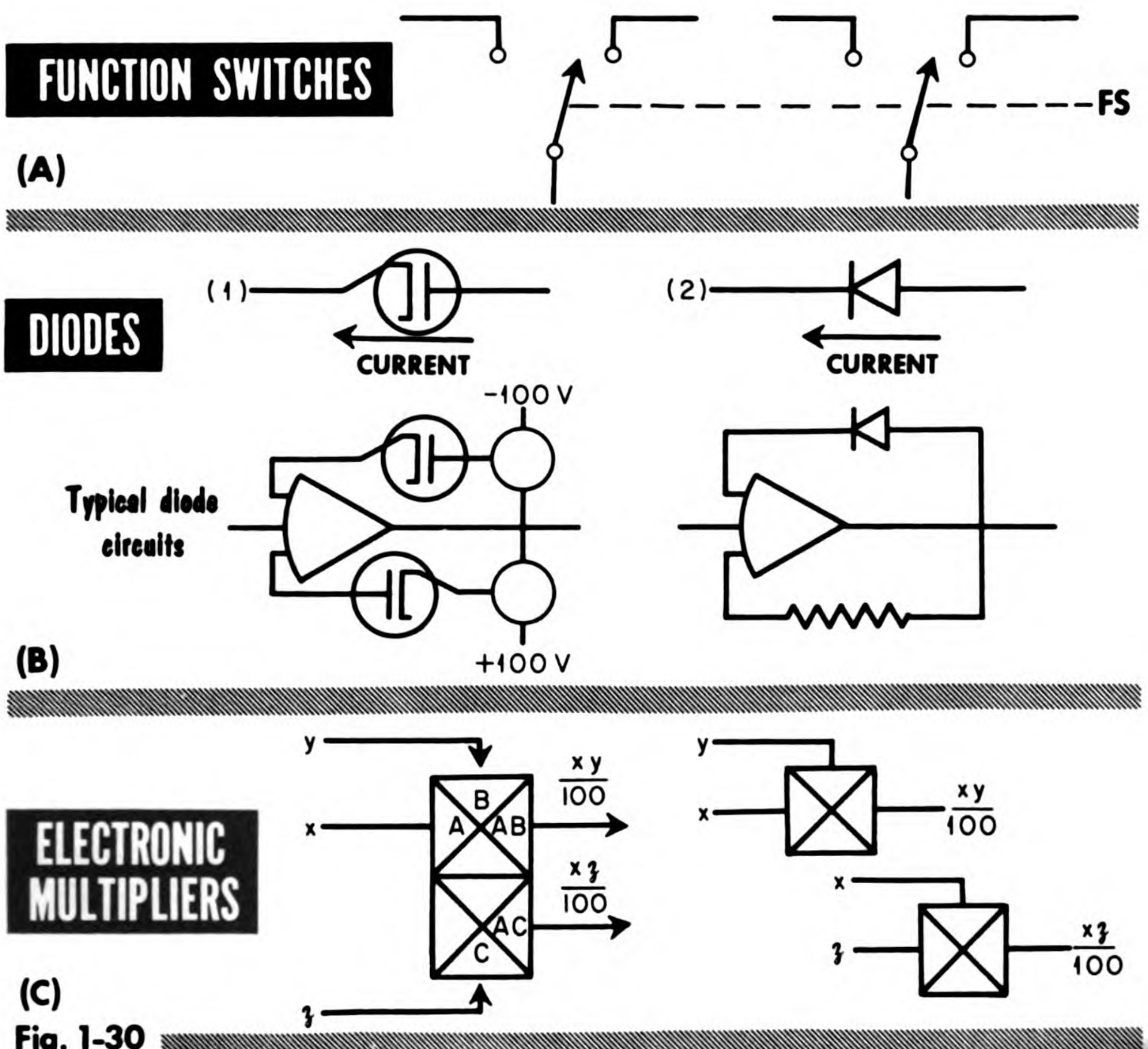


Fig. 1-30

plying *cups*. The connection for multiplication of x and y is shown. "N" stands for "normal connection". "1M" indicates the loading on the follow-up cup is 1 megohm. These terms are explained later. With the servo symbol and its cup symbols properly identified, it is common practice to draw the circles and square wherever convenient for efficient schematic layout, rather than lined up as shown.

The servomultiplier may be used as a function generator [Fig. 1-30 (E)], when used in conjunction with a "pot-padder" or "cup-padder" unit, and when special "tapped cups" are available on the servo-motor shaft. The tapped cup has 17 connections or taps linearly spaced along the potentiometer winding. The cup padder unit fixes the voltage at each tap at some value determined by the desired function, $f(x)$. The wiper arm "sees" only the voltages set at the taps and the linear interpolation between taps. Hence the output is a straight-line approximation to $f(x)$. The symbol only

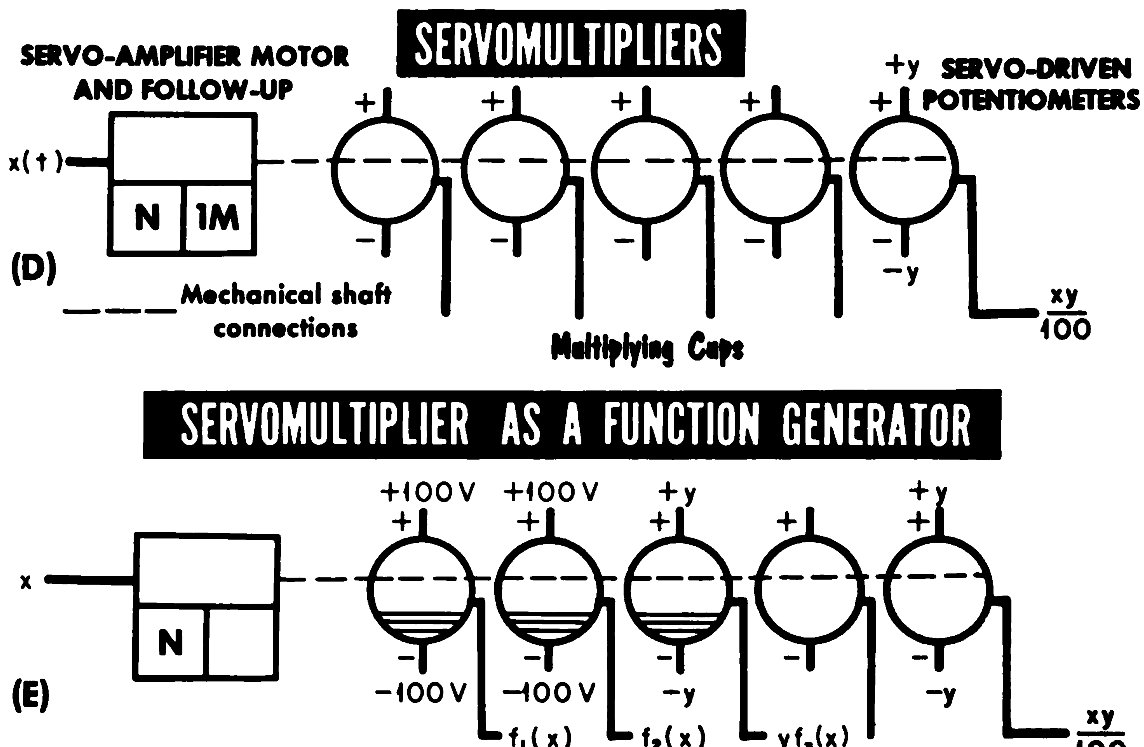


Fig. 1-30

adds three lines to the servo-cup symbol. Several multiplying and padded cups may be on the same servo shaft.

Servo resolvers (Fig. 1-31A) perform the operations indicated here by the symbol. The resolver has two modes of operation: *polar* and *rectangular*, corresponding to input signals representing polar coordinate variables and rectangular coordinate variables.

The *diode function generator* (dfg) [Fig. 1-31B, part (a)] is used to form an arbitrary function of some voltage variable. The actual arbitrary func-

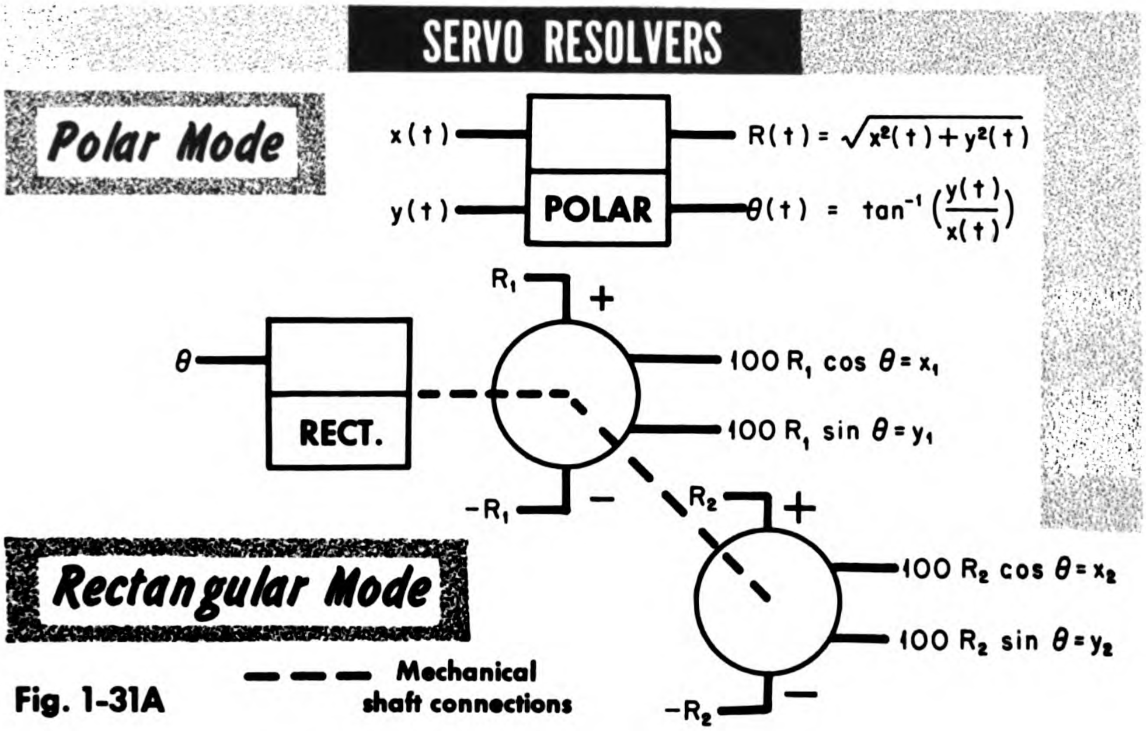


Fig. 1-31A

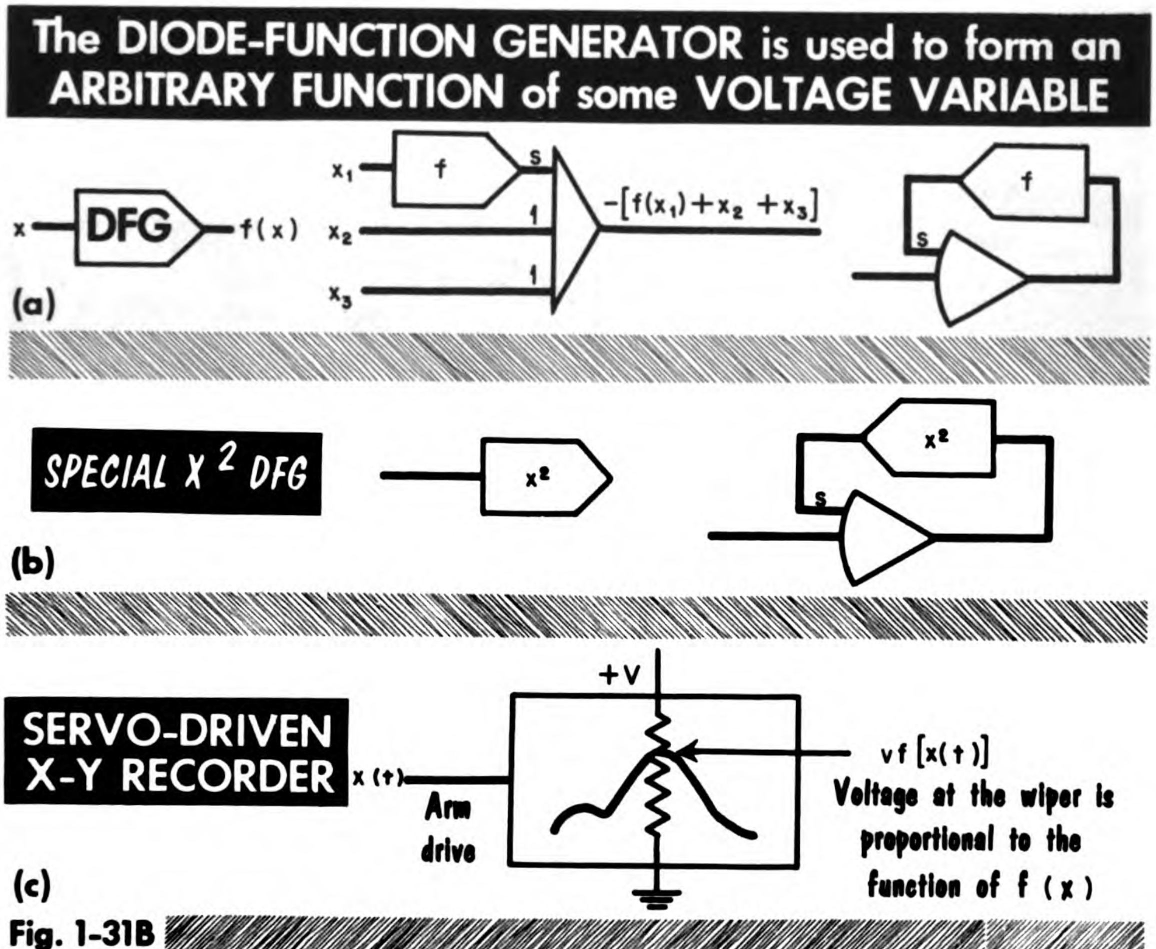


Fig. 1-31B

tion is approximated by 20 straight-line segments. The function is "set into" the dfg by the setting of 40 small potentiometers, and is then left alone for the duration of the problem. A dfg may be placed in the input or feedback path of an amplifier.

If a particular parabolic function is placed on a dfg one can use the unit for *squaring* a variable [Fig. 1-31B, part (b)]. Squaring units are useful enough to warrant production of special, so-called x^2-dfg's with permanent wiring replacing the 40 potentiometers.

Another useful function generator is made from a servo-driven *x-y* recorder or plotter (which has an arm and pen moving at right angles to each other). An attachment is installed replacing the pen with a pickup device. Some electronic circuits cause the pickup to "follow" a line drawn with silver conducting ink as the arm is moved by an input variable, $x(t)$ [Fig. 1-31B, part (c)]. The moving pickup is connected to the wiper of a potentiometer. The voltage at the wiper is proportional to the function of $x(t)$, as described by the drawing of $f(x)$ in silver ink. The voltage supplied to the end of the potentiometer is multiplied times f(x); this voltage may be a variable.

Computer Controls

Besides the usual power controls there is a main computer control switch which determines the mode of operation of the computer. Generally there

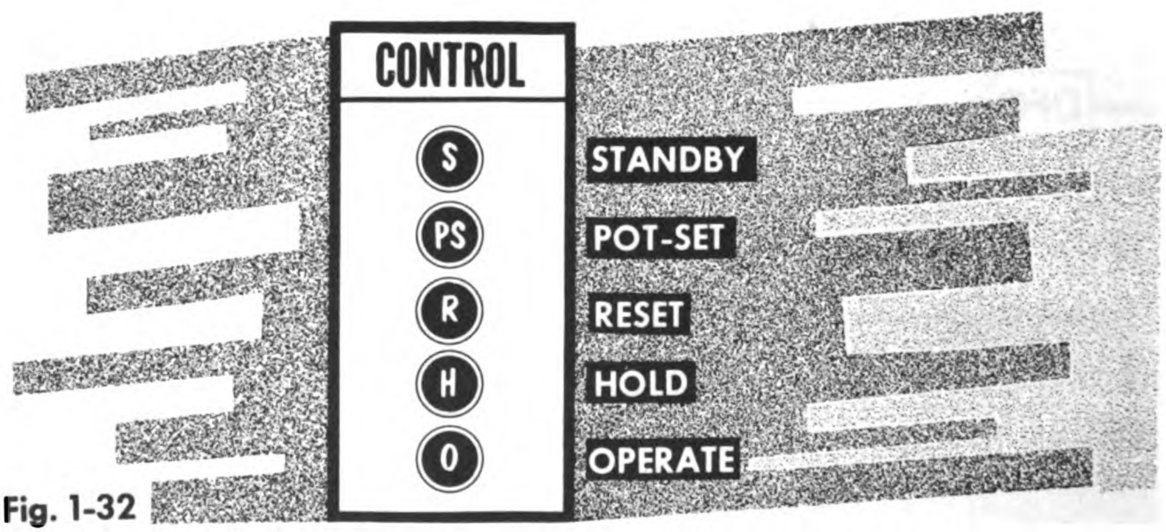


Fig. 1-32

are five modes and the computer can be in only one at a time: STANDBY, POT-SET, RESET, HOLD, OPERATE (Fig. 1-32).

1. STANDBY: All high voltages are turned off. This mode is selected only when the computer is not in use, such as when repairs are being made, or at night. For maximum longevity of electronic components continuous operation of vacuum tube filaments is found to be desirable.

2. POT-SET: In this mode the summing points of all amplifiers are grounded by means of relays so that coefficient potentiometers (pots) can be set without danger of overdriving an amplifier during the process of setting the pot. Such an occurrence would not damage the amplifier but would result in an incorrect pot setting. The danger usually exists only when the pot is in the feedback path of an amplifier; thus the POT-SET mode is not necessarily required for setting of all pots. If one forgets to use this mode when required, the overload alarm (an audible tone) is sure to warn him.
3. RESET: Whenever one switches to this mode the initial conditions (IC's) on each integrator are *set* or *reset*, as the case may be, so that the computer is ready to go, ready to begin computing as soon as the operator switches to the OPERATE mode. Resetting of initial conditions means that the feedback capacitors of all integrators are discharged from whatever voltage they may have had across them, and are charged to some particular set of voltages, representing the initial conditions of the integrals. The voltage inserted at the IC input to an integrator is employed only when the computer is in RESET.
4. HOLD: Sometimes when solving a problem with a computer it is desired to stop the computation before it is completed, "freeze" all the computer variables so they may be observed more carefully before returning the computer to the OPERATE mode to complete the solution. The HOLD mode performs this function, preventing all integrators from integrating, by means of relays which disconnect the inputs from each integrator.
5. OPERATE: In this mode the computer earns its keep.

Output Presentation

The output from a computer, or the results of a particular computer program, may be presented to the operator in several ways. For the purpose of checking that the computer program is correct and that the computer components are performing correctly, a voltmeter is commonly used. A vacuum tube voltmeter is almost always built into the operator's console, with selector switches which direct the reading of the voltages at the inputs and outputs of all building blocks. Furthermore, most large computers have a *digital voltmeter* [Fig. 1-33 (A)], which is similarly provided with selector switches. Digital voltmeters provide a visual display of a decimal representation of the voltage being measured, plus an indication of the decimal point, and of the polarity of the voltage. Typical digital voltmeters (dvm) present four decimal figures, measuring from $\pm$00.01 volt to $\pm$99.99 volts. Additional features for large computers include the ability to make a permanent record of the dvm reading by printing the reading on a strip of paper along with a coded identification of the source of the voltage.

For presentation of the results of an actual computation of a so-called "run" on the computer, some form of graphic presentation is always used, rather than numerical. Three display techniques are very common. In any one computer operation one or all display methods may be used.

Computer Output or Program Results may be presented in several ways

Courtesy Non-Linear Systems, Inc.

(A)

The Digital Voltmeter

provides decimal representation of voltage being measured

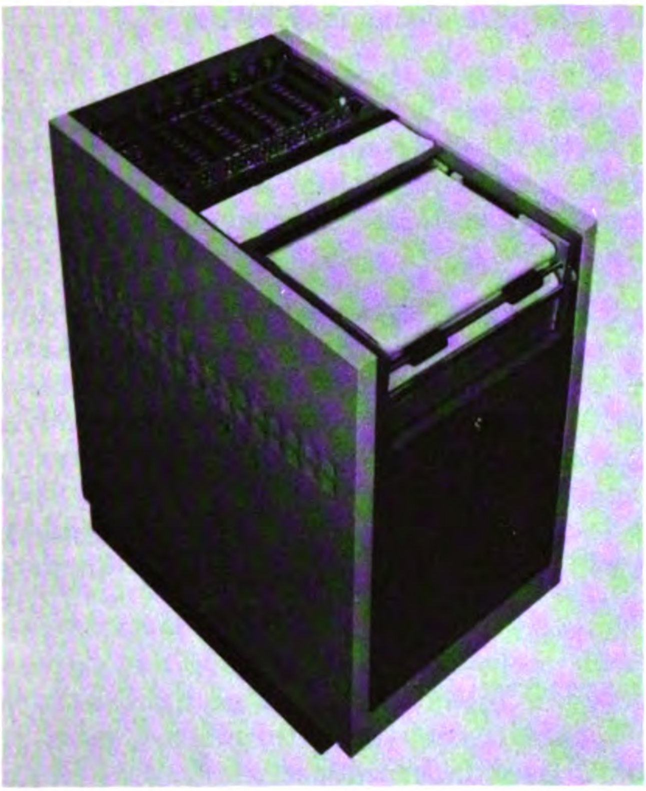
(B)

High-Persistence Large-Screen Oscilloscopes

display a computer variable

Courtesy ITT, Industrial Products Div.

Fig. 1-33 (A) (B)



(C)

Time variations
of computer
voltage variables
are graphed
by the
Strip Recorder

(D)

Servo-Driven
x-y Plotters
are useful for
high-accuracy
recordings

Fig. 1-33 (C) (D)

High-persistence oscilloscope: For a qualitative display of a computer variable an oscilloscope may be useful [Fig. 1-33 (B)]. High-persistence screens are required, since a single sweep of several seconds is involved. If a permanent record is required, photographic means may be used.

Strip recorders [Fig. 1-33 (C)] or time base recorders, are used in all branches of industry. A strip of graph paper is moved at a constant speed beneath one or more pens driven transversely by computer voltage variables, thereby drawing a graph of their time variations. Six- and eight-channel recorders are commonly used, so that many variables from a single computer run can be displayed simultaneously.

x-y plotters. Though restricted to computer variables with only low-frequency variations, servo-driven *x-y* plotters [Fig. 1-33 (D)] are very useful for high-accuracy recordings. One servomechanism moves an arm across a flat table covered with graph paper, while a second servomechanism moves a pen mounted on the arm across the table in the other direction. The result is that a variable can be plotted against any other variable, not just time. Small plotting tables usually have one pen and one arm, while large tables may have two pens and two arms which operate simultaneously, recording two curves.

QUESTIONS

1. Briefly describe the Nordsieck differential analyzer.
2. What is the most difficult building block to design for the a-c analog computer?
3. What operations appear to be easier to accomplish with the a-c analog than with the d-c analog computer?
4. How is a vector represented on an electronic computer?
5. What is a DDA?
6. What is a passive-element analog computer? Name three kinds.
7. What characteristic of a physical problem to be simulated would indicate the desirability of a passive-element computer?
8. What can R-C network analyzers do that other passive-element computers cannot?
9. What can R-*L*-C network analyzers do that cannot be accomplished with an R-C network analyzer?
10. Explain how large R-L-C network analyzers are used.
11. How is analog computer precision usually specified?
12. How is the accuracy of a computer program predicted?
13. What are the computing symbols used for an integrator, servomultiplier, potentiometer, summer, diode, and relay?
14. What modes of operation does a typical analog computer have? What is each used for?

Chapter 2

D-C ANALOG COMPUTER: LINEAR COMPUTING COMPONENTS

Introduction

In Volume 1 of this book, the concept of analogy was presented and many examples of analogous physical systems and their possible uses in the investigation of system behavior described. The major idea to be grasped is that of building a model in one domain that behaves similarly to a device or primary system in another domain, so that it is possible to perform on the model experiments which will forecast the behaviors of the device or primary system. The reasons for wanting to do this are many. They include considerations of cost, flexibility, safety, simplicity of experimentation, and time.

Of all the examples and ideas detailed, one configuration stands above all others in its usefulness, and when one speaks today of the general purpose analog computer (Fig. 2-1) it is this configuration — the electronic differential analyzer employing direct voltages — that is implied. In such a computer direct voltages are used to represent the variables describing the behavior of a primary physical system, and by using standard components within the computer, an electrical model is built whose behavior is analogous to the physical system. By experimenting with the electrical model, by obtaining solutions to specific problem situations, one can appreciate and understand the physical system of interest.

This Volume describes in detail the components of the electronic differential analyzer and how they perform the operations required of them. These components are few in number; the size and complexity of a modern computer results solely from the large *number* of each component required, and their somewhat complicated interconnection, control and monitoring system. The equipment complexity is aimed at simplifying the programming and use of the computer, but it should not be allowed to confuse the reader about the very simple devices which are the important computing parts of the machine. To appreciate modern analog computing principles, it is the

The MODERN COMPUTER is an ASSEMBLY of SEVERAL KINDS of ELECTRONIC DEVICES

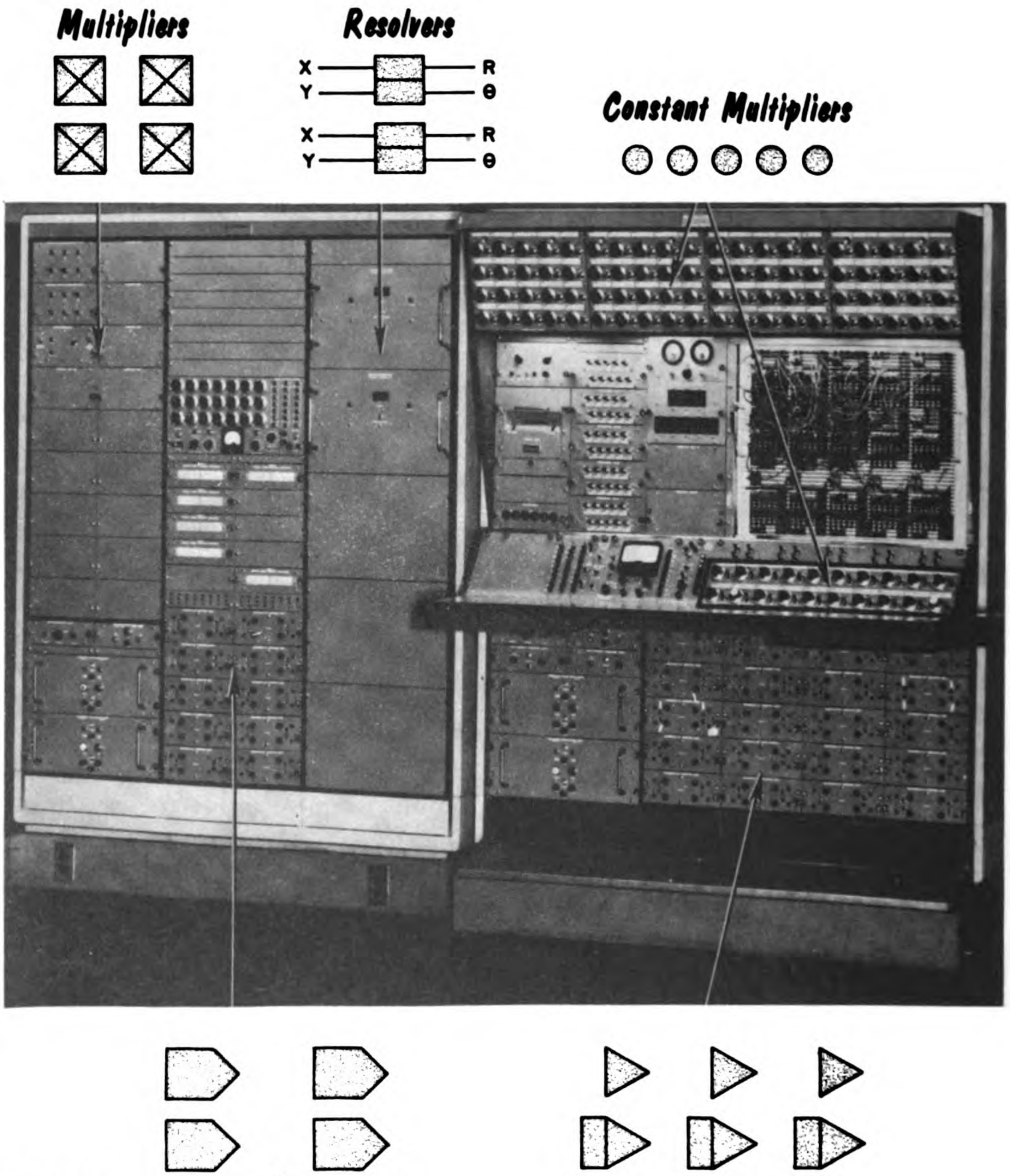


Fig. 2-1

simple components which form the main course for study. Once these are thoroughly understood, the additions which make the programming of a computer less tedious than was previously the case, can be enjoyed at leisure.

The Mathematics of Problem Analysis

The discussion of the mathematical description of a physical system given in Volume 1 showed that by the use of symbols (usually the letters of the

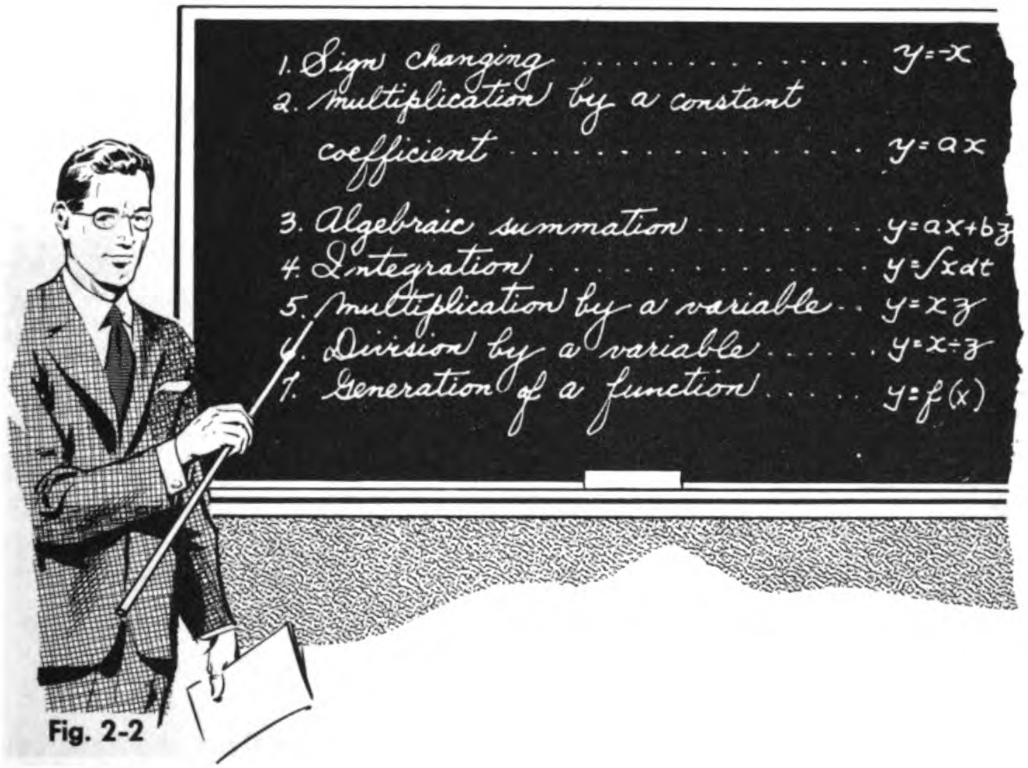


Fig. 2-2

alphabet) to represent the values of the interesting variable quantities, one can set down a neat, concise statement of the interrelation of these quantities. Then the behavior of the primary physical system, as described by the time-history of these variable quantities, can be determined by solving the mathematical equations of the statement. Frequently, the statement describing a system in one domain is similar in form to the statement describing a system in another domain, and then we have analogous physical systems. On the other hand, by setting up a model system that is described by the same or a similar mathematical statement, we can create an analog. If we are to do this it would be helpful to know what mathematical operations are likely to occur in a statement, so that we can plan to have available, devices capable of performing these operations for inclusion in the model.

The basic mathematical operations given in Volume 1 are listed here for convenience (Fig. 2-2). How to perform these operations in an electrical voltage model will be our concern in this volume.

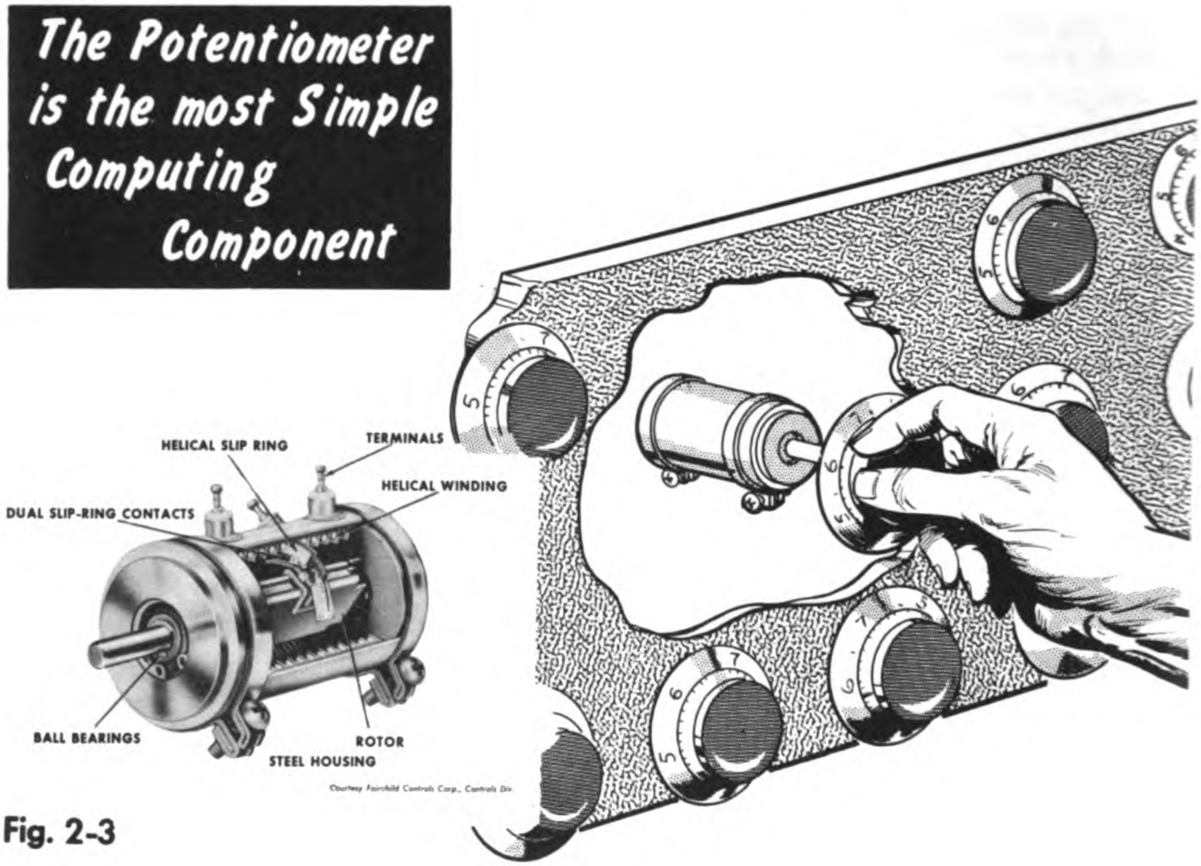


Fig. 2-3

ATTENUATORS

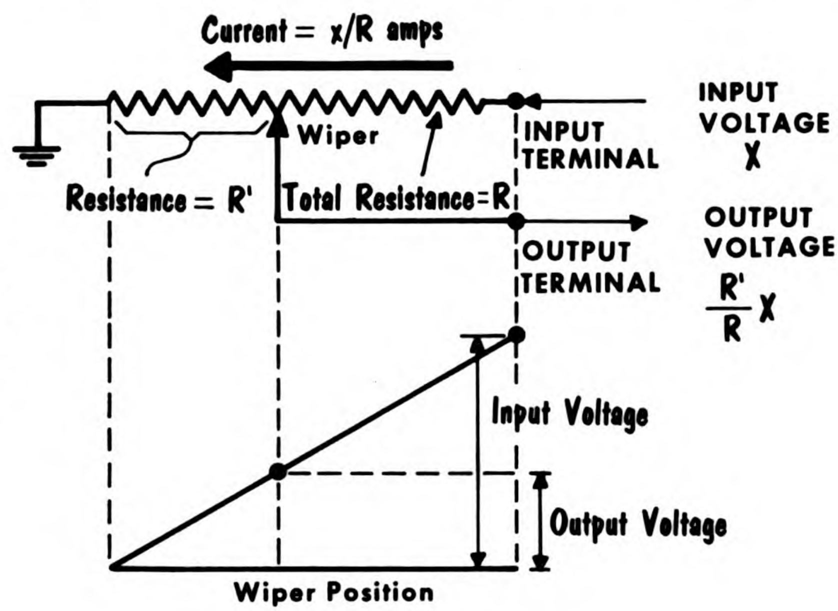
The Potentiometer

The simple operation of multiplying a varying voltage by a constant can be achieved accurately and without difficulty provided the constant is a positive number less than unity. It is obvious that this particular operation is equivalent to decreasing or attenuating the voltage, and the potentiometer does exactly this. The potentiometer is a standard type electronic component that finds uses in all kinds of electronic circuits where an adjustable level of operation is required. The potentiometer used in the analog computer to attenuate (i.e., multiply by a positive constant less than unity) differs from commonly-used potentiometers only in that it is a high-precision, accurately-adjustable device (Fig. 2-3). Resistance wire is wound on a helical mandrel which has 10 turns. Each loop of wire is insulated from its neighbor but can be touched by a slider or wiper that rides on the helix. The position of the wiper can be adjusted by turning a knob attached to a shaft that carries the wiper. Built into the knob there is an indicator which shows accurately to three significant figures, the mechanical position of the wiper as a decimal percentage of its total allowable displacement. Such an arrangement permits the wiper to be positioned so that the resistance from the wiper to one end of potentiometer (usually referred to as the low end) is accurately that percentage of the total potentiometer resistance shown by the indicator.

Attenuating a Voltage

The accurate potentiometer described previously has three terminations, the low end of the resistance wire, the high end of the resistance wire, and the wiper. When it is used as an attenuator, the low end is connected to ground potential, the high end is referred to as the input termination and the wiper becomes the output termination (Fig. 2-4). If a voltage, x volts, is applied to the input termination, then a current of x/R amps, where R is

A POTENTIOMETER MULTIPLIES the VOLTAGE at its INPUT TERMINAL by a CONSTANT RATIO



The RATIO is always LESS than UNITY, hence the unit is often termed an ATTENUATOR

Fig. 2-4

the total resistance of the wire, will flow through the resistance wire. There will be a steadily decreasing voltage potential along the wire from x volts at the high end, to zero or ground potential at the low end. At any point along the wire the voltage will be given by the current x/R multiplied by the resistance between that point and the low end, i.e., $(R'/R) \times x$. Thus the voltage produced at the output will be the voltage applied to the potentiometer multiplied by the constant shown as a decimal reading on the indicator. The potentiometer used in this manner is usually known as an *attenuator*.

Loading Error

As described thus far, the attenuator would allow multiplication of a varying voltage by a positive constant less than unity, and this operation would be performed accurately to three figures. However, in practice this is not

quite the case. In order to be useful, the voltage produced by the attenuator has to be applied to some other device so that further mathematical operations may be performed. Normally the output goes to a summing device. In connecting the output to the input of some other device a current will be drawn through the wiper of the attenuator, causing more current to flow through one part of the attenuator than the other, thus changing the output voltage. The magnitude of the output voltage will be reduced and it will no longer be proportional to the mechanical setting of the potentiometer. This loading error depends on the current drawn through the wiper (Fig. 2-5). Thus it depends on both the setting of the potentiometer (the wiper position) and the input resistance of the following unit. It can be quite serious and certainly must be corrected in some way. A plot of the required output voltage and that obtained when the attenuator is loaded, is shown in the figure for typical modern equipment. Obviously it is necessary to set the wiper of the attenuator to a position closer to the high end if the correct multiplication factor is to be achieved.

Correcting the Loading Error

To overcome the difficulty of the loading error one might use a loading correction curve and set the wiper mechanically to a position higher on the attenuator as dictated by the curve [Fig. 2-6 (A)]. However, it is preferable

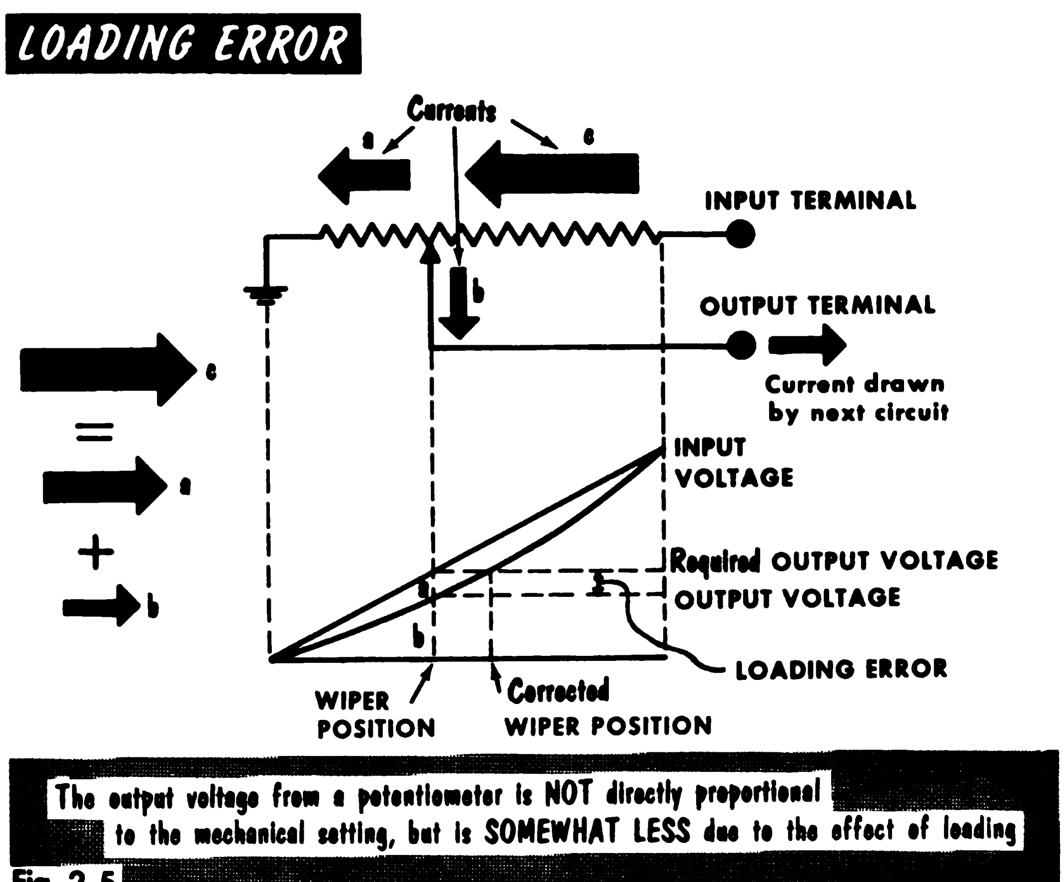


Fig. 2-5

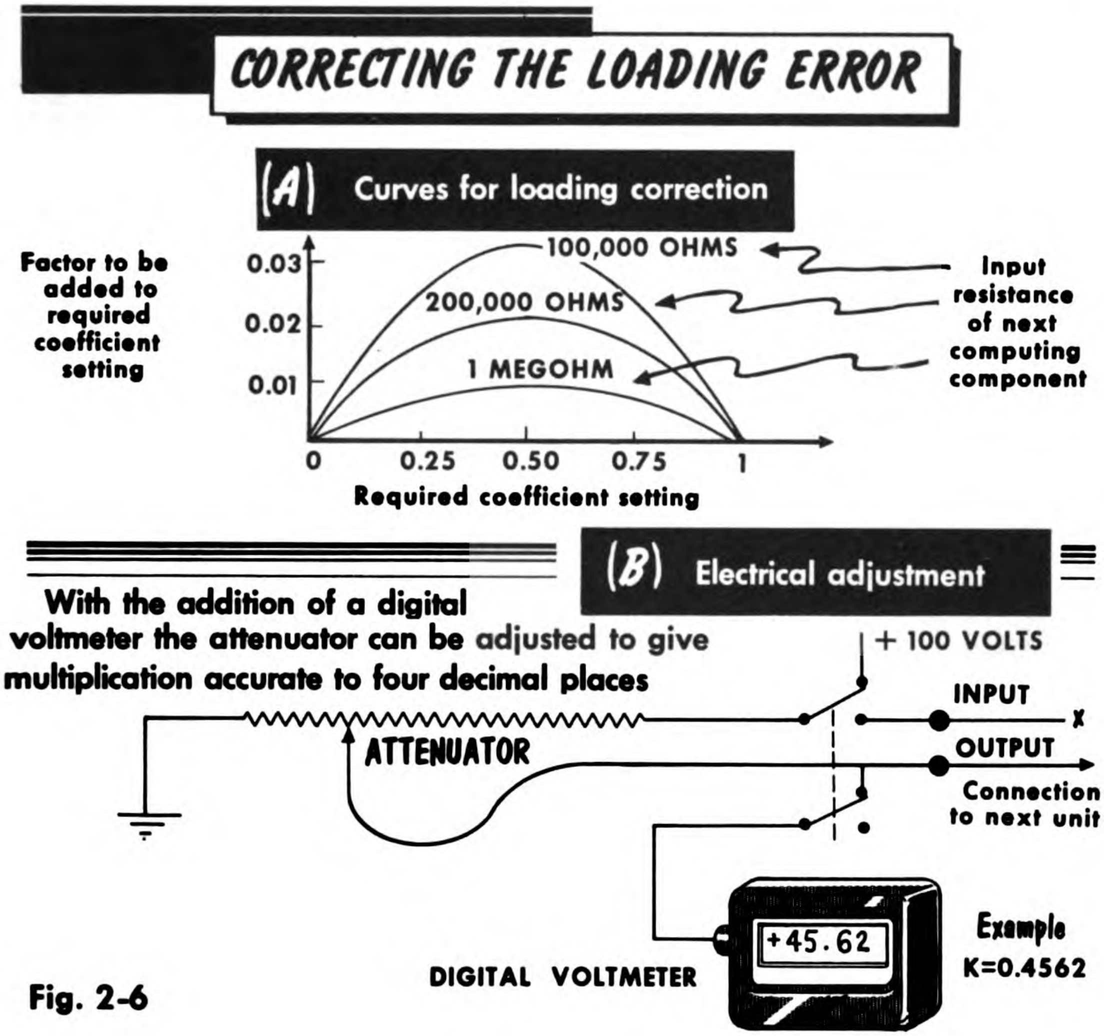


Fig. 2-6

to set the attenuator "electrically" by applying a known standard voltage (usually 100 volts) to the input, measuring the voltage appearing at the output with the required loading, and adjusting the wiper position until the output reaches the appropriate value. Using this technique with a precise digital voltmeter* which is able to measure the output voltage correctly to 10 mv, the attenuator can be adjusted to give multiplication accurate to four decimal places [Fig. 2-6 (B)].

VOLTAGE AMPLIFIERS

Multiplication by a Constant Greater than Unity: Amplification

The multiplication of a variable voltage by a constant greater than unity corresponds to an amplification. We have seen how an attenuation can be achieved quite simply with a potentiometer. No power was required. In

* See p. 3-6

The OPERATIONAL AMPLIFIER is THE KING of ANALOG COMPUTING COMPONENTS

It is used to

- ☆ Change the sign of a voltage
- ☆ Multiply by a constant greater than unity
- ☆ Algebraically sum many voltages
- ☆ Integrate voltages

Fig. 2-7

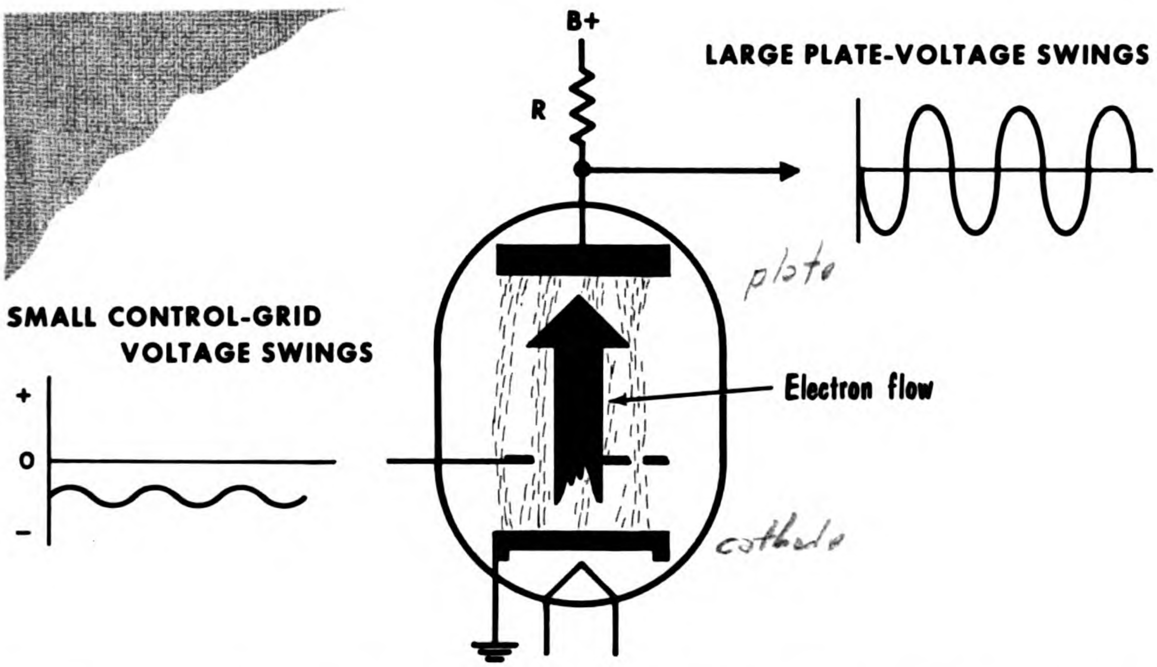
fact, power was being dissipated in the process. Things are different when we wish to amplify a voltage. In place of the purely passive attenuator we require an active (supplied with power) device, called an *operational* or *computing amplifier* (Fig. 2-7).

The operational amplifier is the most essential computing component within the general purpose analog computer. Its successful development was essential to the wide acceptance of analog computation using direct voltages. It is quite versatile in its use, being capable of a number of tasks. As well as being able to multiply a voltage by a constant greater than unity, it can sum a number of voltages algebraically (taking account of their signs), change the sign of any voltage, and also perform the important mathematical operation of integration. It is very simple to use and maintain, and probably ranks as the most reliable of the active computing components.

The size of a general purpose analog computer is usually described by the number of operational amplifiers it contains, for this frequently determines the complexity of the problem that can be solved using the computer. A medium size computer would contain between 50 and 100 amplifiers. Some computers have as few as 20, others have as many as 400.

Single-Stage Electron-Tube Amplifier: Triode Amplifier

An electron tube is an assembly of electrodes within an evacuated envelope, which can be made of either glass or metal. As indicated by its name, a triode contains three electrodes, a cathode, a plate and a control grid. When the cathode is heated to a high temperature by an electrical hot-wire filament, electrons are ejected in useful quantity and are directed to the plate by applying a positive voltage between the plate and the cathode. The number of electrons per second, traveling to the plate, i.e., the plate current, depends on the temperature of the cathode and on the voltage applied to the plate. Now, if a negative voltage is applied between the control grid and the cathode, where the control grid is physically positioned between the

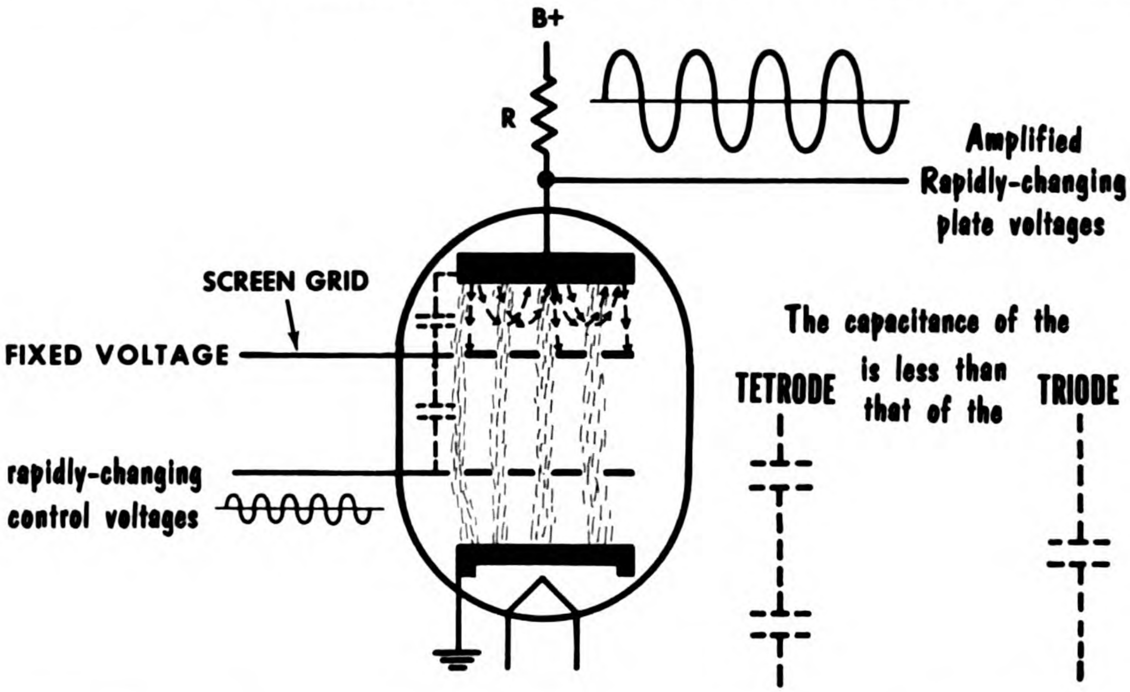


The *TRIODE* is the *SIMPLEST* vacuum tube amplifier

Fig. 2-8

cathode and the plate, then the value of this grid voltage exercises far greater control over the plate current, than does the value of the plate voltage. A small change in grid voltage causes a relatively large change in plate current. By passing the plate current through a resistor, a voltage can be developed whose value is controlled by the grid voltage (Fig. 2-8). The normal arrangement for a triode circuit is shown in the figure, and is such that a small negative change in the grid voltage causes a large positive change in the plate voltage. The voltage gain in such a tube, defined as the change in plate voltage for a given change in grid voltage, is usually somewhere between 20 and 50. By adding other electrodes to the tube between the cath-

INSERTION of a SCREEN GRID between the CONTROL GRID and the PLATE, reduces INTERELECTRODE CAPACITANCE



Lower internal capacitance allows constant amplification at higher frequencies, but positive screen potential attracts some secondary emission electrons and reduces gain at all frequencies

Fig. 2-9

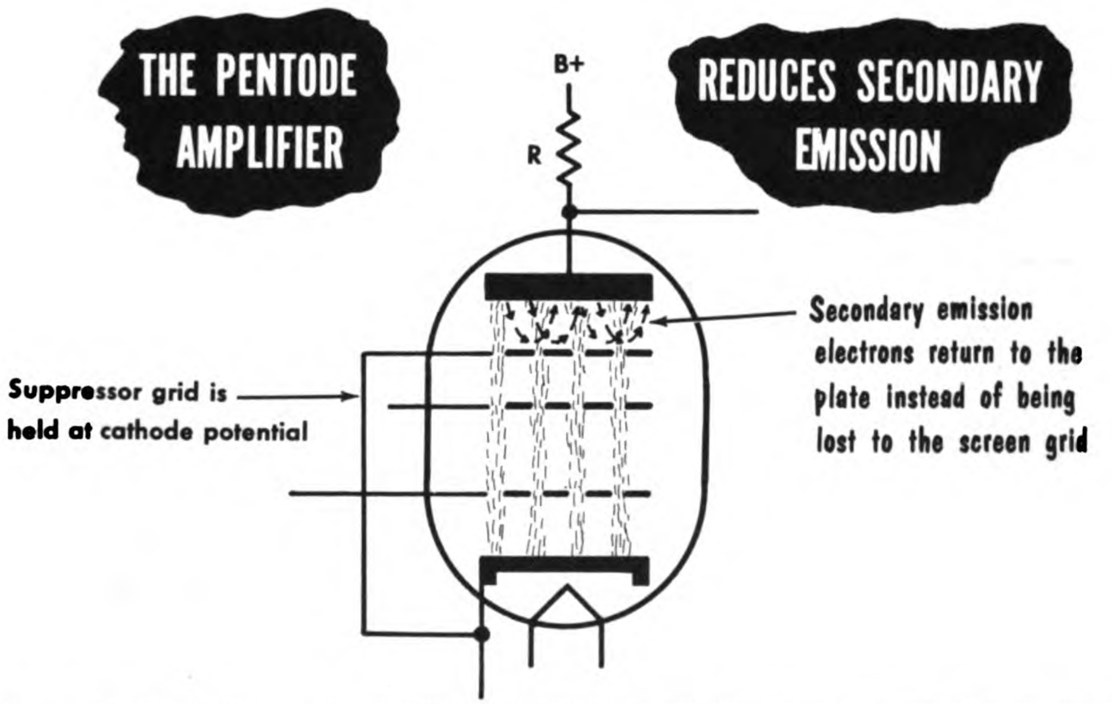
ode and the plate one forms a tetrode (4) or a pentode (5), which with appropriate voltages applied to the extra electrodes produce greater amplification of the control-grid voltage. The tetrode is of little importance for our purposes, but the pentode is used widely in many circuits, and it is worthwhile reviewing the advantages of this tube.

The Pentode Amplifier

When electrons flow from the cathode to the plate of a triode under the influence of the positive plate voltage and the negative control-grid voltage, there is a limit to the speed with which the control-grid voltage can effect changes in the current flowing. This limit is particularly dependent on the electrical capacitance of the structure formed by the metal plate and control-grid electrodes. It makes the triode unable to amplify fast-changing or high-frequency voltages as effectively as small slowly-changing or low-frequency voltages. The situation is improved by reducing the interelectrode capacitance by inserting between the control grid and the plate another open-mesh

electrode called appropriately the screen grid (Fig. 2-9). With this screen grid maintained at a suitable positive voltage somewhere between that of the control grid and the plate, we have two distributed capacitances in series resulting in a reduced total capacitance between control grid and plate. With lower capacitance a more rapid change in plate current is possible and thus higher frequencies can be amplified.

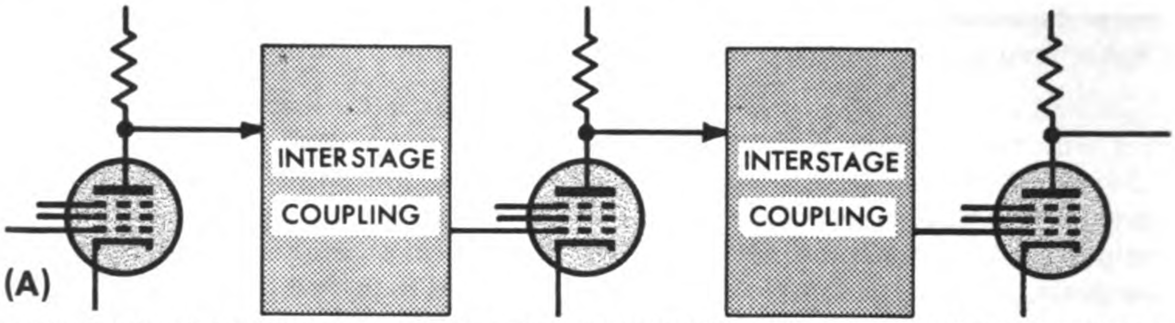
A second effect is present in the triode which was not mentioned previously, and which is increased by the inclusion of a screen grid. When the fast-moving electrons hit the plate they knock other electrons out of the plate forming a cloud of electrons in front of it. In the triode this secondary emission is unimportant, for the electrons are eventually attracted back to the plate, it being the only electrode having a positive, attracting potential.



The PENTODE AMPLIFIER obtains greater GAIN by SUPPRESSING the secondary emission current that flows to the screen grid in the tetrode
Fig. 2-10

When a screen grid is present, however, secondary electrons will be attracted to it away from the plate, reducing the plate current, and thus reducing the amplification (Fig. 2-10). This loss of secondary electrons can be, and is, prevented, by placing between the screen grid and the plate an open-mesh suppressor grid which is given a negative, electron-repelling potential. The suppressor grid forces the slowly-moving secondary electrons back into the the plate. It is normal to connect the suppressor grid directly to the cathode thus producing a large difference in potential between it and the plate, making its action very effective.

MULTISTAGE AMPLIFIERS provide GREATER GAIN WITHOUT OVERLOADING individual amplifiers



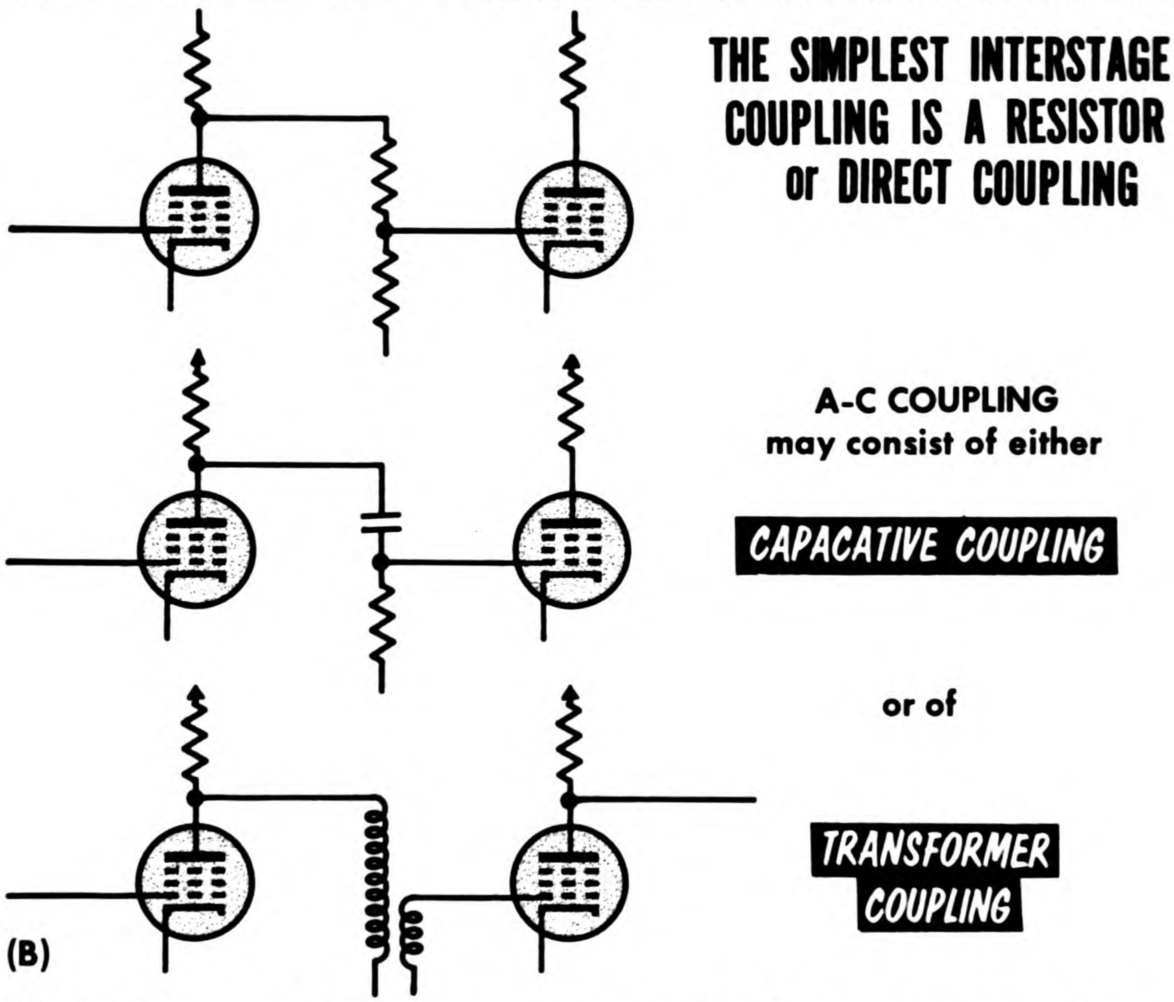


Fig. 2-11 There is no direct-current path through a capacitor or a transformer

The two extra grids in the pentode produce a much higher voltage gain than is possible in the triode, and also enable the pentode to amplify voltage signals with frequencies that are much higher than those amplified by a triode.

Multistage Electron-Tube Amplifier

Frequently, an electron-tube amplifier is required to amplify a very small voltage to a reasonably sensible value. For example, the currents excited in a radio or television antenna by the electromagnetic waves transmitted from the broadcasting station produce only microvolts (millionths of a volt). These have to be amplified to the level of volts so that sufficient power is available to excite the radio loudspeaker or television picture tube. To obtain such a large gain many tubes are used, one following another. We then speak of a multistage amplifier [Fig. 2-11 (A)].

There are slight technical difficulties in the design of such an amplifier, for the plate of a tube operates at a voltage level that is much higher than the control grid that follows it. Thus some method is required to reduce this voltage level without seriously reducing the amplitude of any voltage changes that will occur during the amplifier's operation, and which are of major interest. The interstage coupling used to effect this change in mean voltage level can be either direct-coupling, using only resistors, or a-c coupling which uses either a transformer or a capacitor [Fig. 2-11 (B)]. With the resistive direct-coupling, both constant level voltages and varying voltages are passed from one stage to the next. With a-c coupling, only varying voltages pass through to be amplified by a succeeding stage, for capacitors and transformers do not transmit steady electrical signals. They block the constant or direct-current voltage. Thus if we wish to amplify a voltage which does not change with time we require a d-c (direct-coupled) amplifier, but if the voltage is changing continually, as in a radio, we can use an a-c (alternating-current) amplifier.

The general purpose analog computer makes use of both kinds of amplifiers, but its use of the d-c amplifier is by far the more important.

The Cathode Follower

In the description of the single-tube amplifier the output voltage is obtained by passing the tube current through a plate resistor. The tube current variations which are controlled by the grid voltage cause changes in the plate voltage. It changes positively for a decreasing tube current corresponding to a negatively changing grid voltage, and *vice versa*. There is nothing to prevent the output voltage of the tube from being taken from the cathode rather than the plate (Fig. 2-12). By including a resistor in the cathode circuit a voltage will be present at the cathode which is directly dependent on the tube current, and thus the grid voltage. Such an arrangement, however, gives quite different results from those obtained with a plate resistor.

Firstly, the output voltage will be in phase with the grid voltage — positive changes at the grid cause more current to flow, raising the voltage at the cathode. Secondly, the available gain is considerably reduced. The important controlling voltage in a tube is the relative potential between the grid and the cathode. This is now considerably influenced by the output voltage as well as the input grid voltage. Previously, a change in grid voltage was

The Cathode-Follower Amplifier

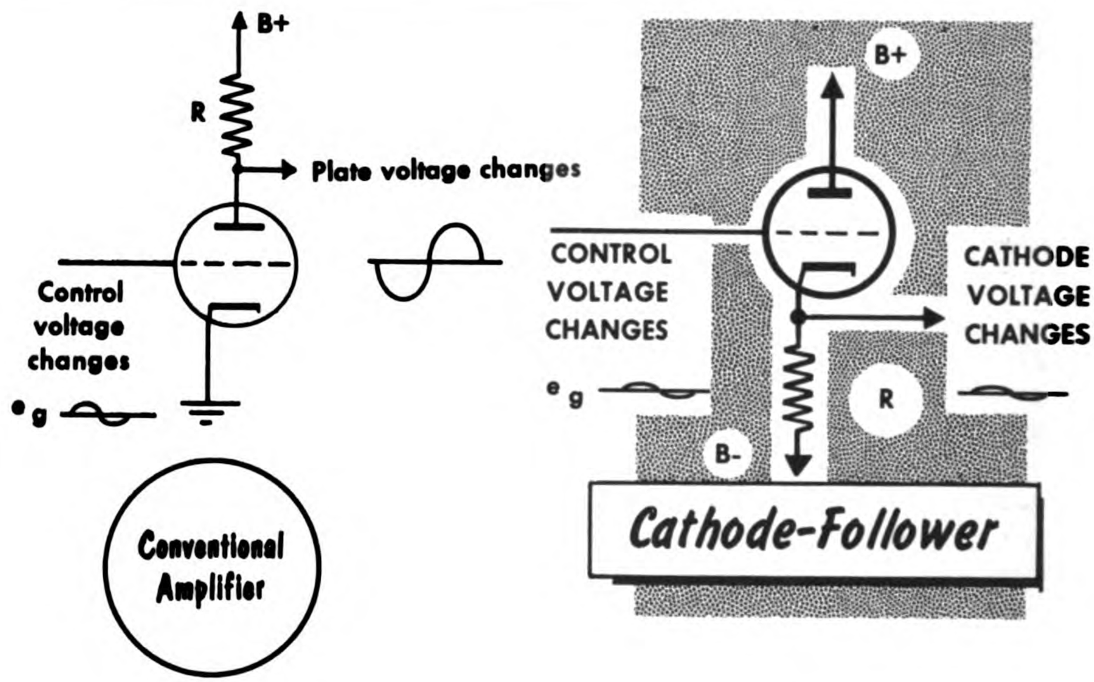


Fig. 2-12 In the cathode follower the gain is always LESS than unity, and the output voltage is IN phase with the control voltage

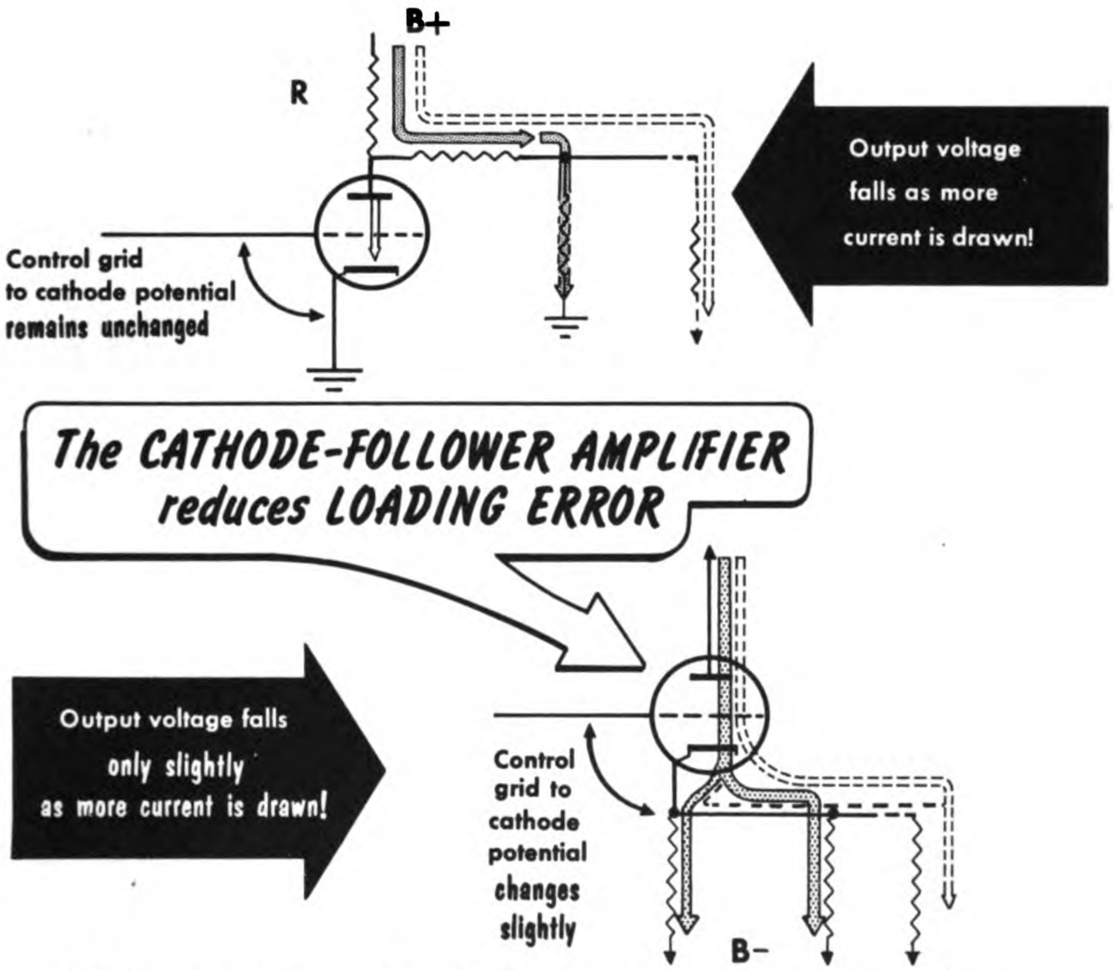
assumed to cause no change in the cathode voltage, but now, as the grid voltage changes so changes the cathode voltage. The cathode follows the grid, maintaining the grid-to-cathode voltage almost constant. In such an arrangement it is impossible for the cathode voltage to change more than the driving-grid voltage. In fact the negative feedback action of the cathode follower produces an overall gain which is less than unity.

No Loading Error

A cathode follower is often used as the output stage of the operational amplifier. Quite naturally, there must be a good reason for using such an apparently wasteful device, wasteful in that it gives no amplification. The operational amplifier, being the universal component that it is, will be used in many different ways in a computer circuit. It must be capable of supplying its output voltage to any other unit without any danger of a computational error being introduced. It must be free from loading error (Fig. 2-13).

Consider what happens in the single-tube amplifier if the current being drawn by a load connected to the plate is increased by decreasing the load

resistance. Then the current passing through the plate resistor increases, and except for a slight correction made by the tube, nothing prevents the output voltage from sagging. Contrast this with what happens in the cathode follower under similar circumstances. The increased current drawn by the load now passes through the cathode resistor, tending to increase the cathode voltage. However, a slight increase in cathode voltage decreases



The immediate feedback effect of a changing output voltage reduces the influence of load changes on the output voltage. A CATHODE FOLLOWER HAS A LOW OUTPUT IMPEDANCE.

Fig. 2-13

the grid-to-cathode potential, causing a relatively large decrease in the tube current. Decreasing the tube current forces down the cathode voltage, restoring its original value. Looked at in another way, one might regard the increased output current to be compensated for by a decreased tube current. As both flow through the cathode resistor, the effect is to maintain the cathode voltage unchanged by the current drawn by the load.

To this negative feedback action of the output stage of an operational ampli-

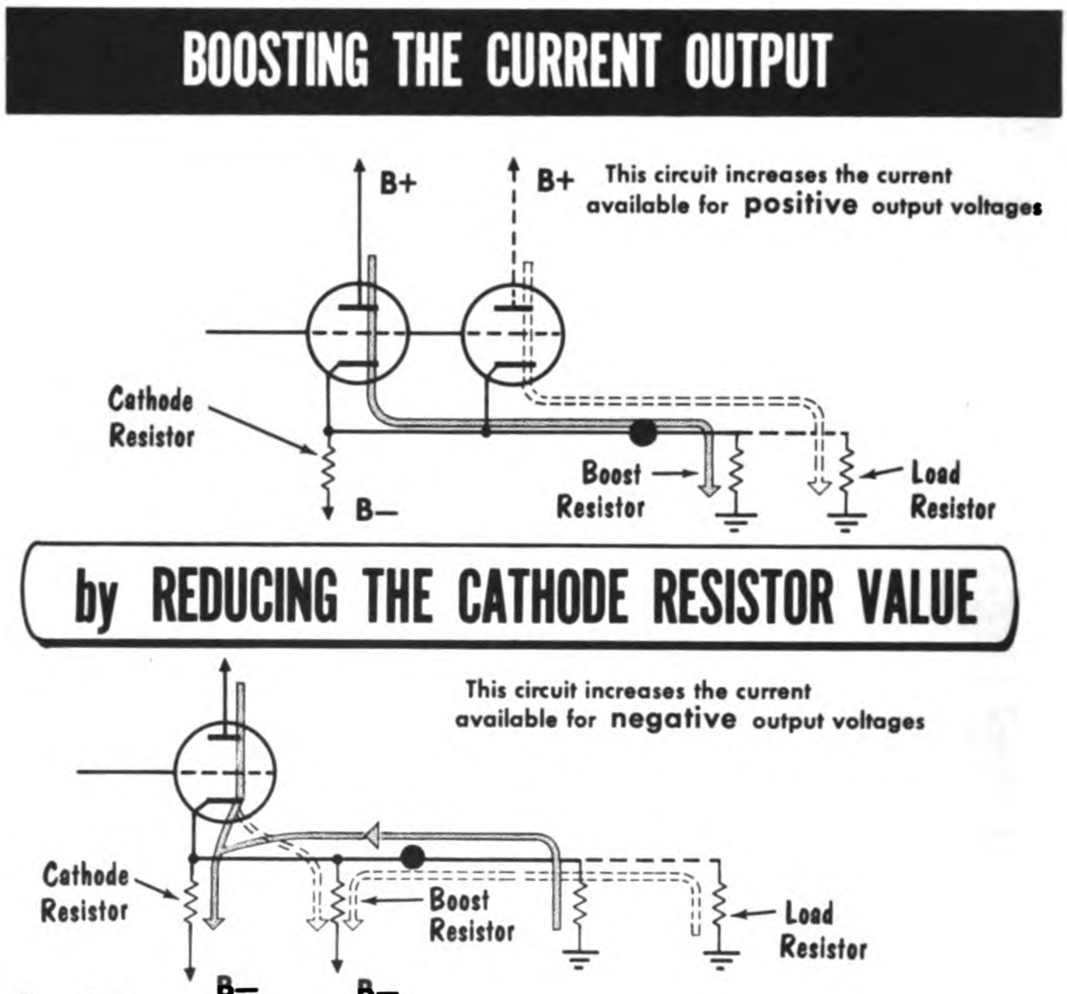


Fig. 2-14

fier there is added the effect of an overall feedback circuit around the amplifier, and the result is that under normal operation, the amplifier has no measurable loading error.

Boosting the Output

Sometimes one amplifier is required to supply many circuits in parallel. For example, the same variable might occur many times in the mathematical description of a problem. It would be generated once only on the computer, and from one amplifier, connections would be made to many other units. In each connection a current would flow, and the total current drawn from the amplifier would be quite large, particularly when the amplifier's output voltage is high. Under these conditions the output circuit might become overloaded, too much current would be drawn through the output circuit, and due to the abnormal conditions the amplifier's output voltage would not correspond to the input voltages. To relieve this situation, one might place another tube in parallel with the output tube to increase the available output power. However, a more usual procedure is to change the operating point of the existing tube by reducing the value of the cathode resistor.

This causes more current to pass through the tube in the region where the overload previously occurred, providing more current for the circuits connected to the cathode and preventing an inaccurate voltage from occurring. To reduce the cathode resistance requires a parallel resistor to be available, and the connection of such a resistor is made very conveniently on the computer. This "boost" resistor is not used unless required, for it causes far more current to pass through the tube than is usually necessary (Fig. 2-14).

An amplifier is normally able to supply sufficient current (25 ma) for half-a-dozen attenuators.

Balancing

One of the most stringent requirements of a computing amplifier is that when the input voltage applied to the amplifier is zero, then the output volt-

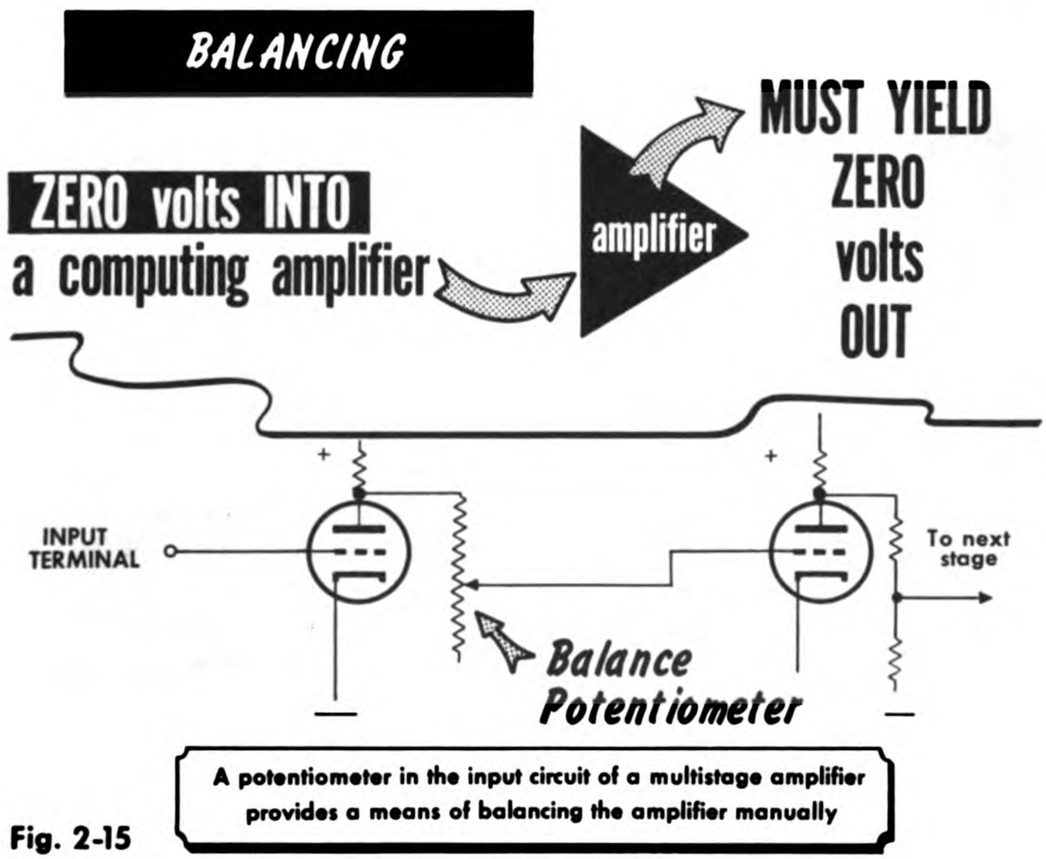


Fig. 2-15

age must be zero. If this were not the case we would not have a device capable of performing with accuracy the operations outlined previously, and the computer would thus be unable to solve even the most simple algebraic problems. This *zero correspondence* between the output and input voltages is known appropriately as *amplifier balance*, and the adjustment that is made to achieve it is termed *balancing* (Fig. 2-15).

Circuits for balancing the amplifier can be included in a number of ways. However, each would be equivalent to the simple circuit shown in the figure. The interstage coupling at one point in the amplifier — usually between the 1st and 2nd stages — includes a potentiometer, by means of which the operating potential of the grid of the succeeding tube can be adjusted. With zero voltage at the input of the complete amplifier, the potentiometer is adjusted until the amplifier output is zero. This zero adjustment must be made accurately.

Distortion and Noise

Unfortunately, nothing in this world is perfect and electronic components are no exception. For the components of a multistage amplifier to be perfect, their properties would have to remain constant for all time, and they would have to act on all voltages in the same way. But the gain of a tube, the value of a resistor, the capacitance of a capacitor, all change randomly with time, causing the overall amplification of a voltage amplifier to change with time [Fig. 2-16 (A)]. Furthermore, these same components do not operate on

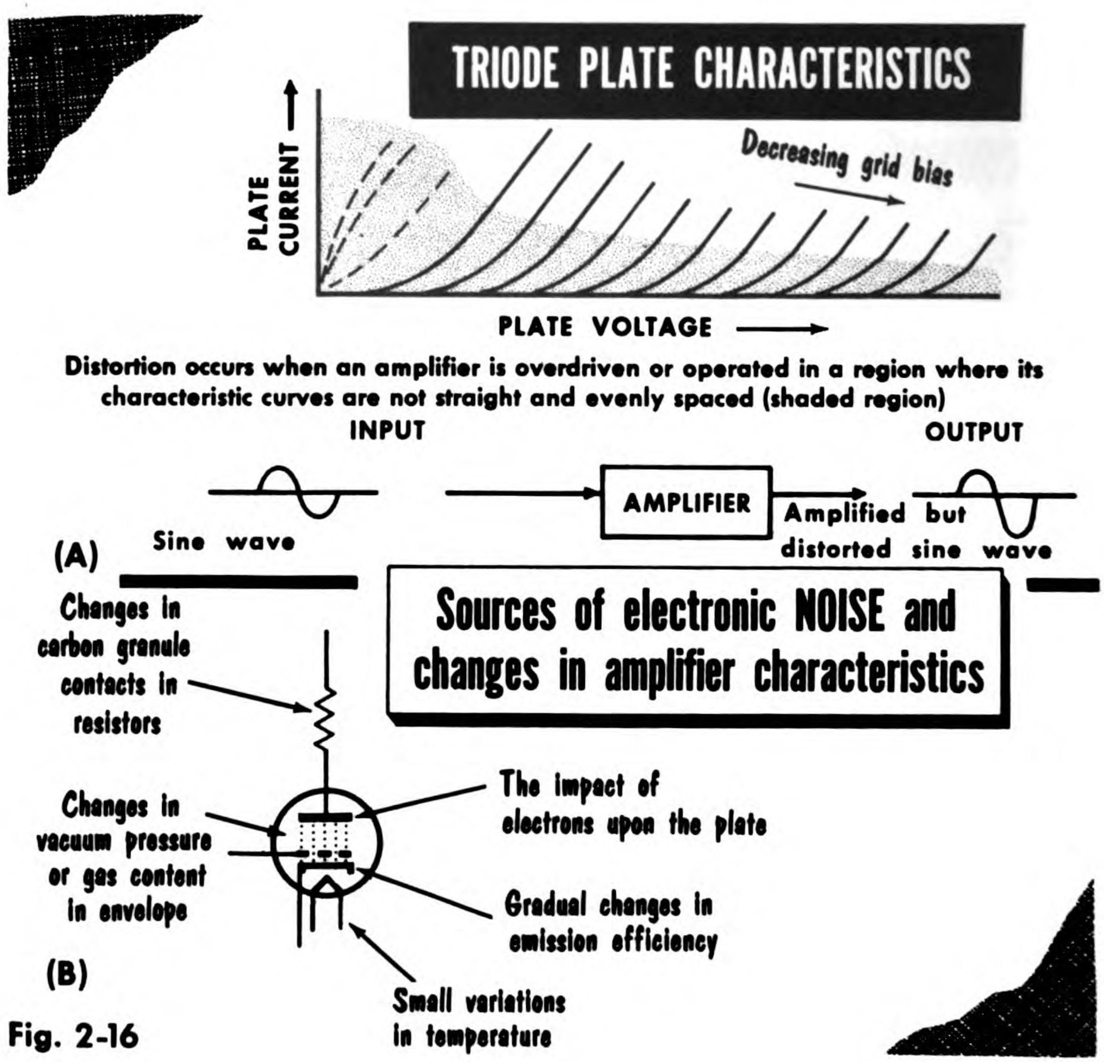


Fig. 2-16

A NEGATIVE FEEDBACK AMPLIFIER

employs the high gain of the amplifier to improve the accuracy of the device

even if the net result is NO amplification but only a changing of the sign of the input voltage

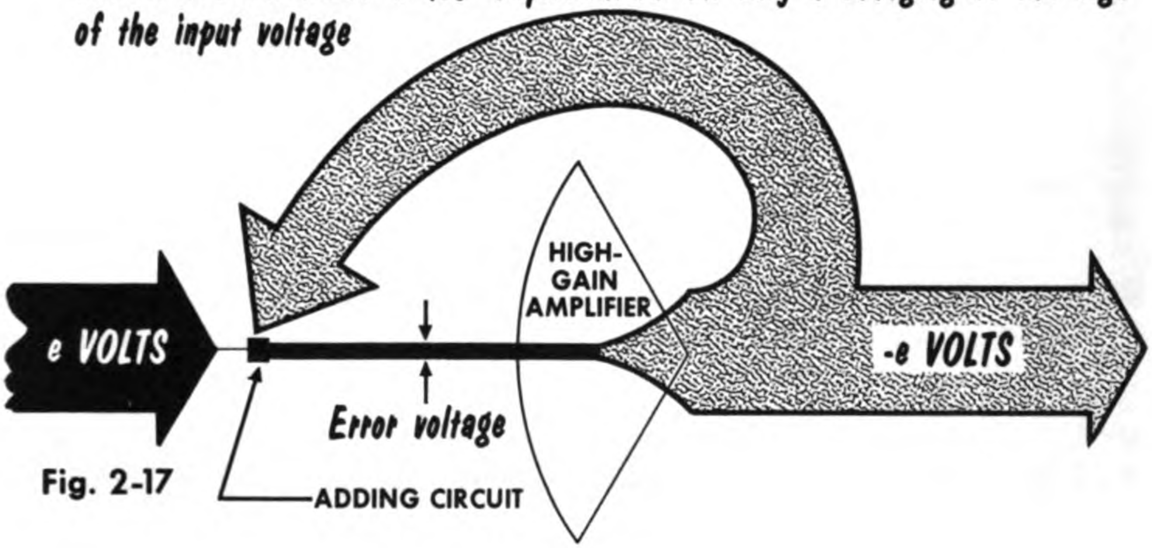


Fig. 2-17

different voltages in the same way. For example, a tube may amplify a d-c voltage by a factor of 30, and yet at the same instant amplify a rapidly changing voltage by only 25. This distortion is unwanted, and in a computing amplifier it must be overcome.

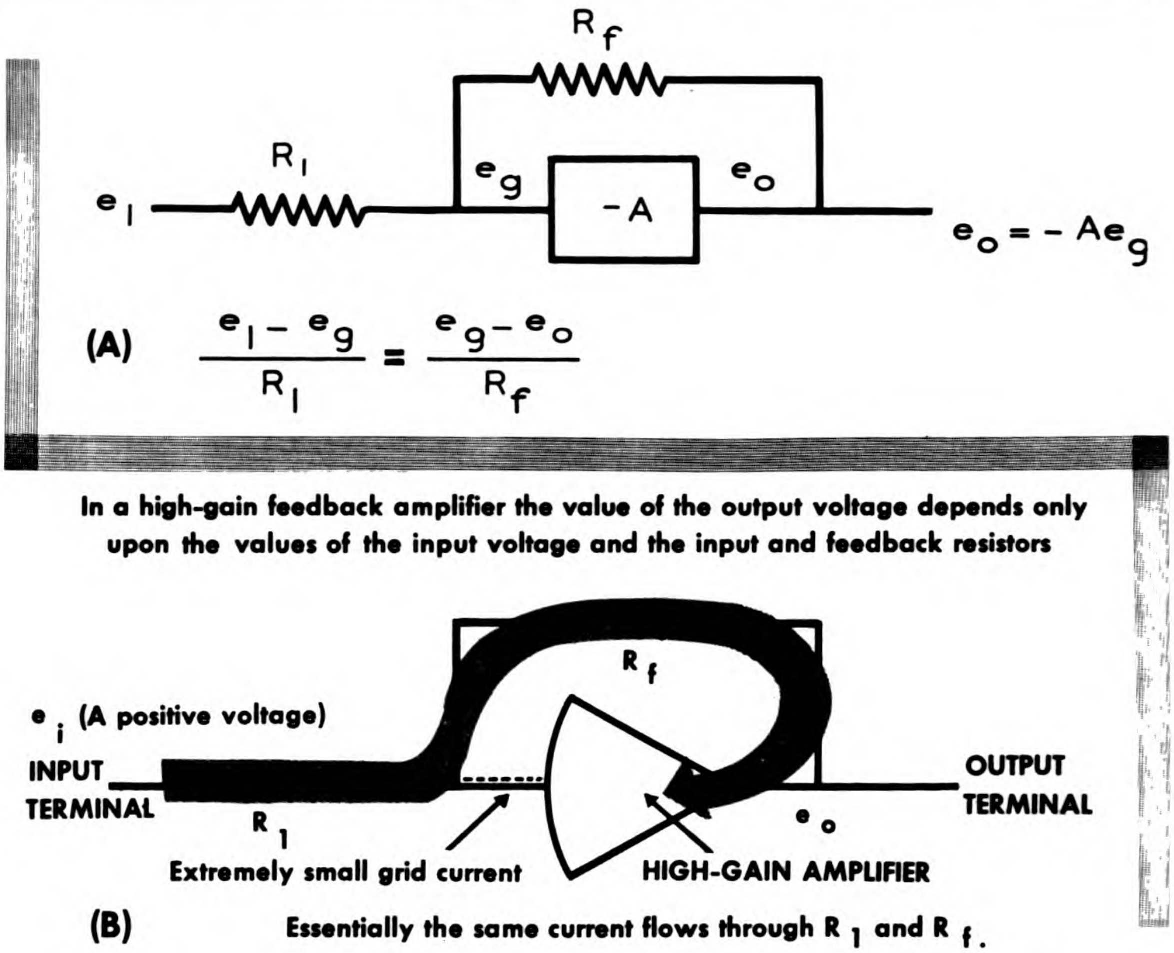
Electronic components are imperfect in yet another fashion. They generate unwanted, randomly-varying voltages (noise) [Fig. 2-16 (B)]. In the amplifiers so far described, the output voltage is not a simple multiple of the input voltage, for added to this desired multiple there will be a noise voltage. The noise voltage is normally rapidly varying (containing high frequencies) and in many applications it can be greatly attenuated, compared with any desired slowly-varying (low-frequency) voltages, by circuit arrangements within the amplifier. These low-pass filters depreciate the amplifier performance and can only be used with care in the computing amplifier.

THE SUMMING AMPLIFIER

Negative Feedback

The computing amplifiers used in a general purpose computer must perform their tasks accurately. Due to the random variations in component characteristics that occur in its operation, the simple series, multistage voltage amplifier is quite inadequate for the job. It must be improved in a number of ways, but especially must the noise and distortion be reduced to a negli-

THE D-C OPERATIONAL AMPLIFIER



Essentially the same current flows through R_1 and R_f.

The grid is at zero potential, hence e_o must equal $\frac{-R_f}{R_1}$ e_i.

Fig. 2-18

gible level. Fortunately, the multistage amplifier does have one advantageous property which is not wanted in its use as a computing amplifier. It has a gain which is far in excess of that normally required for computation purposes. This gain which is easily made to be 100 million, large indeed, compared with the normally required multiplication factors of 5 or 10, can be turned into improved quality. We trade quantity for quality (Fig. 2-17).

By arranging that the output voltage is opposite in sign to the input voltage, we may feed it back to the input, and by adding the two voltages algebraically (taking account of sign) at the grid of the input tube, we reduce to a very low level the driving-grid voltage of the input stage. This negative feedback has a quite marked effect on the operation of the amplifier. Of course, reducing the driving-grid voltage reduces the magnitude of the output voltage. In fact, it is customary for the output voltage to be not more than 20 times the input signal voltage. However, distortion and noise are re-

duced to extremely low values and the quality of performance of the amplifier is now very little dependent on most of the components of which it is made. Component values, operating points, voltage supplies, etc., can change appreciably within the amplifier, and provided the amplifier's multistage gain without feedback remains very high, the output voltage will be related to the input voltage by a constant negative factor. The high quality of constant gain and good operation replace the high gain that is present without feedback.

The D-C Operational Amplifier

In determining the operation of the d-c operational amplifier it is not necessary to consider all of the many components of which the unit is made. The confusing details of the physical make-up of each stage of amplification can be omitted, and the whole high-gain amplifier can be replaced by a "black box" which accepts an input grid voltage, e_g, and gives out a very much larger voltage e_o, where the two are related by the amplifier gain, $-A$. The minus sign is included, for there is a sign change through the amplifier so that when e_g is a small positive voltage, e_o is a large negative voltage, and when e_g is a small negative voltage, e_o is a large positive voltage. The output of the amplifier is connected through a resistor R_f to the input grid, and the input voltage e_1 to the completed device is applied through a resistor R_1 to the same point [Fig. 2-18 (A)].

The input grid of the amplifier draws a negligible current from the circuits connected to it. Thus the current flowing through the input resistor must flow through the feedback resistor.

$$\frac{e_1 - e_g}{R_1} = \frac{e_g - e_o}{R_f}$$

Or rearranging,

$$\frac{e_1}{R_1} + \frac{e_o}{R_f} = \frac{e_g}{R_1} + \frac{e_g}{R_f}$$

Also, we know that $e_o = -A\ e_g$, or dividing by $-A$

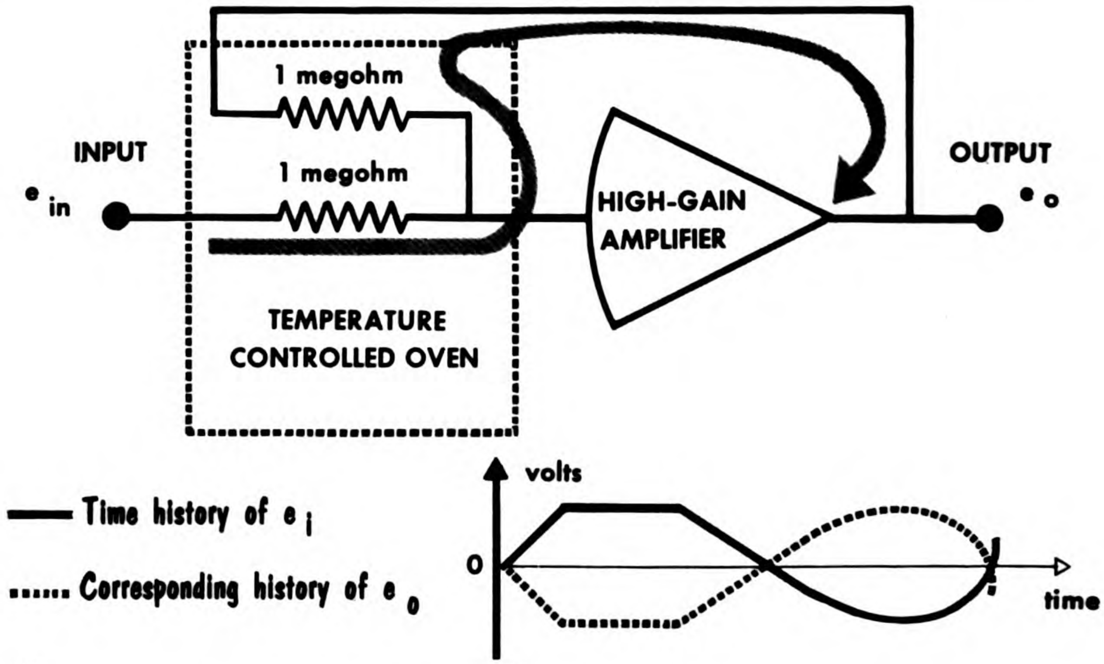
$$\frac{e_o}{-A} = e_g$$

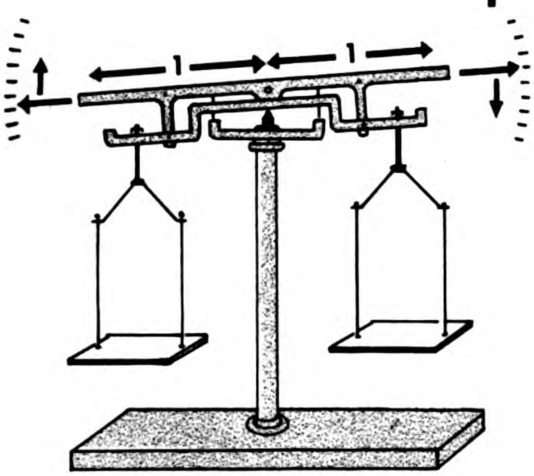
But A is very large compared with any of the other values involved (when e_g is 1 μv, e_o will be 10 volts or more), and therefore we can say that for all practical purposes e_g is zero. Thus, setting e_g to zero and rearranging, we have,

$$\frac{e_o}{R_f} = -\frac{e_1}{R_1}$$

and we have the amazing result that provided the multistage d-c amplifier has a very high gain and no grid current is drawn by the input tube, then by using negative feedback the value of output voltage does not in any way

The INVERTING AMPLIFIER changes the SIGN of a VOLTAGE





The operational amplifier may be regarded as a balance pivoted at some fulcrum with equal arms representing the resistors

Fig. 2-19

depend on the components of which the amplifier is made. We see from the above relationship that the ratio of output to input voltage is equal to minus the ratio of feedback resistance to input resistance, and the accuracy of this ratio therefore depends solely on the accuracy of the input and feedback resistors.

In a high-gain feedback amplifier the value of the output voltage depends

only upon the values of the input voltage and the input and feedback resistors. Essentially the same current flows through R_1 and R_f. The grid is at zero potential, hence e_o must equal $-(R_f/R_1)\ e_1$ [Fig. 2-18 (B)].

The Inverting Amplifier

10^6 ohms

If the feedback component is made a simple 1-megohm resistor, and the input component is also made a 1-megohm resistor, then their ratio will be unity. Thus with this arrangement the output voltage will have the same amplitude as the input voltage, but it will have the opposite sign. Applying +10 volts to such a device will produce −10 volts at the output. The accuracy of this sign changing, or inverting, operation, depends only on the precision of the two resistors used, and not on any property of the electronic amplifier save its gain. There are no component fluctuations, power supply changes, tube deterioration to worry about continually. We simply require two precision resistors, which can be accurately balanced one against the other. Their values can be maintained by mounting them in a temperature-controlled oven in the computer (Fig. 2-19).

It is frequently found convenient to regard the operational amplifier when used in this manner as a balance pivoted at some fulcrum with equal arms representing the equal resistors. The high gain of the amplifier is represented by the strength of the arms, and the voltages are equivalent to the displacements of the two balance pans. As one goes upwards (positive) the other goes downwards (negative), and provided the arms are equal and strong the displacements are equal.

Multiplication by a Constant Greater Than Unity

Changing the value of either of the resistors in the inverting amplifier will change the ratio from unity. For example, if the feedback resistor is made to have a value of 500,000 ohms, when the input resistor is 1 megohm or 1,000,000 ohms, the ratio will become 1/2, and the output voltage will then be minus one half of the input voltage (Fig. 2-20). Such an arrangement is rarely, if ever used, because as we have seen, the multiplication of a voltage by a constant less than unity can be achieved quite simply and cheaply with a potentiometer. On the other hand, if the feedback resistor is maintained at a value of 1 megohm, and the input resistor is reduced to 200,000 ohms, say, then the ratio becomes 5. The output voltage will now be −5 times the input voltage, and we have achieved a multiplication by a constant greater than unity. If the constant required is +5 rather than −5, following the multiplying amplifier by an inverting amplifier produces the desired result.

It is becoming much more common than was previously the case for operational amplifiers to be arranged to have gains of only −1 and −10. The −10 is achieved using a 100,000-ohm input resistor. Any other constant of multiplication can be achieved by using a combination of an attenuator and one or two amplifiers. For example, with an attenuator set to 0.7365, and an amplifier having a multiplication factor of −10, we can multiply a voltage by −7.365 [Fig. 2-21 (A)].

MULTIPLYING A VOLTAGE BY POSITIVE OR NEGATIVE CONSTANTS

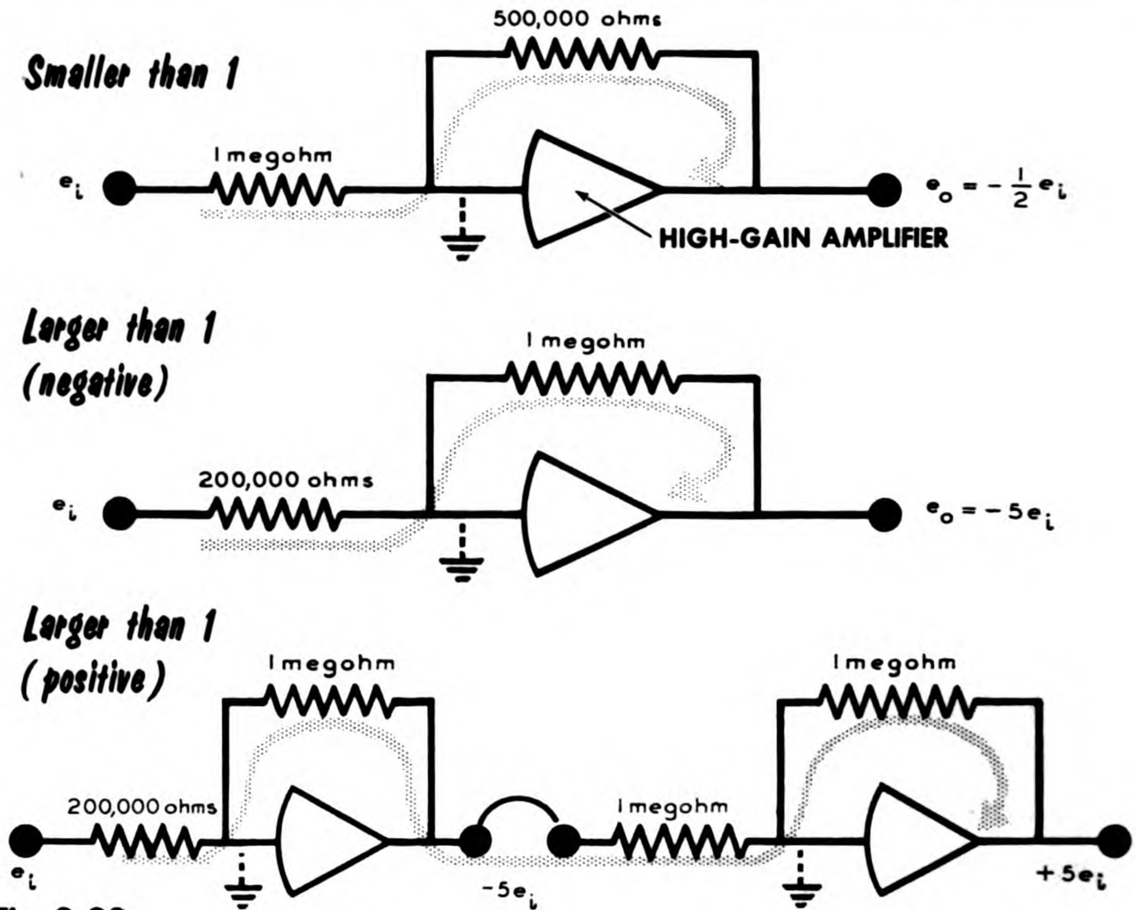


Fig. 2-20

As a second example, with an attenuator set to 0.5731, and two amplifiers each having gains of −10, we can multiply a voltage by 57.31. This would be the constant of multiplication required to change a voltage representing an angle in radians, to one representing the same angle in degrees [Fig. 2-21 (B)]. A radian is an angular measure equal to 57.31°. A full 360° angle is measured as 2π radians.

Frequency Response

In most of what is written in this volume, the computing voltages described may appear to have constant values. Sometimes in practice they do have constant values, but far more frequently in the solution of a problem the voltages are changing considerably with time. It is their changes with time under the influence of certain disturbances that is of particular interest to the investigator. Now, the mathematical operations performed by the computing components on these voltages must remain the same, no matter how quickly the voltages change. If an amplifier (Fig. 2-22) is to multiply a

MULTIPLYING A VOLTAGE BY POSITIVE OR NEGATIVE CONSTANTS

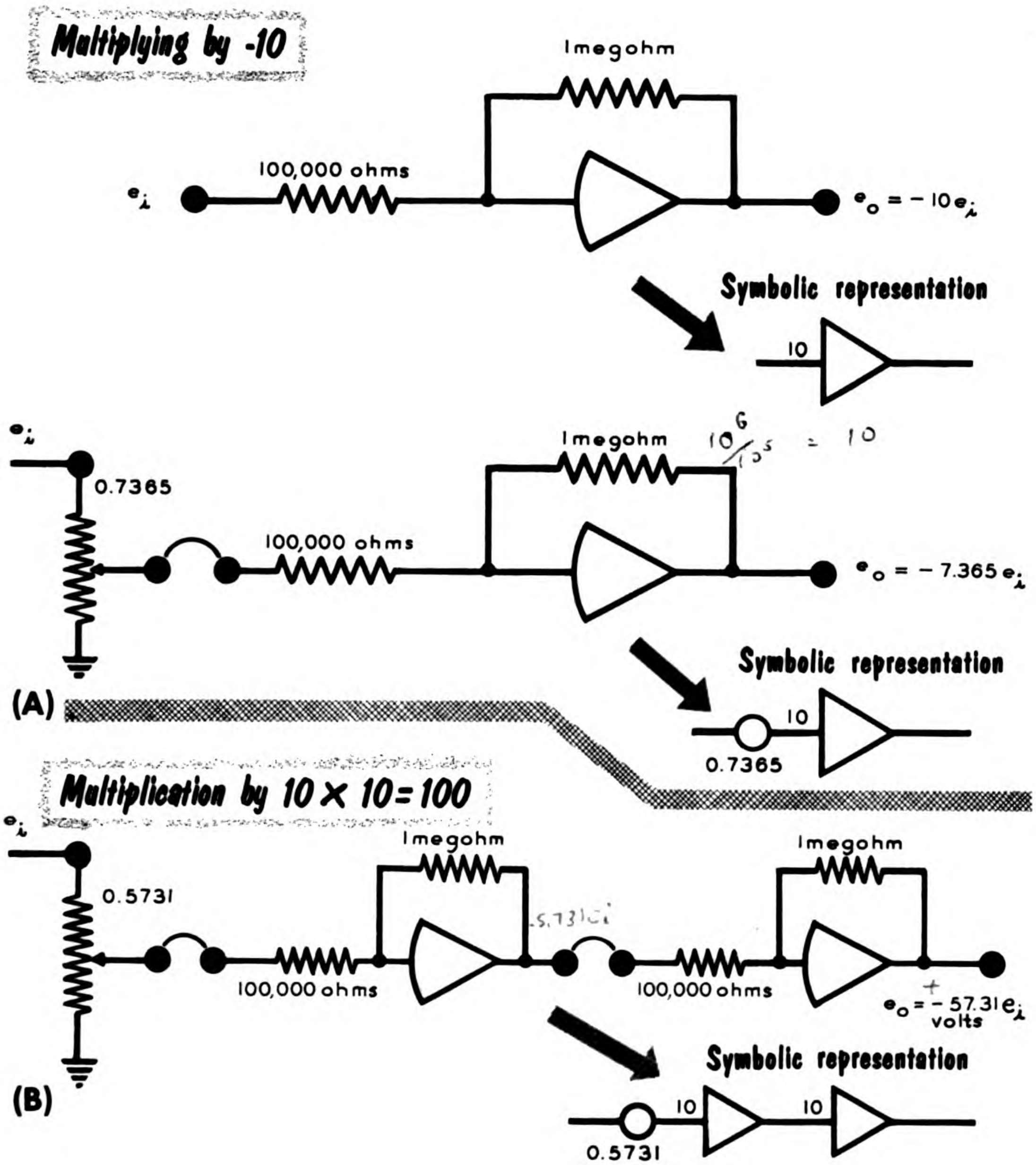


Fig. 2-21

voltage by a factor of −10, then this factor must remain −10 for a voltage alternating at 100 cps, just as it is for a steady voltage. Furthermore, there must be no delay in the multiplication. If the input voltage has an instantaneous value of 5 volts, then at that very same instant the output voltage must be −50 volts. Errors would be introduced into the problem solutions if this were not the case.

There is no problem in this respect in a passive element like a potentiometer. However, delays and reduction in the amplitude ratio do occur in normal electronic amplifier (active) circuits. These delays and reduced amplitude ratios become worse as the voltages change more rapidly, because the frequencies increase. The design of the amplifier is aimed at maintaining the correct mathematical operation on the input voltages for as high a frequency as possible. In this task the negative feedback used to improve the quality of the amplifier in other respects, helps considerably. It is not unusual for the operation of a computing amplifier to be quite constant for input voltages containing frequencies up to many thousands of cycles per second, which

For VERY HIGH-FREQUENCY INPUT SIGNALS a COMPUTING COMPONENT will PRODUCE an ATTENUATED OUTPUT due to the limited frequency response characteristic of the component

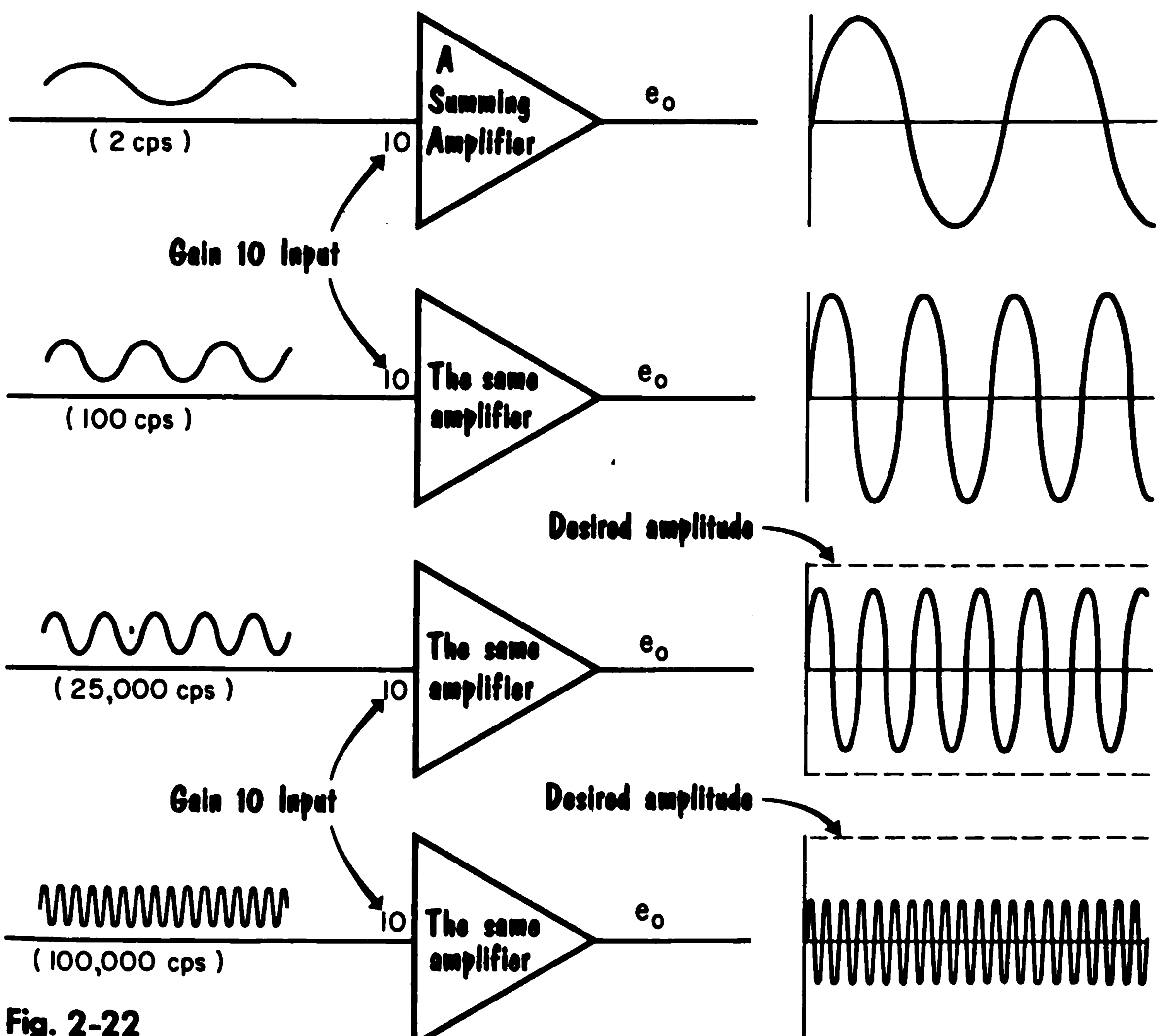


Fig. 2-22

THE SUMMING AMPLIFIER

With a number of equal input resistors, the amplifier sums the input voltages

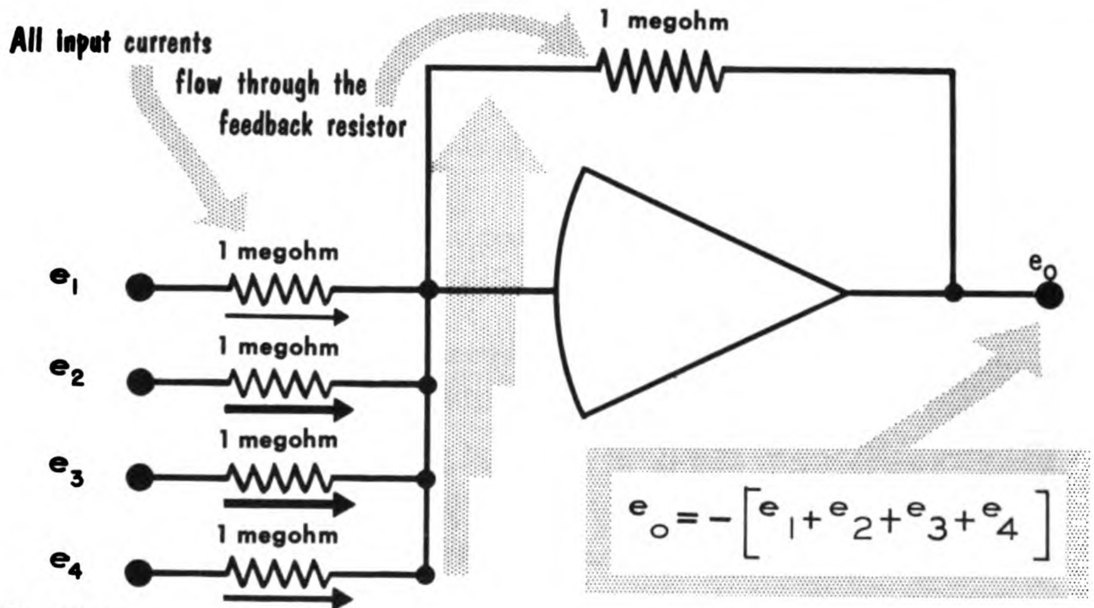


Fig. 2-23

is far above those normally present in the variable voltages investigated on the computer.

The Summing Amplifier

Consider what happens to the output of a computing amplifier if with the single 1-megohm feedback resistor a number of 1-megohm input resistors are used, and to each there is applied an input voltage. Assume for simplicity that all the voltages are positive. Then, as the grid voltage is negligibly small compared with the normal voltages used as inputs, the current passing through each input resistor will be equal to the corresponding input voltage divided by 1 megohm. All these currents will add together and pass through the feedback resistor, for no accumulation of charge can occur at the input grid. Thus as the feedback resistor is also equal to 1 megohm, the voltage developed across it will be the sum of the input voltages, and with the input grid voltage at very nearly zero, the output voltage is forced to be as far below zero as the sum of input voltages is above zero. *The output voltage is equal to minus the sum of the input voltages* (Fig. 2-23).

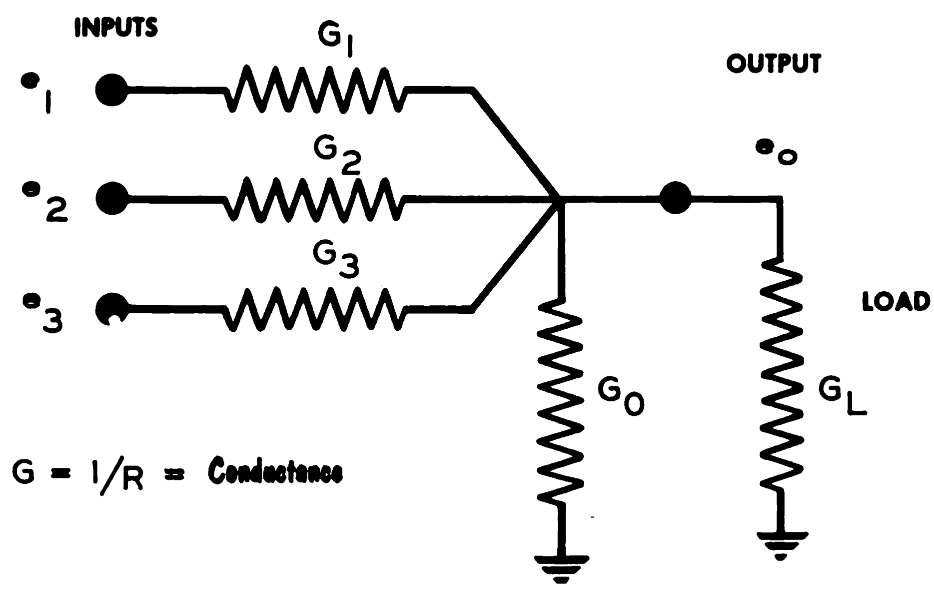
Had any of the input voltages been negative, it would have contributed a current flowing in the opposite direction to the positive ones, and this would have been subtracted from the total, reducing the current flowing through the feedback resistor and thus decreasing the amplitude of the output voltage. Thus the summing amplifier recognizes the sign of an input voltage and produces minus the algebraic sum of the applied voltages.

Why Not Use a Simple Resistive Node for Summation?

The thoughtful reader might question from this explanation of voltage summing with an operational amplifier, whether the amplifier is really necessary. Why not simply use the resistors, and by an algebraic summing of the driving currents through the output resistor obtain the desired result! Of course, the resistor network would work (Fig. 2-24) as already indicated in Volume 1, but it would not be convenient or accurate enough for computational purposes. Its operation would be very susceptible to the kind of circuit attached to the output resistor, and there would be interaction between the input voltages.

The accuracy of any summation process achieved using the operational amplifier depends on two requirements other than the accuracy of the re-

In SIMPLE RESISTIVE SUMMING the MULTIPLICATION CONSTANTS a,b, and c, DEPEND on ALL RESISTOR VALUES and the LOAD connected to the OUTPUT



$$\left[G_0 + G_1 + G_2 + G_3 + G_L\right] e_0 = G_1 e_1 + G_2 e_2 + G_3 e_3$$

Thus

$$e_0 = a e_1 + b e_2 + c e_3$$

Fig. 2-24

sistors involved: (1) the voltage at the mutual connection of all input resistors must remain sensibly at zero; (2) the current drawn by any loading circuit must not change the current flowing through the feedback resistor. Neither of these conditions could be satisfied without the amplifier.

Summation and Multiplication in One Unit

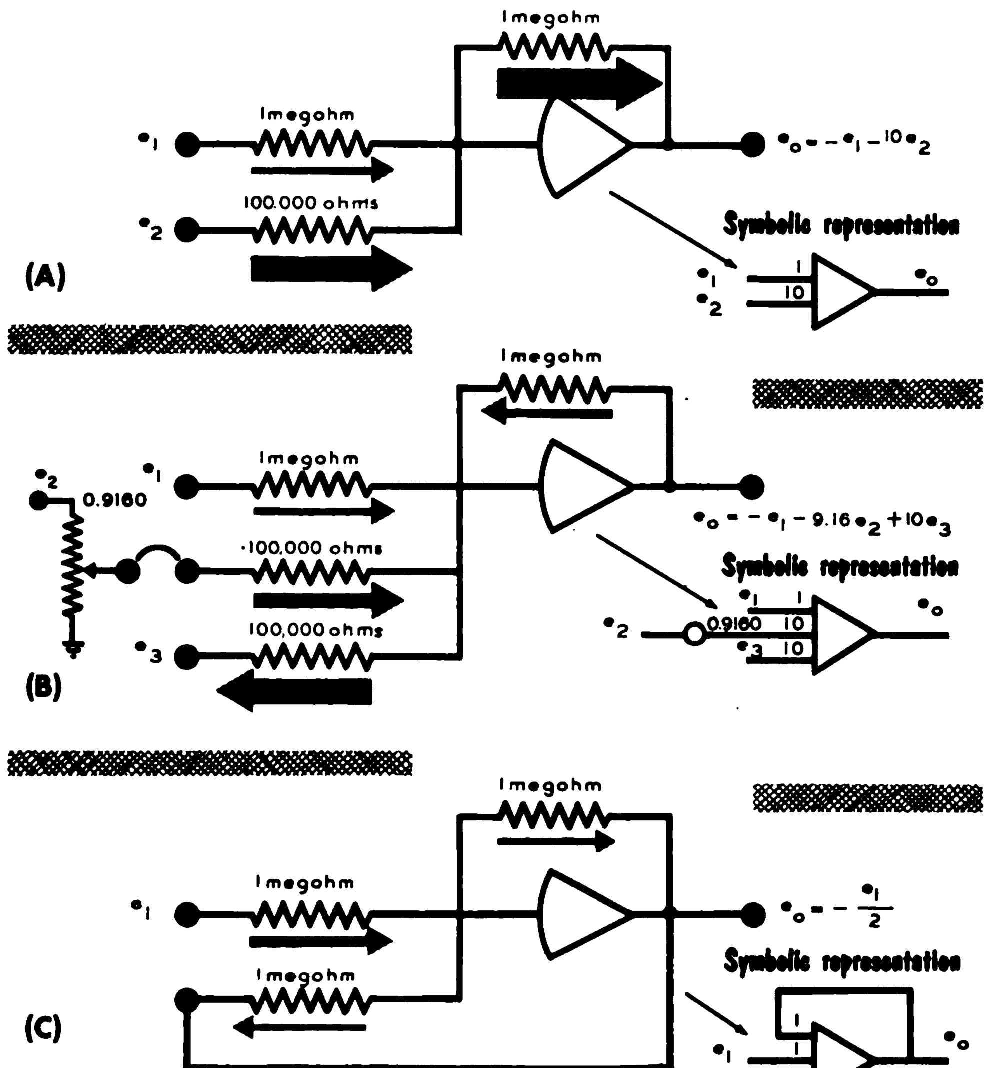


Fig. 2-25

Multiplication Plus Summation in One Unit

If in one amplifier there are both 1-megohm and 100,000-ohm input resistors, it is possible to multiply voltages by constant factors as well as summing them. A voltage applied to a 100,000-ohm resistor will supply ten times the current to pass through the feedback resistor as it would if it were applied to a 1-megohm resistor. Thus its contribution to the total output voltage will be multiplied by a factor of 10. By using attenuators in association with the assorted input resistors it is possible to sum together a number of voltages each multiplied by a factor which can have any convenient value between 0.001 and 10. If factors outside this range are required, then other amplifiers will be needed (Fig. 2-25).

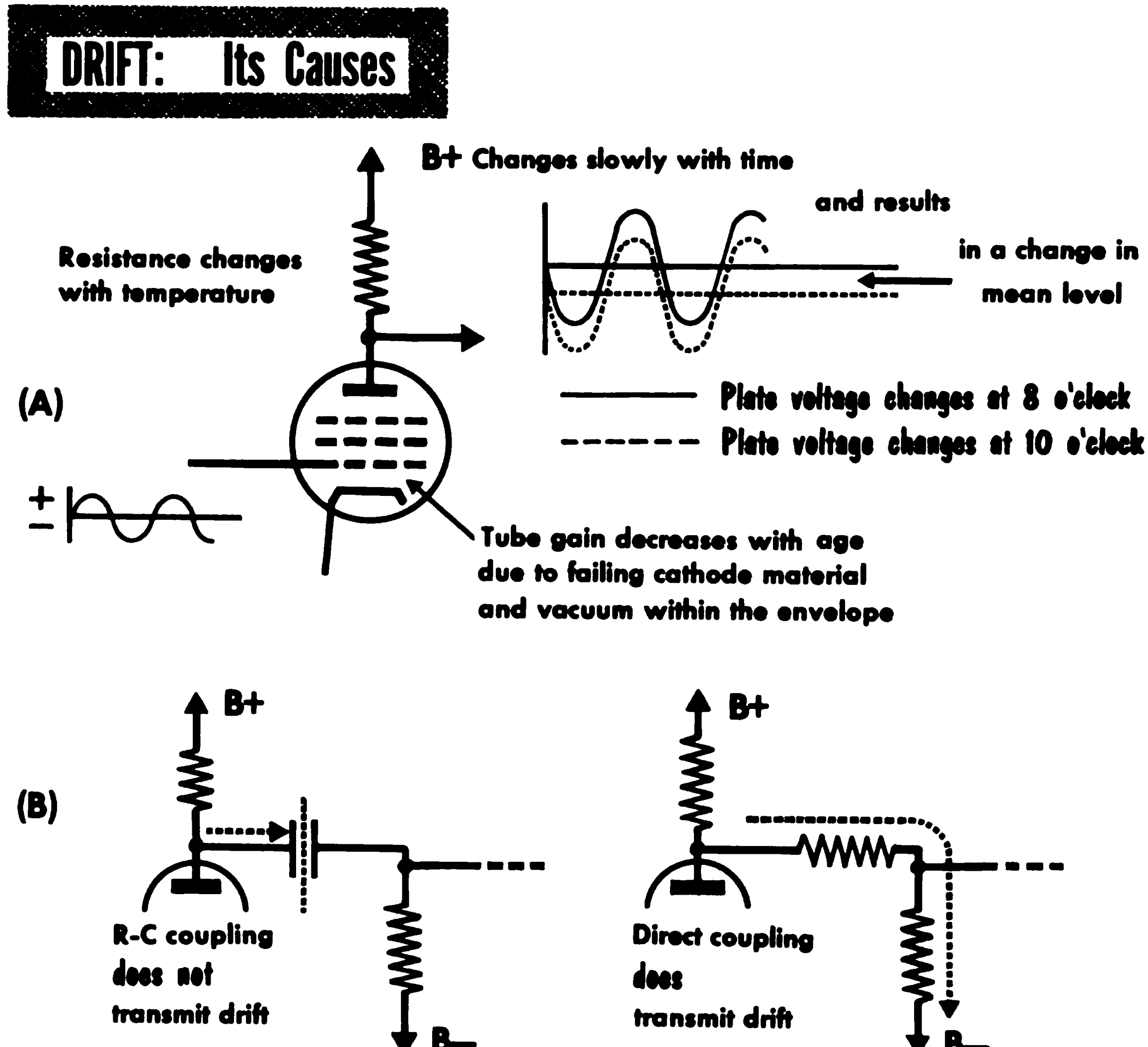


Fig. 2-26

There is no reason why the output voltage should not form one of the inputs. For example, by feeding back the output voltage to an input resistor of 1 megohm, the effective feedback impedance will be reduced to one half of its original value, or 500,000 ohms. This will cause the factors by which the true inputs are multiplied in the amplifier to be effectively halved.

THE PROBLEM OF DRIFT

What is Drift?

In all that has been stated so far it has been assumed that due to the use of a high-gain negative feedback amplifier, the output voltage of a computing amplifier is directly proportional to the sum of the input voltages multiplied by appropriate factors. Very particularly has it been assumed that when the input voltages are all zero, then so will the output be. No offset is permitted in the operation of the unit [Fig. 2-26 (A)]. Unfortunately, the cir-

cuits so far described are unable to fulfill this requirement adequately. Although the balancing circuit permits the output voltage to be adjusted to zero, the amplifier balance changes unpredictably with time, and very soon an offset would exist, once more preventing accurate operation. Such a random variation of the output zero is known as *drift*.

The components of which any electronic amplifier is made have characteristics that change with time. In particular, vacuum tubes age, and their operating levels change. Thus all voltages throughout the circuit may change with time. The changes are usually small. They can occur quickly or slowly, but there will be a long-term wander in the operating level of each stage of amplification. In an a-c amplifier the change in operating level of one stage is not passed on to the next stage, for it is blocked by the interstage coupling network. In an amplifier using direct coupling, the changes are passed on from stage to stage, and unfortunately they are amplified in the process [Fig. 2-26 (B)]. Thus drift in a d-c amplifier is quite serious, particularly when that amplifier is to be used in an analog computer. It cannot be eliminated, for it is a natural phenomenon. It can be compensated for, and its effect on the zero level of the output voltage can be made negligible.

Drift in a Computing Amplifier

Consider what happens to the voltages in a computing amplifier when the operating level of any stage of the amplifier drifts. For clarity, let us assume that the effect on the output voltage is to cause it to be more positive than it should be. This error in the output will be transmitted back to the input grid of the amplifier by the feedback circuit, causing the grid to be more positive than it should be. But with the grid becoming more positive, the output voltage is forced in the opposite direction, cancelling in some measure the original drift voltage (Fig. 2-27A). Thus the presence of negative feedback around the computing amplifier considerably reduces the effect of drift. However, the reduction is not sufficient for all the uses to which the computing amplifier is applied, and more effective drift correction is required.

In the computing amplifier one might look upon the major effect of drift as being to move the input-grid voltage away from its value of zero. The mathematical operations that the amplifier performs are critically dependent on the grid voltage remaining for all practical purposes, zero. Thus some circuit arrangement is required to maintain this condition in spite of the drift. In the analogy of the balance previously presented, the displacements of the pans can only be equal and opposite provided the position of the fulcrum (here the grid voltage) remains zero (Fig. 2-27B).

Sampling the Drift Voltage

It is known that a-c amplifiers do not amplify drift voltages. Thus it is logical that a drift-compensating circuit should use an a-c amplifier. However, the drift voltage which is to be corrected is either a direct voltage, or at most, a slowly-changing voltage. Before any correction can be effected the

Negative Feedback Reduces the Effect of Drift

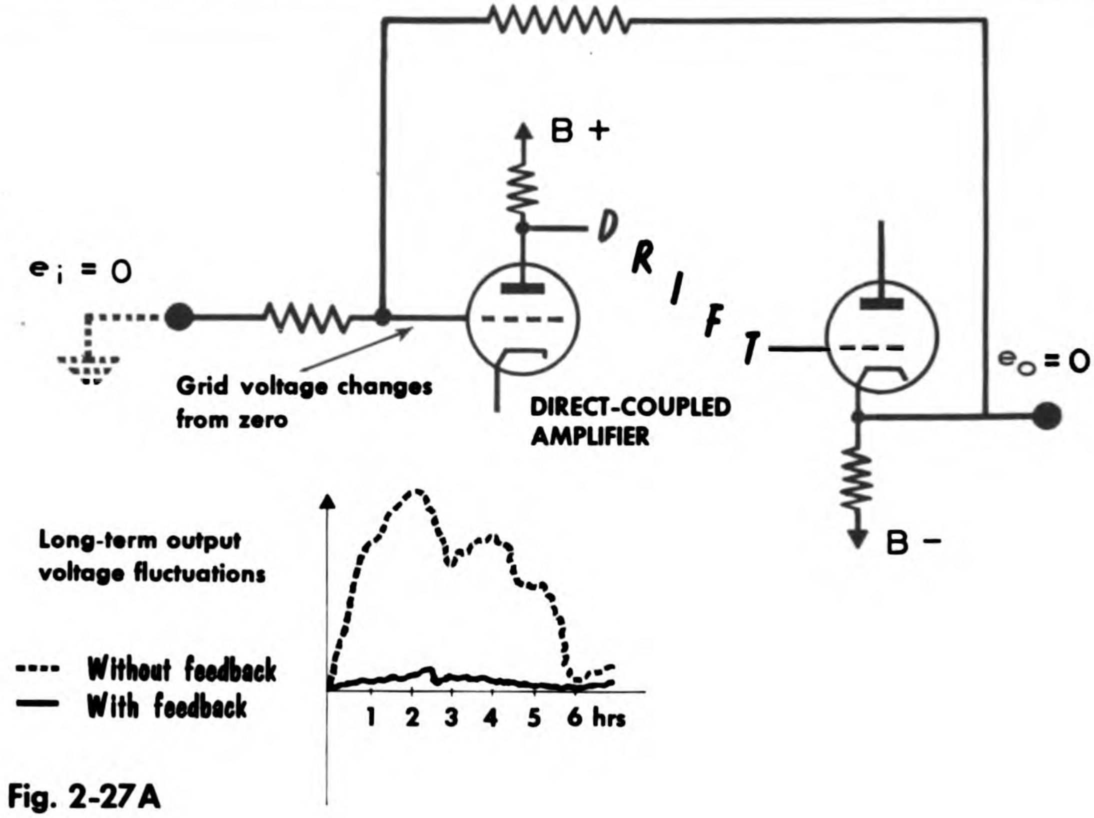


Fig. 2-27A

drift voltage must be measurable as an alternating voltage. This is made possible with the use of an electromechanical vibrator (Fig. 2-28).

The grid voltage of the d-c amplifier is sampled by connecting to the grid a vibrator, which by periodically grounding the a-c amplifier input, produces

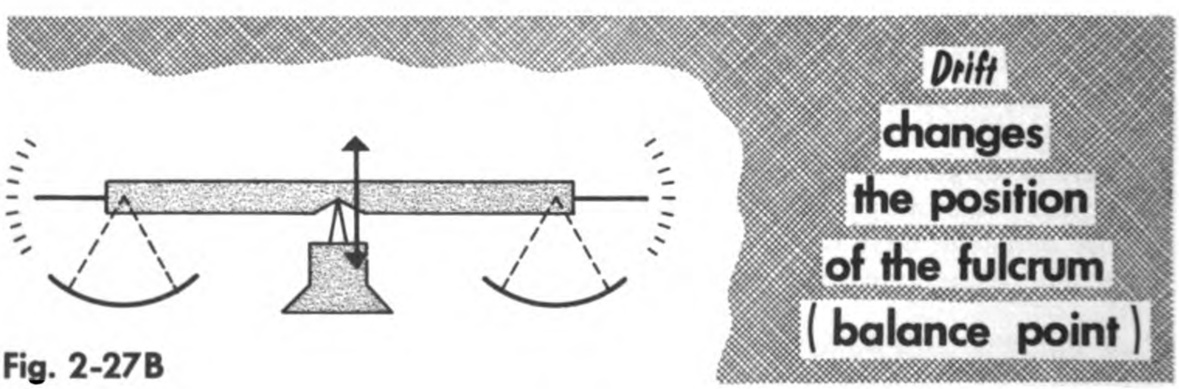


Fig. 2-27B

a square-wave voltage at some suitable frequency, say, 100 cps. For 5 msec the output from the vibrator will be zero. Then for the next 5 msec it will equal the grid voltage, and so on. The amplitude of the square-wave voltage

is equal to the instantaneous value of the drift voltage. The phase of the square-wave voltage, with respect to the driving 100-cps voltage is determined by the sign of the drift voltage. For example, if for the half cycle during which the driving voltage is positive the square wave is also positive, then the drift voltage is known to be positive. On the other hand, if the

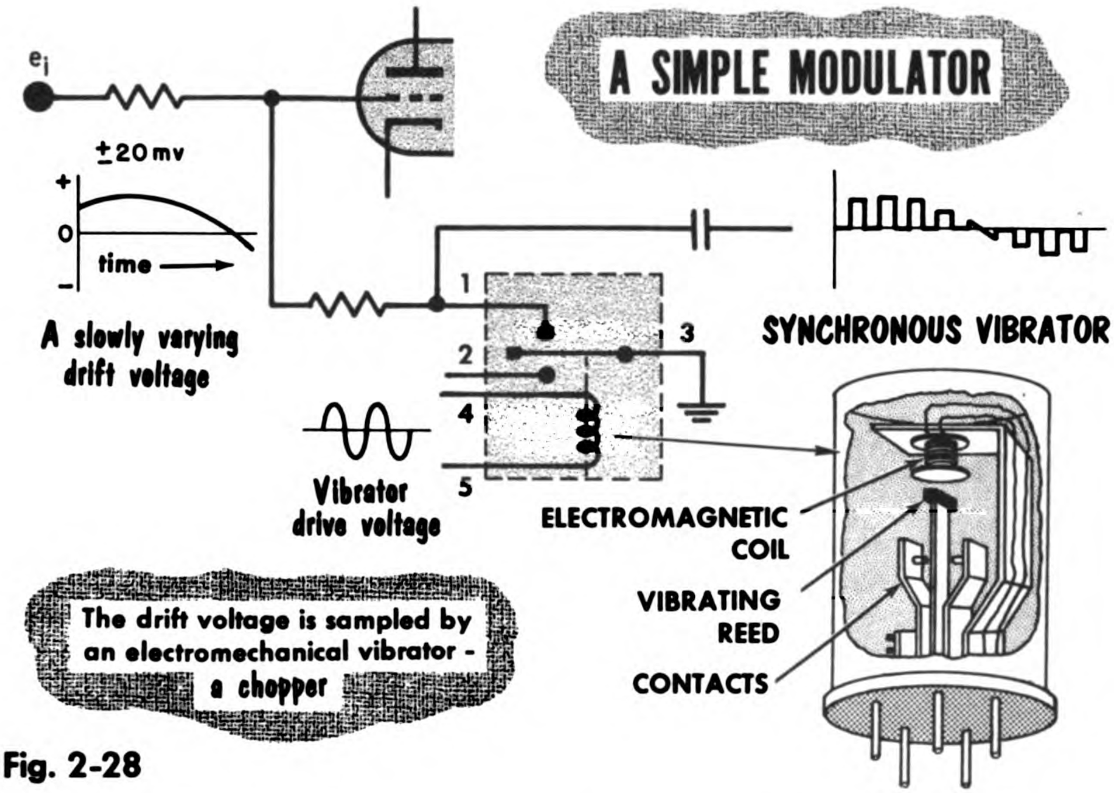


Fig. 2-28

square wave is negative during this time, then the drift voltage must also be negative.

Amplifying the Sampled Drift Voltage

The square-wave output of the mechanical vibrator is coupled to an a-c amplifier. The amplifier used for this purpose typically consists of two pentode stages of amplification with R-C coupling between them. It has a gain in the range 1000 to 5000, and produces an output voltage in phase with the applied voltage. The output voltage will not be as square in waveform as the input voltage due to the smoothing effect of the interstage circuitry. This, however, is of no consequence, for the amplified alternating voltage has to be rectified and filtered to a steady d-c voltage proportional to the rms value of the a-c amplifier output, before it can be returned to the d-c amplifier (Fig. 2-29).

The level about which the output voltage of the a-c amplifier alternates does not depend at all on the input voltage to the amplifier. It is solely determined by the values of the components used in the output stage of amplification, and it can therefore be arranged to be at some convenient level. It

DRIFT-FREE AMPLIFICATION

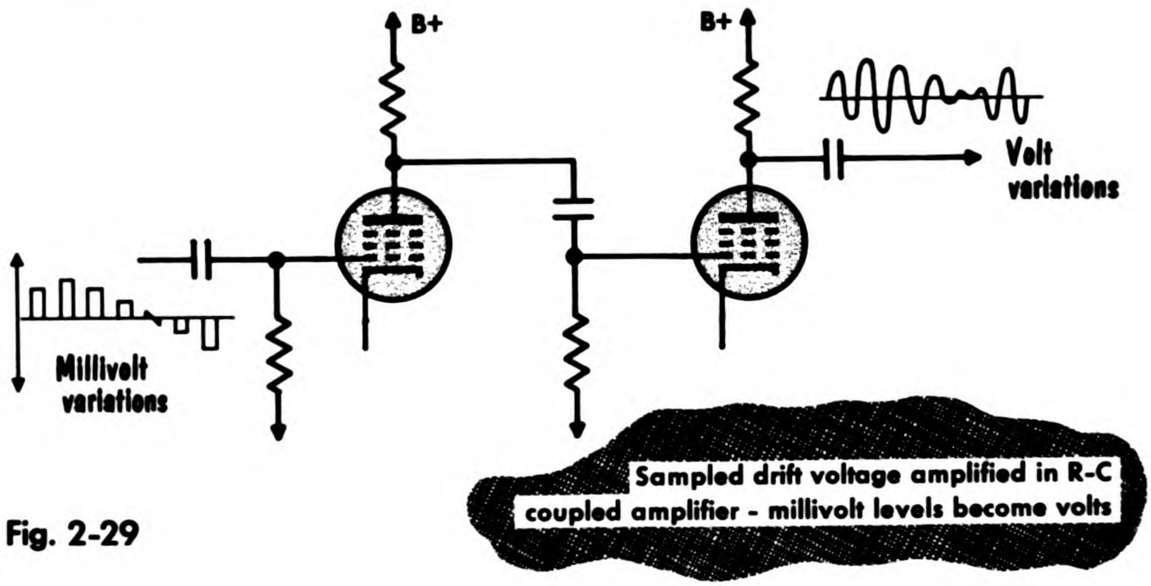


Fig. 2-29

is necessary to establish a zero reference for the demodulated signal obtained from the a-c amplifier output. A second pair of vibrator contacts as part of the phase-sensitive demodulator is used to achieve this.

Phase-Sensitive Demodulation and Smoothing

Just as a vibrator can be used to add a square-wave carrier waveform to a d-c voltage, it can be used to perform the opposite task of phase-sensitive

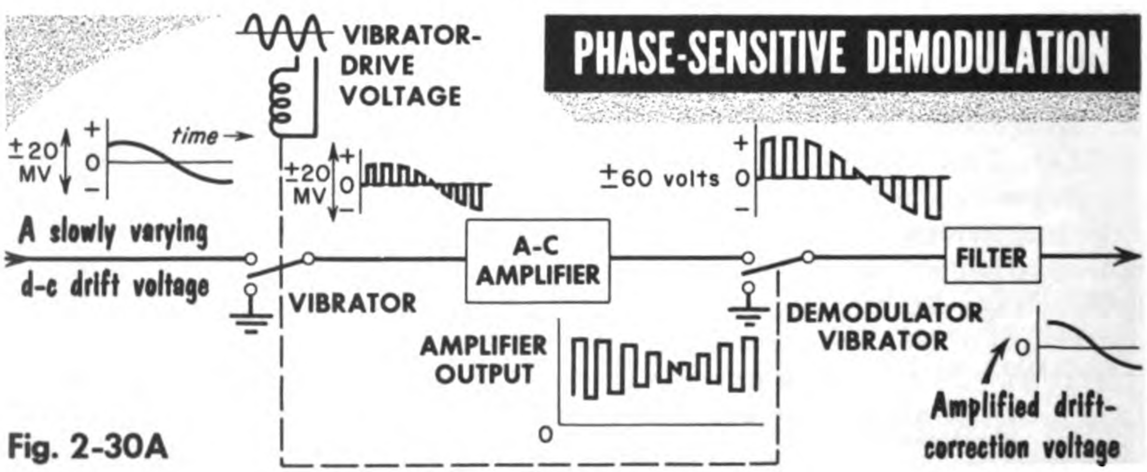


Fig. 2-30A

demodulation. It is the alternating voltage at the output of the a-c amplifier which is of importance to us, not the level about which this alternation occurs. Thus it is necessary to produce a d-c voltage whose amplitude is

dependent on the amplitude of the alternations, and whose sign is dependent on their phase with respect to the phase of some reference voltage. To do this, a synchronous demodulation is necessary. The output of the amplifier is connected through a blocking resistor to a low-pass filter, and to a second leaf of the vibrator which causes the input voltage to the filter to be at ground potential during one half cycle of the voltage alternations (Fig. 2-30A). If this half cycle corresponds to the positive part of the alternating voltage, then the input to the filter will be only negative half waves and *vice versa*. Thus the voltage built up across the capacitor of the low-pass filter will be due to contributions which are positive or negative, depending on the phase of the amplified voltage and therefore on the sign of the original drift voltage. The charging time of the filter circuit is made extremely long compared with the time of each alternation, and therefore the amplitude

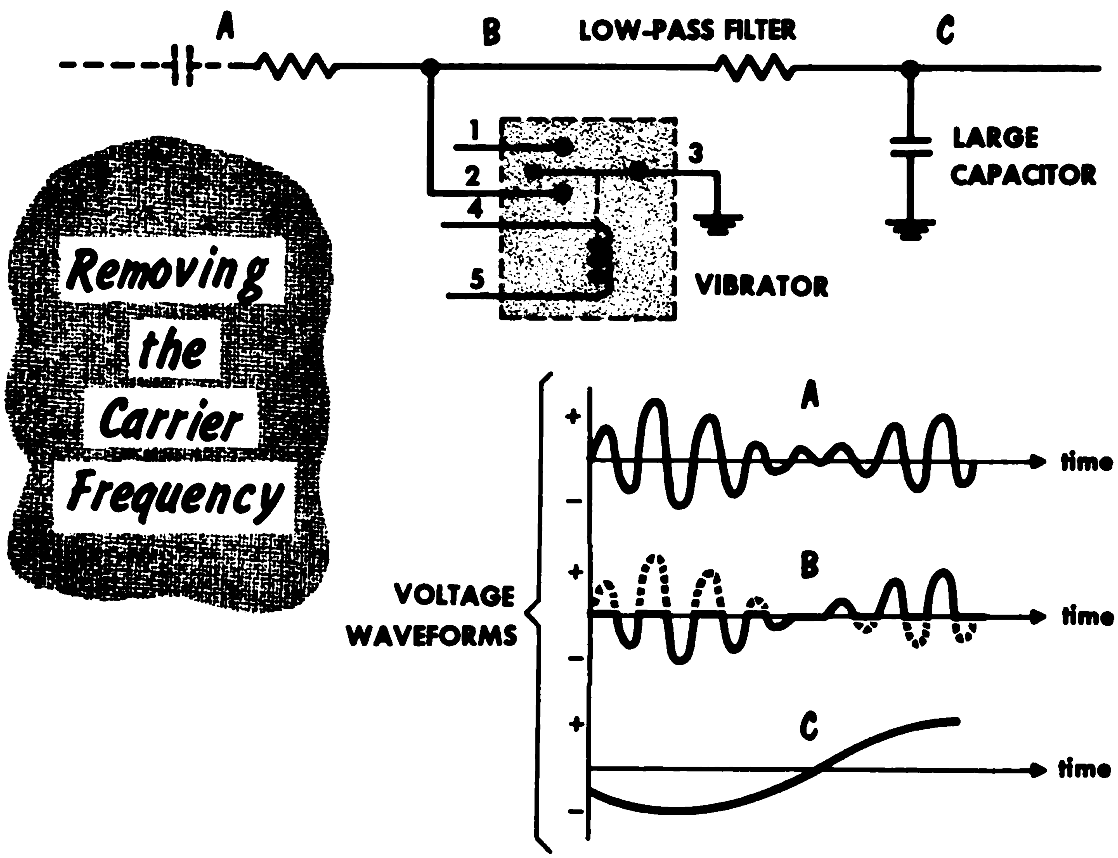
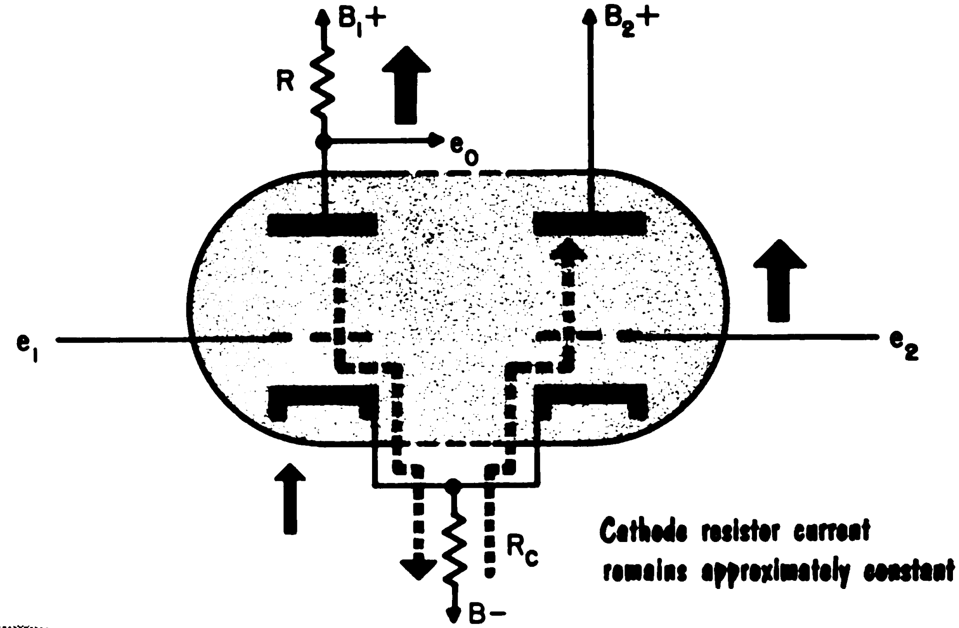


Fig. 2-30B Synchronous demodulation using a vibrator

of the voltage across the capacitor does not follow the alternations, but takes a relatively constant value which is proportional to the rms value of the a-c voltage (Fig. 2-30B).

The output of the filter, i.e., the voltage across the capacitor, is equal to the original d-c drift voltage multiplied by a large negative factor, and this multiplication has been achieved without further drift by using an R-C coupled amplifier. Note that this technique of driftless amplification is only

The DIFFERENCE Amplifier



A difference amplifier produces a voltage e_0 approximately proportional to $(e_2 - e_1)$

Fig. 2-31

possible because the voltage to be amplified changes slowly. A low-pass filter included in the grid circuit prevents any rapidly changing voltages from reaching the vibrator and a-c amplifier. Were this not present, any frequencies in the operational amplifier input voltages close to the vibrator drive frequency would cause errors in amplifier balance by beating with the latter, and producing a contribution to the stabilizer output. This same low-pass filter prevents the square wave produced by the vibrator at the input to the a-c amplifier from reaching the d-c amplifier.

The Difference Amplifier

Having obtained a greatly amplified drift voltage, how is this used to good effect in the operational d-c amplifier? To answer this question it is necessary first to describe the operation of a difference amplifier. Consider the double-triode circuit in the figure, where both cathodes are connected to the same cathode resistor, and the second half of the tube is used as a cathode follower. The output of the circuit will react quite normally to changes in the first-grid voltage provided the second-grid voltage is maintained at some constant level.

What happens if the first-grid voltage remains constant and the second-grid voltage is changed? A positive change in this grid voltage causes more

current to pass through the second half of the tube and thus more current passes through the common cathode resistor. The cathode voltage follows the second grid, decreasing the grid-to-cathode voltage of the first half of the tube. Less current flows through the first half and therefore the output plate voltage rises. Thus, whereas a positive change applied to the first grid causes a negative change in the output voltage, applied to the second grid it causes a positive change in the output voltage. The circuit produces a differencing effect and is commonly known as a *difference amplifier* (Fig. 2-31). Of course, if we have two voltages of opposite sign, by applying them to the two grids of a difference amplifier we will obtain an output proportional to their sum. Thus a difference amplifier can be used to sum voltages, and this is its use in the drift-compensation circuit of the operational amplifier.

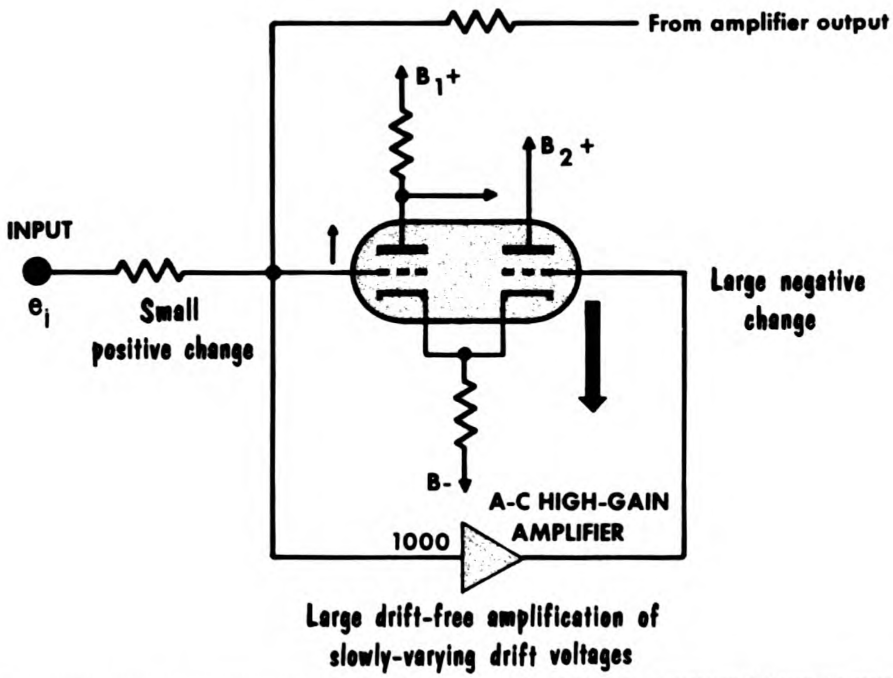
Adding the Amplified Drift Voltage

A difference amplifier can be used to add to the drift voltage at the input of the high-gain d-c amplifier, the amplified drift voltage. The vibrator operation is such that a positive drift voltage at the input grid of the high-gain amplifier produces a negative voltage at the output of the low-pass smoothing filter. This negative voltage, which we will assume quite reasonably to be 3000 times the original drift voltage, is applied to the second grid of the difference amplifier, which forms the input stage of the d-c amplifier (Fig. 2-32). Its effect will be added to the original drift voltage which in turn produces a large negative output voltage from the high-gain amplifier, cancelling in large measure the original positive drift. In fact, the effective drift in the unit will be reduced by a factor equal to the gain of the a-c amplifier, here 3000. The drift compensation achieved in this way permits the unit to be used for at least many hours, and normally days, before very long-term drift effects cause the difference amplifier operation to exceed its usable range. Occasional amplifier balancing is always necessary to cancel the accumulated drift. Without the compensation, drift would make necessary the balancing of the amplifier at frequent intervals, every few minutes, seriously interfering with the use of the computer.

A Second Look at Drift Compensation

A careful study of the preceding pages on the method of drift compensation will show that the technique used effectively places two stages of *drift-free* amplification in front of the input tube of the d-c amplifier. These two stages of amplification employing a carrier frequency, an a-c amplifier, and a demodulation and smoothing circuit, are effective only at very low frequencies, as the passband of the smoothing circuit is very narrow (approxilately 1/100 cps). Thus they are only capable of amplifying constant voltages and very slowly-changing voltages like those due to drift (Fig. 2-33). For such voltages the open-loop gain of the operational amplifier is considerably increased, and the negative feedback action thereby cuts to a negligible level the output fluctuations of the drift voltages in the later, direct-coupled stages.

The DIFFERENCE AMPLIFIER is used to ADD TWO VOLTAGES



Drift-free compensation amplifies the effect of negative feedback, and permits satisfactory operation for many hours

Fig. 2-32

At LOW FREQUENCIES the DRIFT-COMPENSATING AMPLIFIER and the HIGH-GAIN, DIRECT-COUPLED AMPLIFIER are IN SERIES

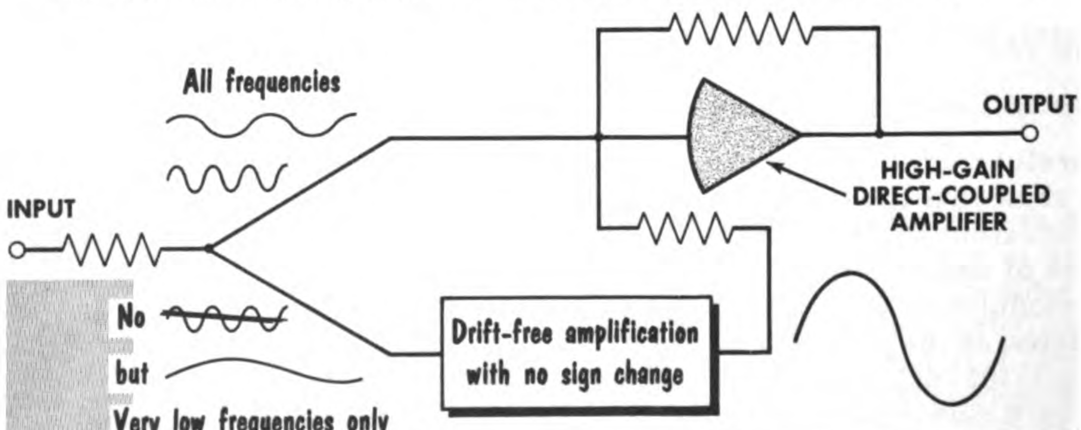


Fig. 2-33

The drift-compensating circuit provides a large amplification for very low-frequency signals only

The OPERATIONAL AMPLIFIER has a BUILT-IN CHECK of its PRECISION

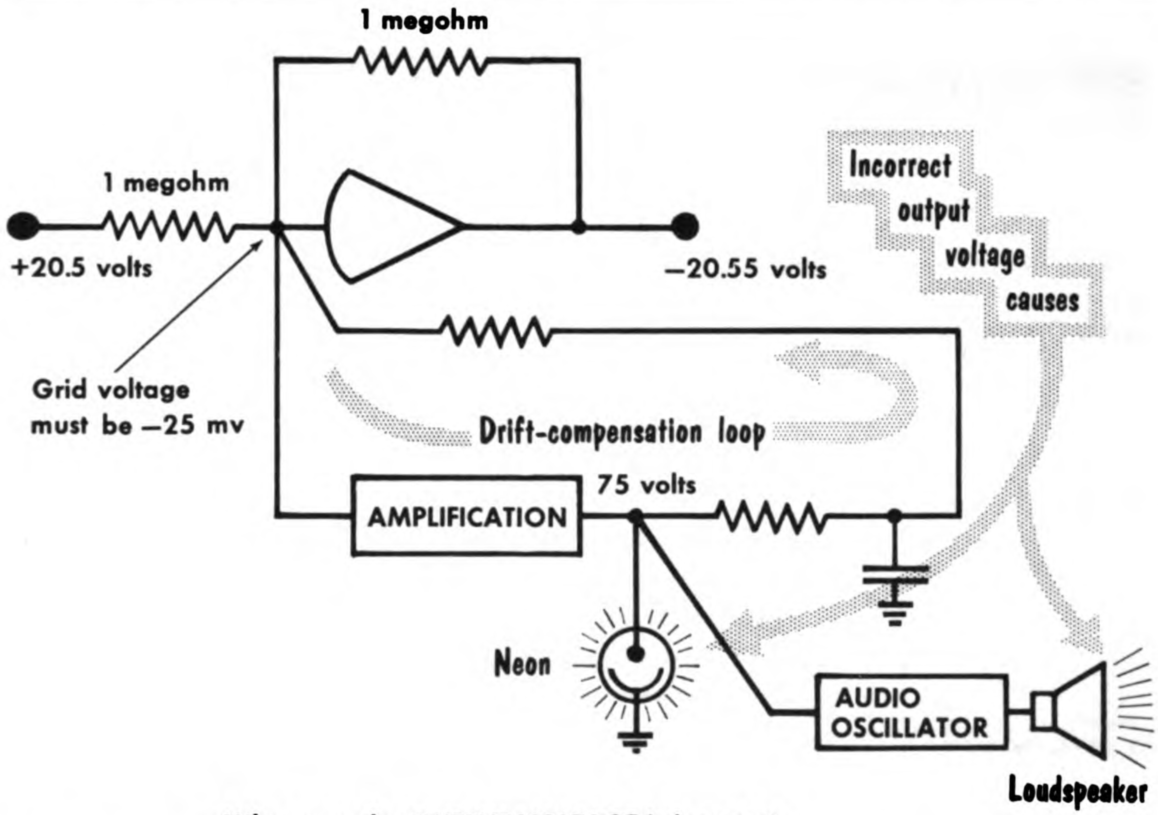


Fig. 2-34 Whenever the OUTPUT VOLTAGE is incorrect, the RESULTING NON-ZERO GRID VOLTAGE excites an alarm system

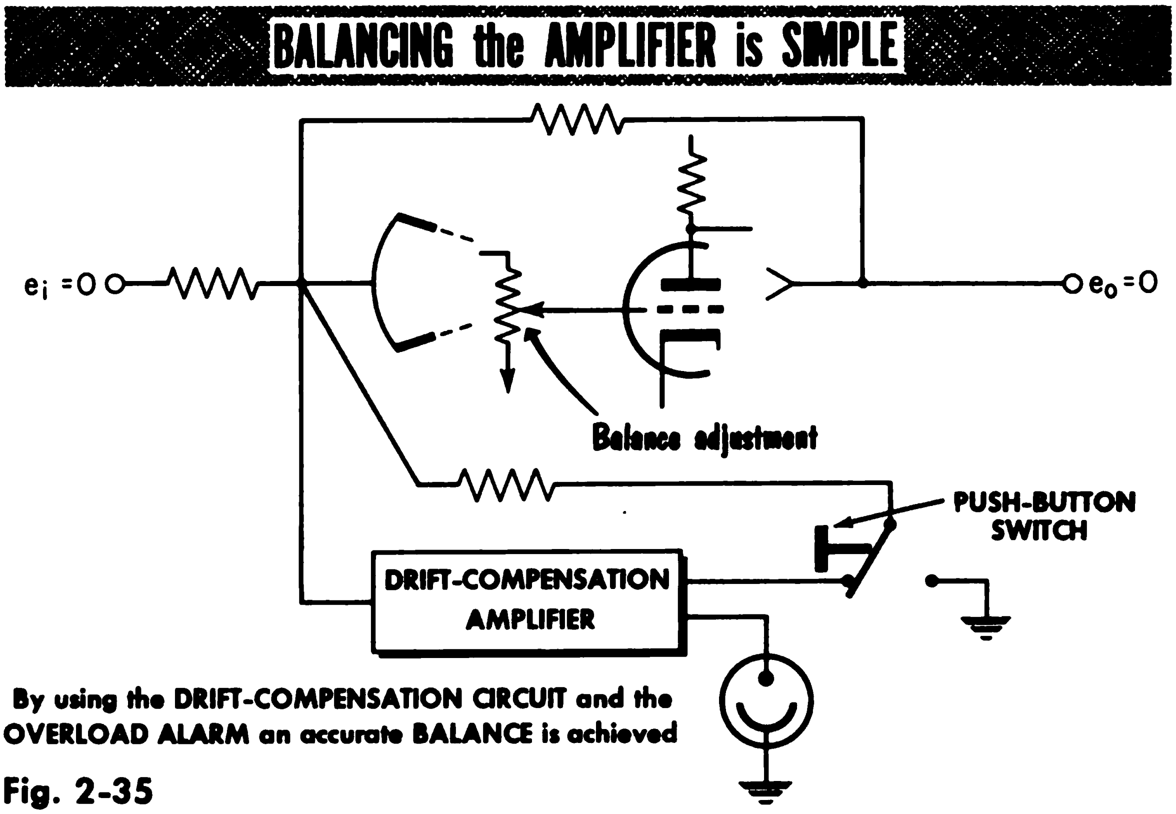
A direct connection between the input resistors of the amplifier and the direct-coupled stages is required, so that voltages outside the narrow band of the drift-compensation circuit can be acted upon without serious attenuation. The overall frequency characteristic of the amplifier is one having a flat response of from an exceptionally low frequency, to 10–50 kc.

ADDITIONAL FEATURES OF THE OPERATIONAL AMPLIFIER

Overload Warning System

In an investigation of a physical system with an analog computer there may be dozens of amplifiers in use. It is impossible to check continually that each is operating as it should and not introducing errors into the study. To overcome this difficulty, a warning system is built into the computer which warns of any inadequate operation of a component. The usual cause of such a failure is an overload condition where an amplifier is being required to supply either too high a voltage, or too much current for its capabilities.

In the operational amplifier any change in operation which causes the output voltage not to correspond to the input voltage(s) must be detected. Except for this possibility, the circuit generally performs satisfactorily. A difference between the output voltage and that demanded by the input



By using the DRIFT-COMPENSATION CIRCUIT and the OVERLOAD ALARM an accurate BALANCE is achieved

Fig. 2-35

voltage(s) causes the input-grid to move away from zero, for its value is determined by the potential divider formed by the feedback resistor and the input resistor(s). This change in the operating level of the input grid can be used to sound the alarm (Fig. 2-34). In practice, the alarm circuit is placed at the output of the a-c amplifier of the drift compensator, for the input grid circuit is not a suitable point from which to obtain the alarm excitation. When a current overload or voltage imbalance occurs and the grid voltage changes from its near-zero value to 10 or 20 mv, the output of the a-c amplifier is raised to 60–70 volts, which is sufficient to trip the alarm. In a computer with many amplifiers it is common to have just one audible alarm fed in parallel from all amplifiers. To identify the troublesome amplifier once the alarm has been sounded, neon bulbs are connected to each a-c amplifier output, and these indicate where the overload is located.

Balancing the Operational Amplifier

With the use of the overload warning system on each operational amplifier as described in the previous section a simple method of balancing the operational amplifier presents itself. In our previous discussion of amplifier bal-

ancing, we stated that the output of the operational amplifier was adjusted accurately to be zero with no input voltages present. This would require measuring the output voltage accurately with a meter. With the proposed form of overload alarm system, such a balancing procedure is not necessary.

Balance of the amplifier requires the grid voltage to be extremely close to zero always. It can be checked and adjusted by disconnecting the drift-compensating voltage to the d-c amplifier and adjusting the balance potentiometer until the overload alarm is not excited (Fig. 2-35). Thus with the a-c amplifier output at some voltage less than 70 volts, and a gain of 3000 in the drift-compensation circuit, the grid voltage must be within a few hundredths of a volt of zero. Reconnecting the compensating circuit introduces into the feedback loop an additional gain, and thus grid voltage is reduced to a few microvolts. With the grid voltage negligibly small, the amplifier-output voltage correctly corresponds to the negative sum of the input voltages, and the amplifier is well balanced.

Physical Layout of the Operational Amplifier

The high-gain amplifiers to be used in the analog computer require no special care either in their location or in the power supplies which they use. For this reason they can be placed in any convenient console of the com-

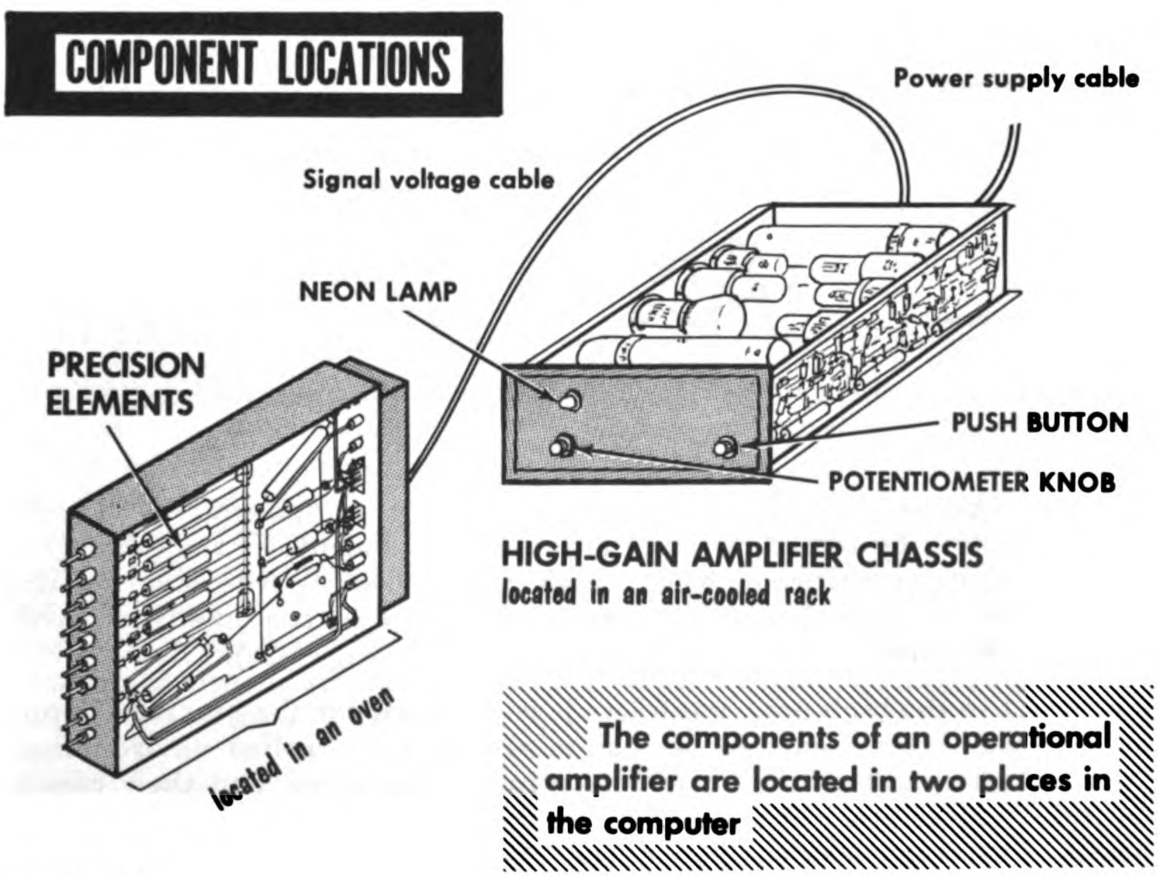


Fig. 2-36

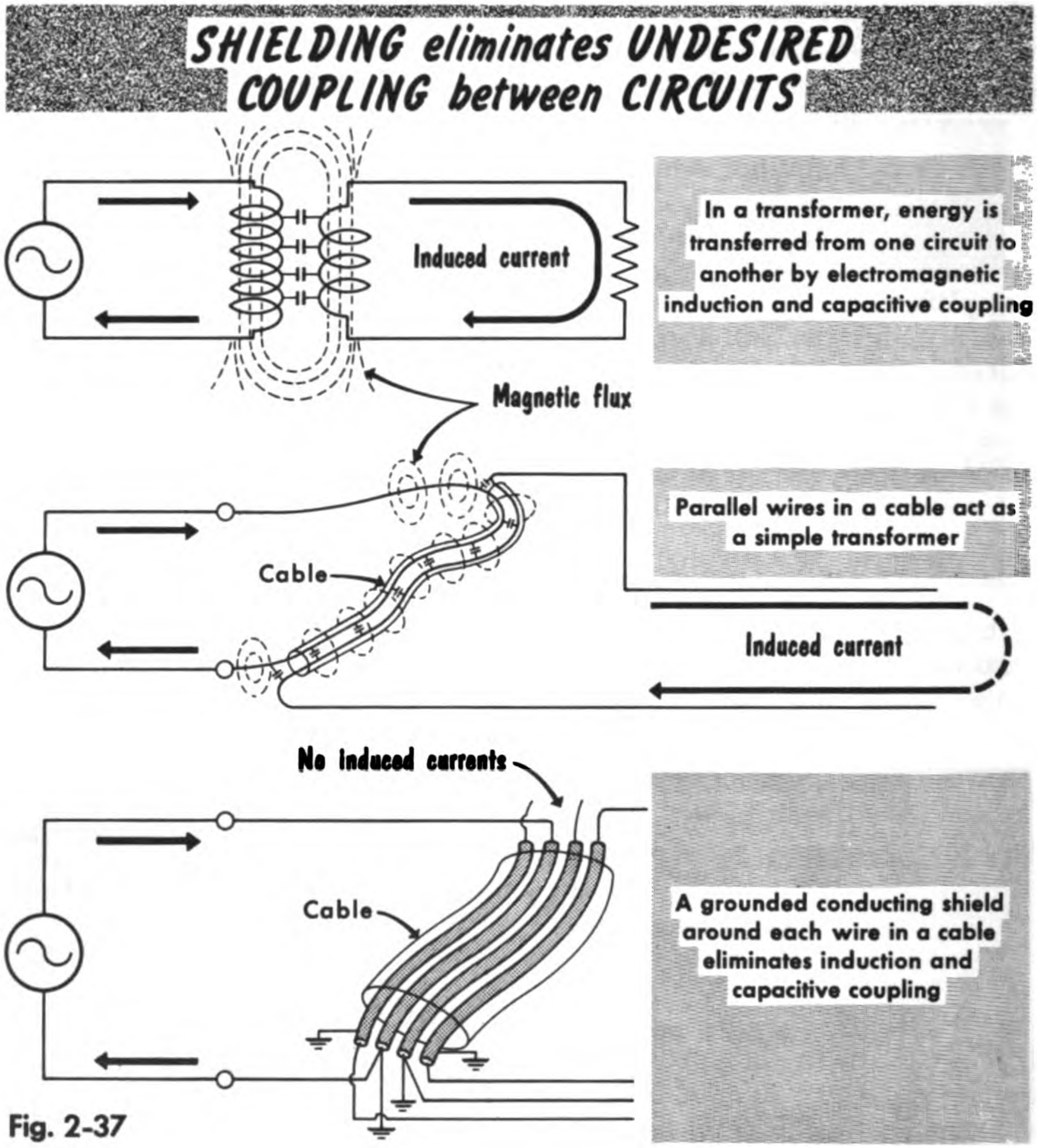


Fig. 2-37

puter. Of course, as with all large-size electronic equipment, the console requires cooling by forced-air circulation as the filaments of the tubes give out appreciable quantities of heat. To aid in this cooling it is usual for the amplifier chassis to be shaped in such a way that the air can pass freely over the tubes. A convenient arrangement is shown in (Fig. 2-36).

In contrast to the high-gain amplifiers it is essential for the precision input and feedback components to be maintained in a controlled environment. Their values would change in a fluctuating temperature, and their condition would deteriorate in the normal surroundings of an electronic circuit chassis. Therefore these components on which the precision of the computer voltages depend are located in a temperature-controlled, low humidity oven. Care is taken that their values are adjusted at the oven's operating tempera-

ture of about 110°F. This temperature is selected to be above room temperature so that it can be maintained by heating elements rather than cooling coils.

Shielding

When connected together it is essential that nothing interfere with the operation of the amplifier and its associated precision components. However, the high-gain amplifier is frequently far removed from the oven (see p. 2-91, Fig. 2-36). Connections must be made by long cables which are bound into harnesses with other cables performing similar tasks. When a current flows in a cable an electromagnetic field is set up around the cable and this field can induce into other cables small voltages which produce their own currents. In this way the current flowing in one cable can interfere with the currents flowing in another cable close by. *Crosstalk* is the common term given to this interference. It must be prevented in the case of the cables connected to the input grid of the high-gain amplifier, for it would seriously change the output voltage (Fig. 2-37).

Now an electric field is unable to penetrate a conducting sheet of metal. By surrounding the important grid cables individually with grounded conducting shields of wire mesh, all crosstalk due to electric fields can be eliminated from these cables. In practice, the cables have built into them the shielding required, and by grounding this to the chassis of the computer, effective electric shielding is achieved. For voltages to be induced by a magnetic field, the conductor must encircle the field. Thus to reduce magnetically-induced crosstalk the cables are made into harnesses without any turns or loops — all cables are maintained as straight as possible.

THE INTEGRATING AMPLIFIER

Capacitive Feedback

Now that all the details of the amplifier's circuitry have been presented, we can return to a consideration of the operational amplifier as a computing component. In the previous simple development of the relationship between the voltage output and the voltage input(s) to an operational amplifier we found that it depended only on the feedback resistance and the input resistance(s). The summing amplifier makes use of this fact to achieve the algebraic summation of a number of voltages. However, the original development which makes use of the well-established Kirchhoff's laws for electrical network analysis does not require that the input and feedback components be purely resistive. The output/input voltage relationship

$$\frac{e_o}{e_i} = -\frac{R_f}{R_i}$$

can be expanded to apply to reactive components, and this fact is of considerable value (Figs. 2-38A and 2-38B).

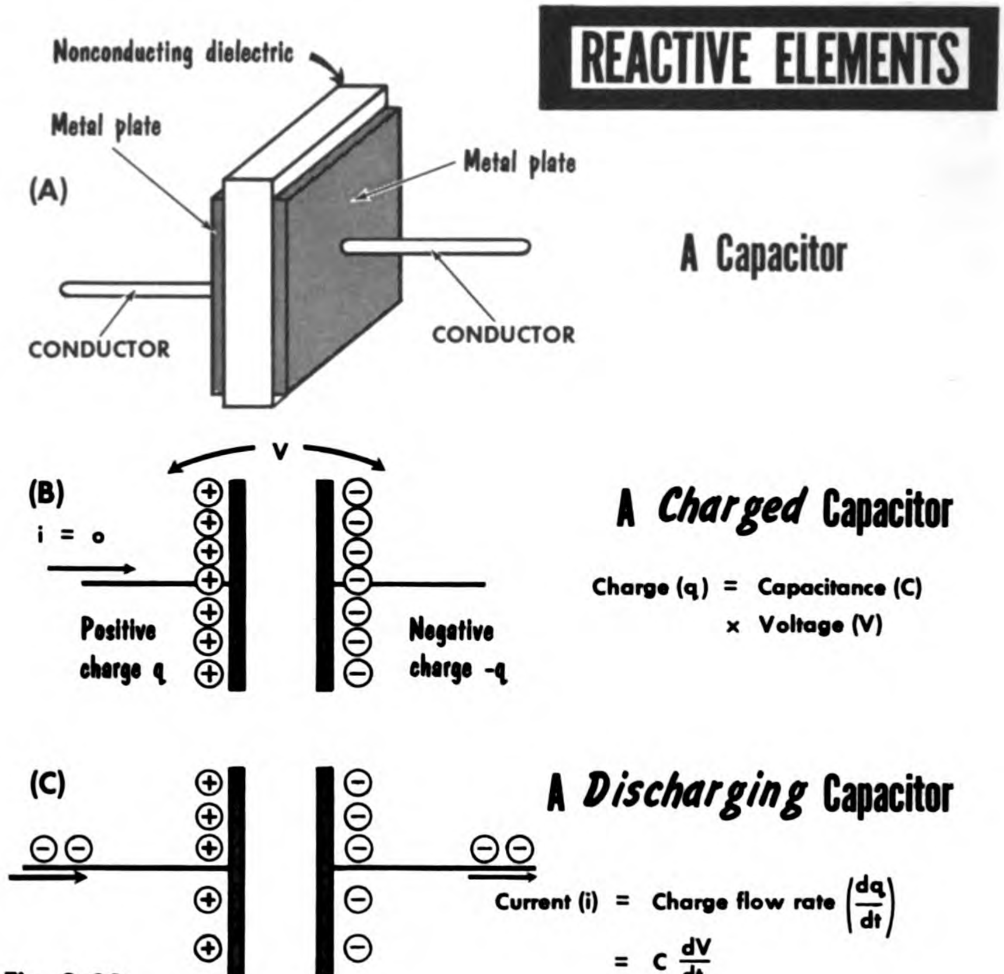


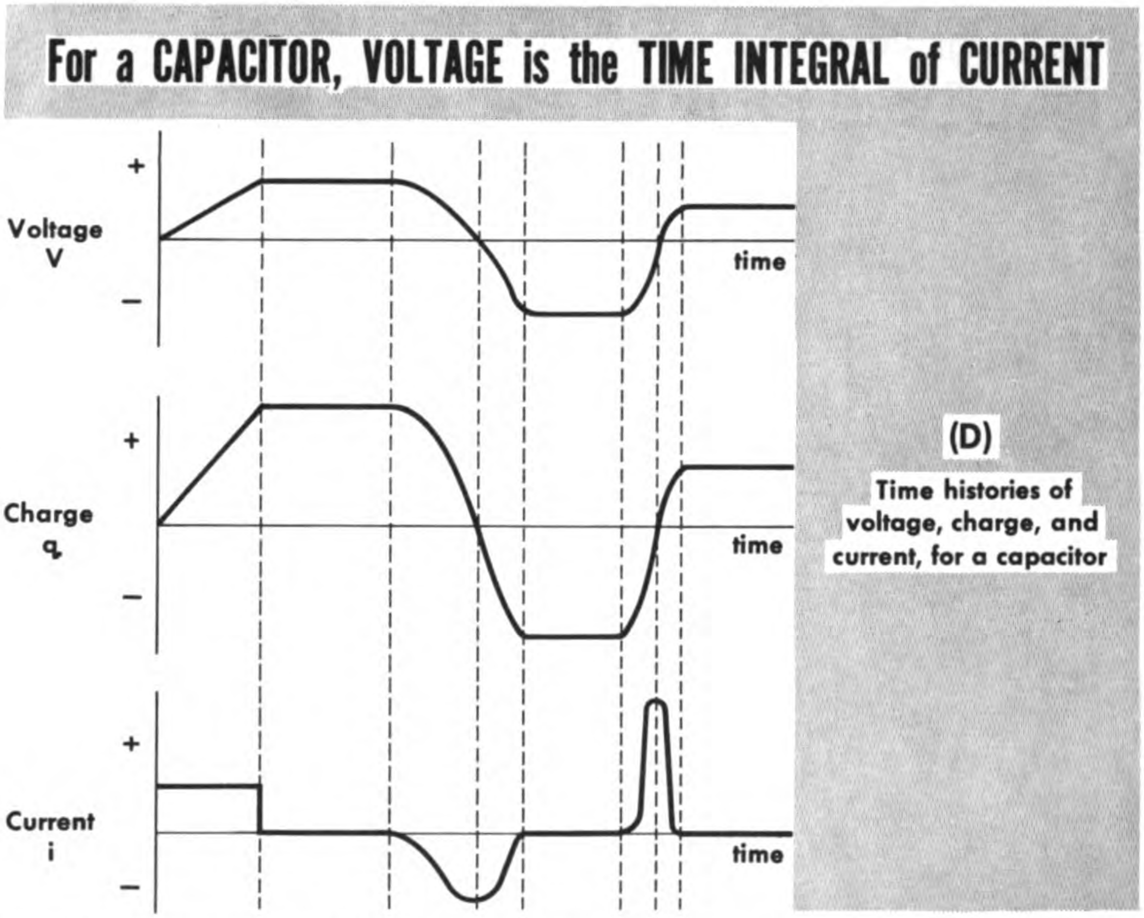
Fig. 2-38

Let us consider a simple example of the use of a reactive component in the feedback circuit of an operational amplifier. What will be the result of using a 1-μf capacitor as the feedback element in place of the 1-megohm resistor employed so far? Still the sum of the currents flowing through the input resistors is forced to flow through the feedback circuit. Thus the voltage difference between the grid of the amplifier and the output must be that required to maintain this current flow. A current will only flow through a capacitor if the voltage across it is changing, and then the current is equal to the capacitance multiplied by the rate of change of the voltage (Fig. 2-38C). For example, the current flowing through a 1-μf capacitor across which the voltage changes in a regular way from 10 to 20 volts in 5 seconds has a value of:

$$\frac{1}{1{,}000{,}000} \times \frac{20 - 10}{5} \text{ amps}$$

or 2 microamps.

Conversely, if the combined current flowing through the input resistors is a constant 2 microamps, the voltage difference across the capacitor will be



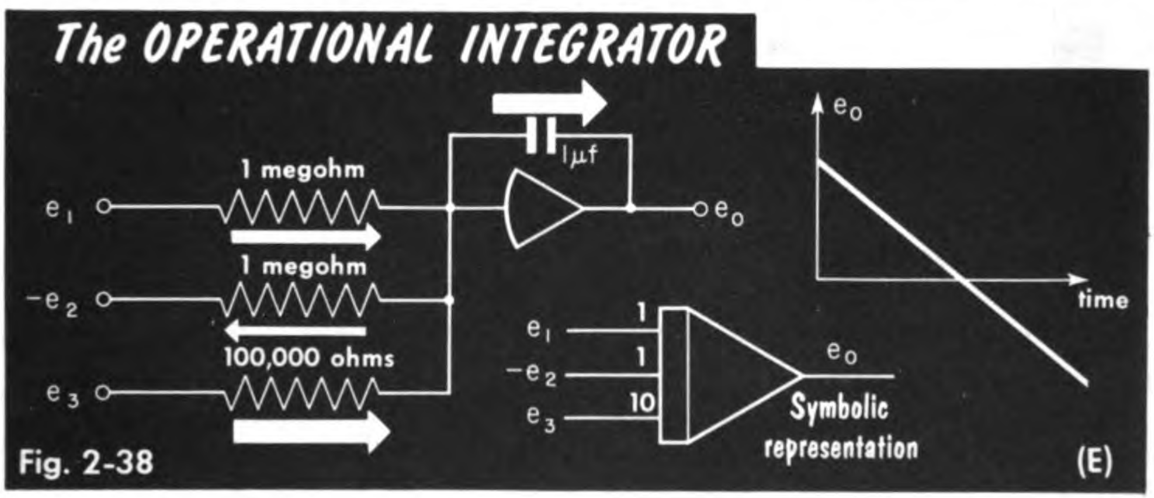


Fig. 2-38

changing at a rate of 2 volts per second. Then the voltage output of the amplifier will be changing at this same rate. At any instant the value of the output voltage will depend not on the present value of the input voltage(s) but on the accumulated (or integrated) total of past values and the periods of time for which those values existed. One can imagine that the sum of the input voltages multiplied by the period for which they exist is stored in

THE INTEGRATING CAPACITOR ACCUMULATES CHARGE
A method of discharging the capacitor must be available

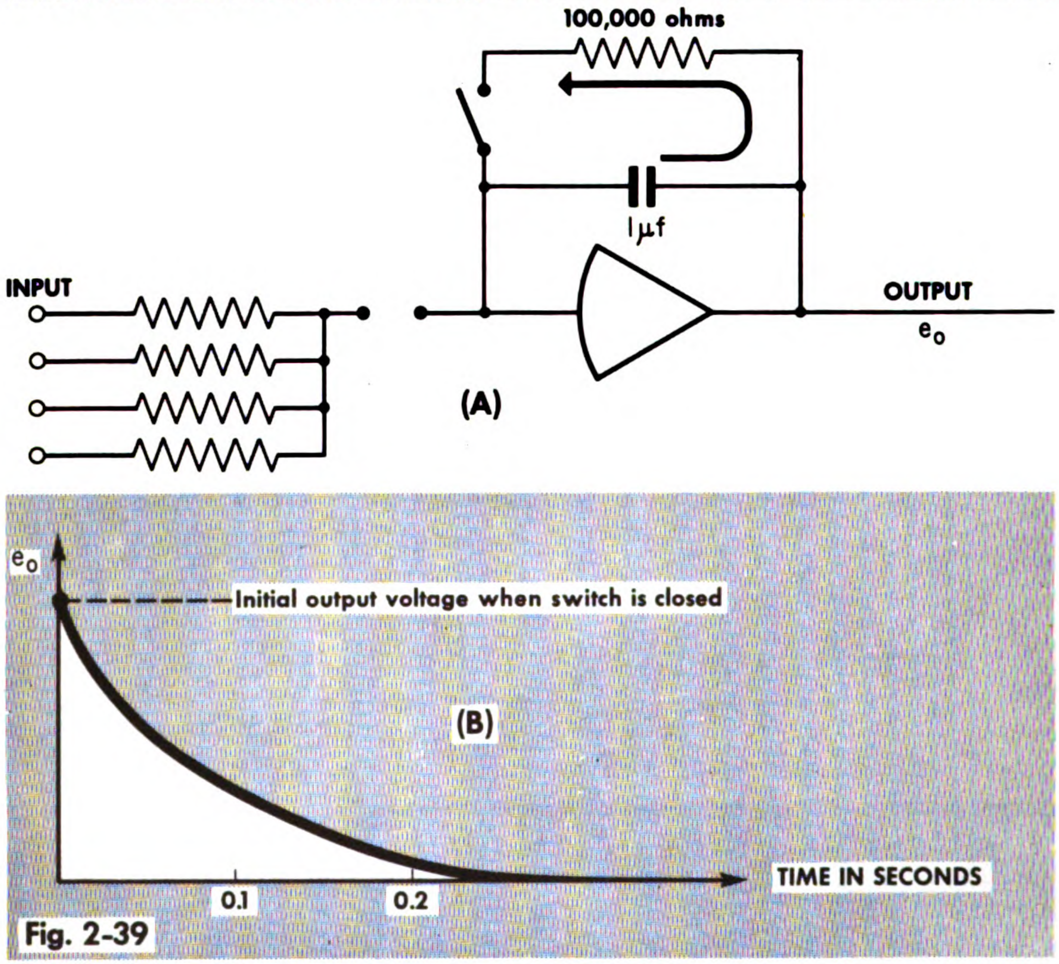


Fig. 2-39

the capacitor, and thus, as the grid of the amplifier is at zero the value of the integrated voltage appears at the amplifier output. Only when the sum of the input currents is zero will the output voltage remain steady.

This operation of voltage integration is of utmost importance in the use of the analog computer for investigating the behavior of physical systems, and it is the ease with which it can be performed by the use of a capacitor as the feedback component around an operational amplifier, that makes the computer so desirable (Figs. 2-38D and 2-38E). With a 1-μf capacitor and 1-megohm input resistors, the integrating amplifier effects the mathematical operation of integration with respect to time,

$$e_0 = -\frac{1}{C}\int_0^t \left[\frac{e_1}{R_1} + \frac{e_2}{R_2} + \ldots\right] dt$$

$$= -\int_0^t [e_1 + e_2 + \ldots] \, dt$$

INTEGRATING DEVICES have INITIAL CONDITIONS

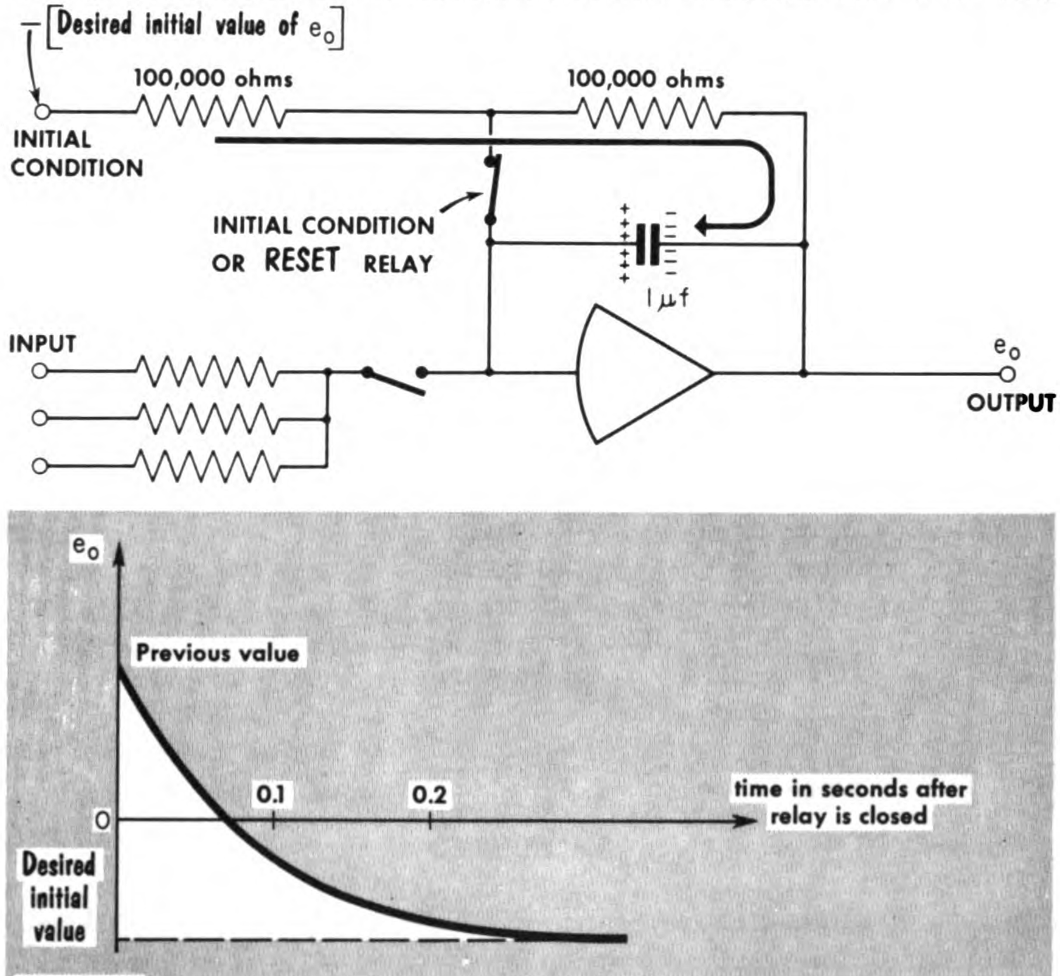


Fig. 2-40

The presence of 100,000-ohm input resistors simply multiplies the effect of the voltages applied to them by 10.

Removing the Integrated Voltage

As the output voltage from the integrating amplifier depends on the past history and not the instantaneous value of the input voltage(s), a means is required to destroy the integrated voltage, so that a computer circuit can be returned to a zero condition between consecutive problem solutions (Fig. 2-39). Destroying the integrated voltage means removing the accumulated charge by discharging the capacitor. This can be done quite easily by connecting across the capacitor a suitable resistor through which the accumulated charge may flow. The value of the resistor must be such that the time taken for the exponential decay of the charge is short, and it is usual to use a 100,000-ohm resistor for this purpose. The time constant for the decay process is then one-tenth second, and within one second very little of the charge would remain.

It is of course necessary to disconnect the input voltages to the amplifier as the capacitor is discharged, for if this is not done the amplifier acts in part as a simple summing amplifier with a feedback resistor of 100,000 ohms. Under these conditions the output voltage and thus the voltage across the capacitor is brought to a value one tenth that of a summing amplifier, rather

The INTEGRATORS CAUSE the ANALOG COMPUTER MODEL to PERFORM. To halt the performance they must be halted

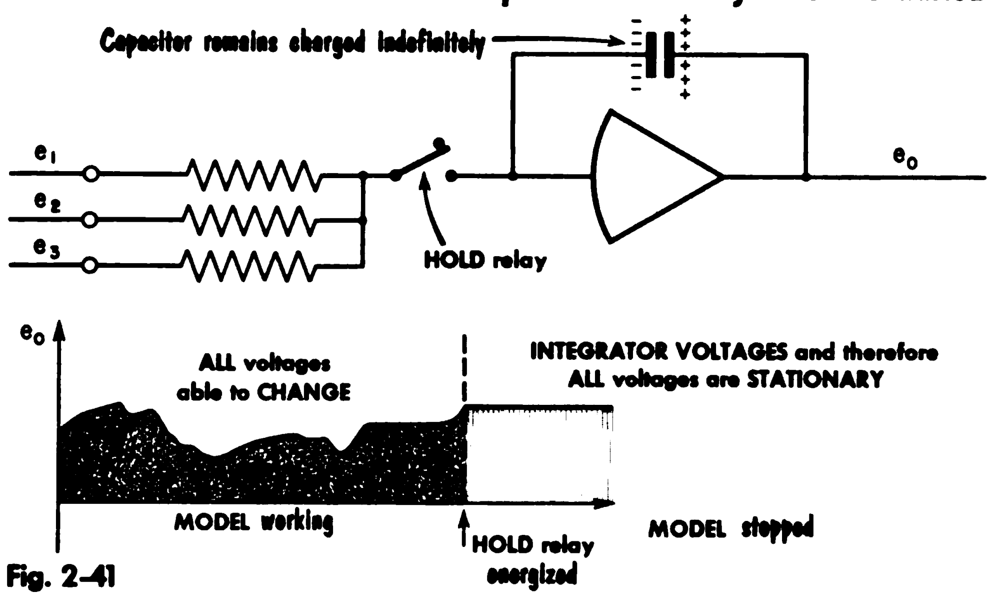


Fig. 2-41

than to zero. It is most convenient to disconnect the inputs from the amplifier by breaking the grid connection of the resistors. Then although the input voltages are still present they will not act on the amplifier circuit.

Applying Initial Values to the Integrating Amplifier

Frequently, the value of the integral of a number of variables is not zero at the beginning of a study. Probably the simplest example of this fact is the reading of the odometer in your car whenever you set out from home. The odometer integrates the speed of your car, and you might consider each journey as a separate study in which the mileage you travel is an interesting variable. Then although with a new car, the mileage begins at zero on the first study, no other study has a zero mileage to begin it. In this case, an analog computer circuit, producing a voltage proportional to mileage by integrating another voltage proportional to speed, would need a method to set the output of the integrating amplifier to the appropriate voltage for each day's initial mileage.

Just as we can *discharge* a capacitor by using a resistor, we can *charge* the capacitor using a resistor (Fig. 2-40). The circuit shown here accomplishes the charging operation. Before obtaining a problem solution from the computer the resistors are disconnected from the capacitor leaving it charged

to the required initial value, and thus ensuring that the voltage at the integrator output has the required *initial condition*.

Halting a Problem Solution

The components of an analog computer are interconnected so that required operations are performed on the voltage variables, and their behavior with time corresponds with that of the variables in the corresponding physical system. Of all the mathematical operations performed, only that of the integrating amplifier *produces* a change with time. All the other components have output voltages which for steady values of input voltages, are steady. With the integrating amplifier the output changes with time for steady input voltages. Thus the dynamic, changing behavior of the variables in a problem depend for their initiation on an integrating amplifier. The integrating amplifiers give life to the solution.

Frequently, one wishes to halt a problem solution for a while to interpret what is happening and then to continue it from the point at which it halted (Fig. 2-41). Obviously such a halting of the solution can only involve the integrating amplifiers, and by disconnecting the inputs to all such units their outputs will remain steady and unchanging, and thus all voltages throughout the computer will remain steady. After making the observations and interpretations the problem solution can be allowed to continue as though no interruption had occurred.

QUESTIONS

1. Write down the fundamental operations found in the mathematical description of a physical system, and give examples of each.
2. Describe why the mechanical adjustment of a potentiometer is insufficient for its accurate use in an analog computer. How do we overcome this difficulty?
3. Draw the graph of output voltage against wiper position for a 50,000-ohm attenuator supplied with a constant 100 volts and connected to a load of 100,000 ohms to ground. Determine the maximum percentage loading error that could exist in the use of such an attenuator.
4. What do you understand by "negative feedback"? Describe the advantages gained by its use in the operational amplifier.
5. Describe in detail the operation of a summing amplifier. What factors determine the accuracy of the summation?
6. What is drift? How is an R-C coupled amplifier used to overcome the difficulties created by drift in a d-c amplifier?
7. Why does one need an overload alarm in an operational amplifier? Describe the operation of a typical alarm circuit.
8. What do you understand by the term integration? Describe how voltages are integrated with respect to time on an analog computer.
9. Suggest why one would wish to halt a problem solution being obtained on an analog computer. How is the solution halted?

Chapter 3

D-C ANALOG COMPUTER: MULTIPLYING COMPONENTS

Multiplying Devices

Frequently the mathematical description of the behavior of a physical system includes the products of variable quantities. If the behavior of such a system is to be investigated on a computer then it will be necessary to multiply together the two voltages representing the factors in a product. One example of a physical system where the multiplication of voltages actually occurs is the common radio receiver (Fig. 3-1). There the process of mixing the received signal with a locally generated signal to produce the i-f signal is equivalent to multiplication. A number of circuits have been devised to achieve this operation, but none are suitable for the precise multiplication at relatively low frequencies required in an analog computer. They use mixing tubes and filter circuits that distort the true product in a way suitable for the frequency-selective circuits to follow, but this distortion is completely unacceptable on a computer.

The requirement on an analog computer multiplier is that it must produce (without distortion or delay) a voltage that is directly proportional to the product of two input voltages. Many ideas for producing a product were discussed previously. A number of devices are available commercially, and their nature permits them to be divided into two groups: electromechanical multipliers and electronic multipliers. We will describe the operation of a number of these components.

THE SERVOMULTIPLIER

The Electromechanical Multiplier

We saw earlier, that any voltage can be effectively multiplied by a positive constant whose magnitude is less than unity, by using a potentiometer. The voltage is applied across the potentiometer and the wiper is positioned so

Multiplying Devices

A commonplace example of the multiplication of physical variables

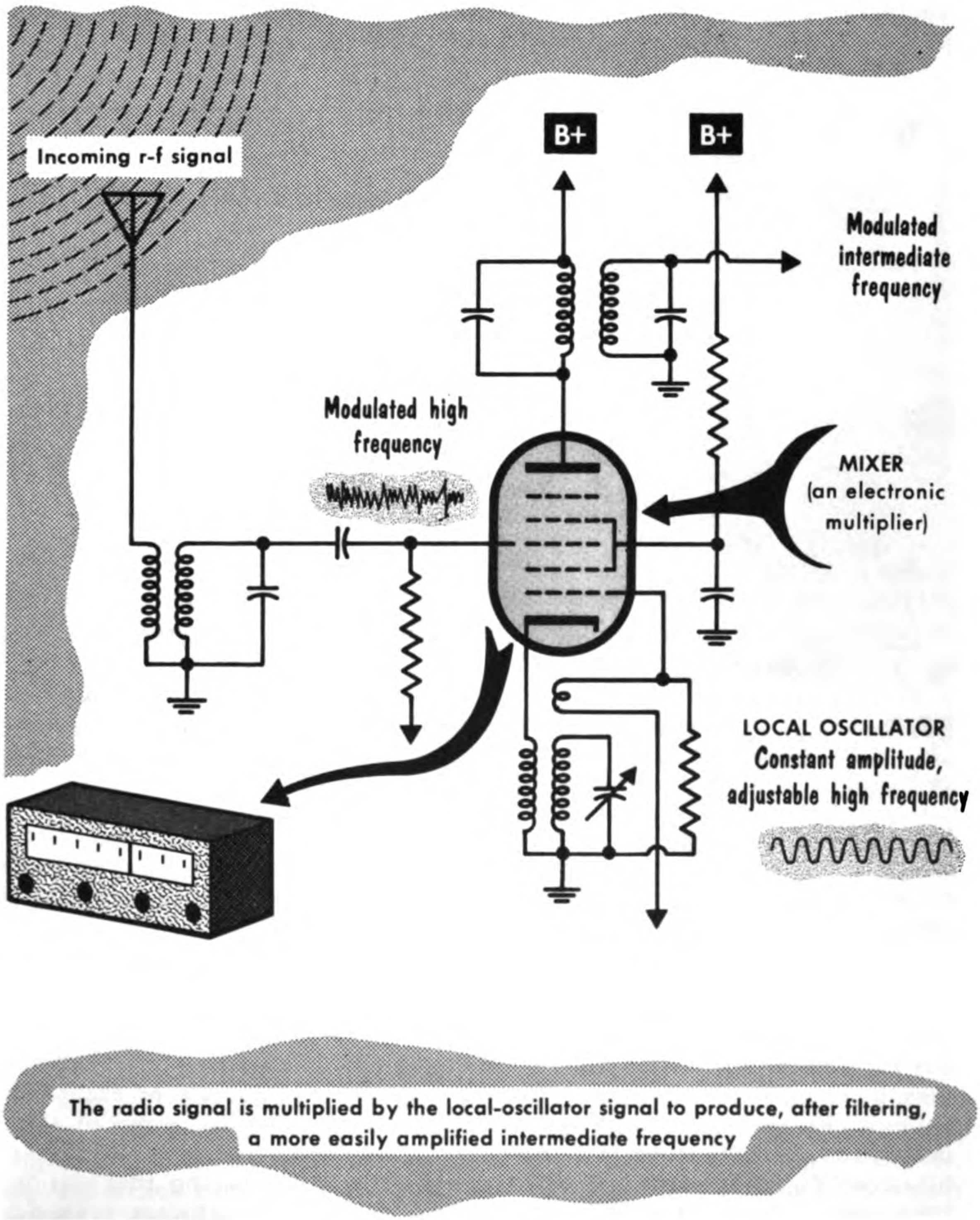


Fig. 3-1

that the desired multiplication is achieved. Now with this same simple device we can imagine multiplication of two variables, one, the voltage applied to the potentiometer; the other, the position of the potentiometer wiper. Each could be changing with time and the output voltage would depend on the instantaneous values of both. It would be directly proportional to the applied voltage and, if we overlook the loading error for the moment, it would also be proportional to the mechanical position of the wiper. Thus we could achieve the required product of two voltage variables, by a scheme

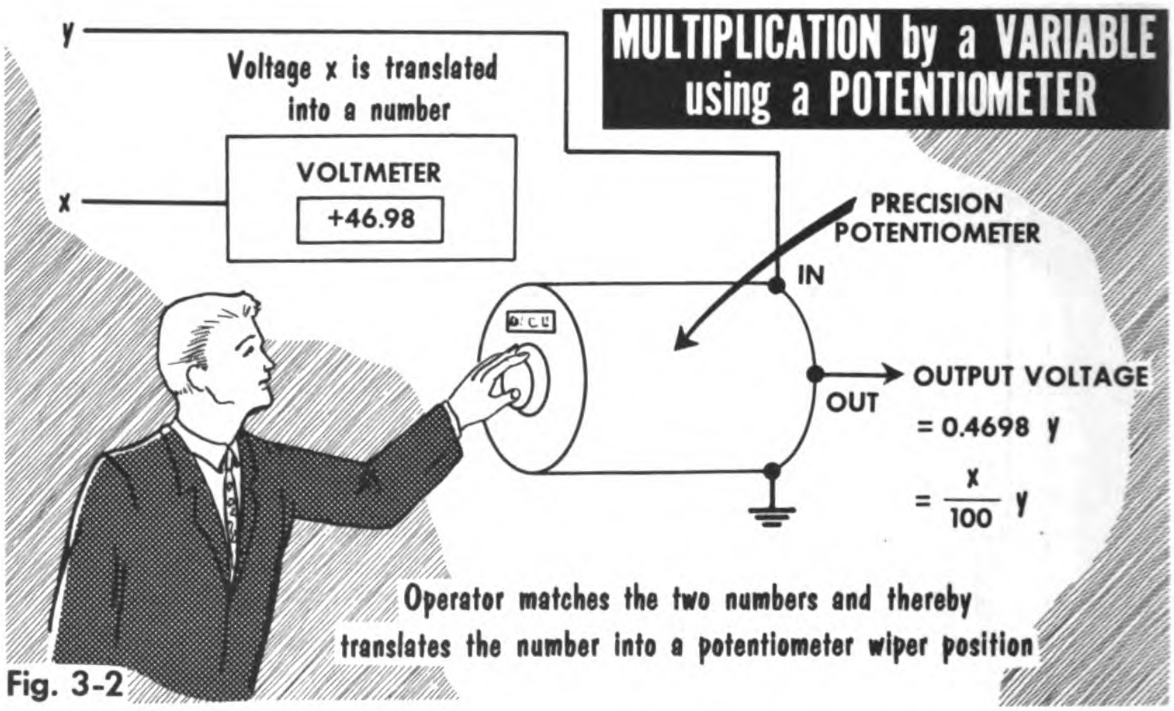


Fig. 3-2

like that shown in the figure (Fig. 3-2). It would not be too efficient, and would be rather fatiguing on the operator, but it demonstrates the scheme of a suitable electromechanical unit.

Note that one of the variable quantities is translated from a voltage to a mechanical position; also, that the value of the voltage product can never exceed the value of either of the applied voltages, for the potentiometer always acts as an attenuator.

Positioning Automatically

The electromechanical scheme which we have demonstrated above would not succeed in practice, of course, for the human operator is not able to follow the changes in value of the second input voltage quickly or accurately enough. Furthermore, one requires in a modern computer many of these multipliers, and the labor cost of employing human operators would be prohibitive (Fig. 3-3). However, it is worthwhile remembering that not too long ago — during the World War II — equivalent schemes involving human-links were almost the rule, rather than the exception, in aircraft-

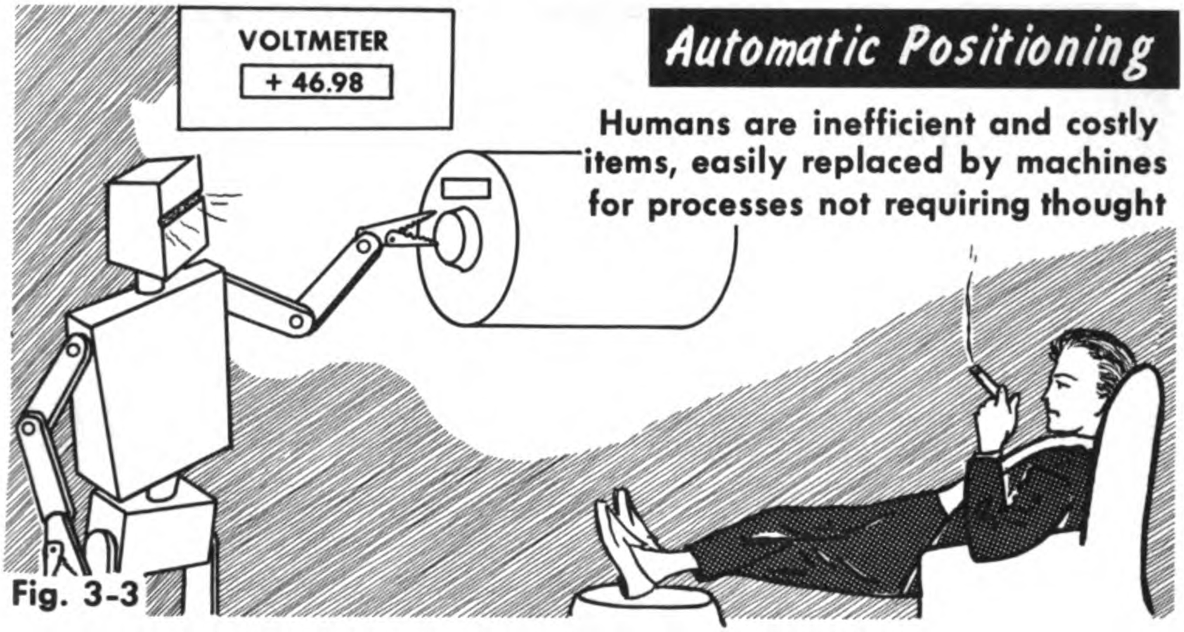


Fig. 3-3

tracking devices, etc. Now, by the use of automatic positioning systems we are able to eliminate the expensive and inadequate human operator. The wiper of the potentiometer can be positioned without difficulty to correspond to the second input voltage.

Open-Loop Position Control

One scheme that might be used to position the potentiometer wiper automatically, simply replaces the operator by a calibrated electromechanical following system. To each value of the input voltage there would correspond a setting of the potentiometer wiper, and using some form of electromechanical transducer, probably similar to the movement of a direct-voltage meter, the wiper would be continually adjusted to follow the voltage (Fig. 3-4). Such a mechanization has several failings that are equivalent to those of an open-loop d-c amplifier which one might wish to have used in place of the operational amplifier discussed previously. Its accuracy is very sensitive to the calibration; its operation is susceptible to component variation, power supplies, drift, etc. In fact, the system would not be good enough for our purpose, and we must find a better method for positioning the potentiometer wiper. Once again, as with the amplifier, we make use of negative feedback and build a closed-loop position control.

Closed-Loop Position Control

We saw previously that many, if not all the difficulties associated with the design of an accurate, reliable computing amplifier were overcome by feeding back the output variable so that it subtracted from the input variable, the difference being used to drive the amplifier. Using a very high-gain amplifier, we traded quantity — in the form of available gain — for quality of the output variable. The same technique can be used here to obtain pre-

OPEN-LOOP POSITION CONTROL is INACCURATE DUE TO....

1. Changing load forces
2. Nonlinear component characteristics
3. Calibration changes
4. Component dead-zone

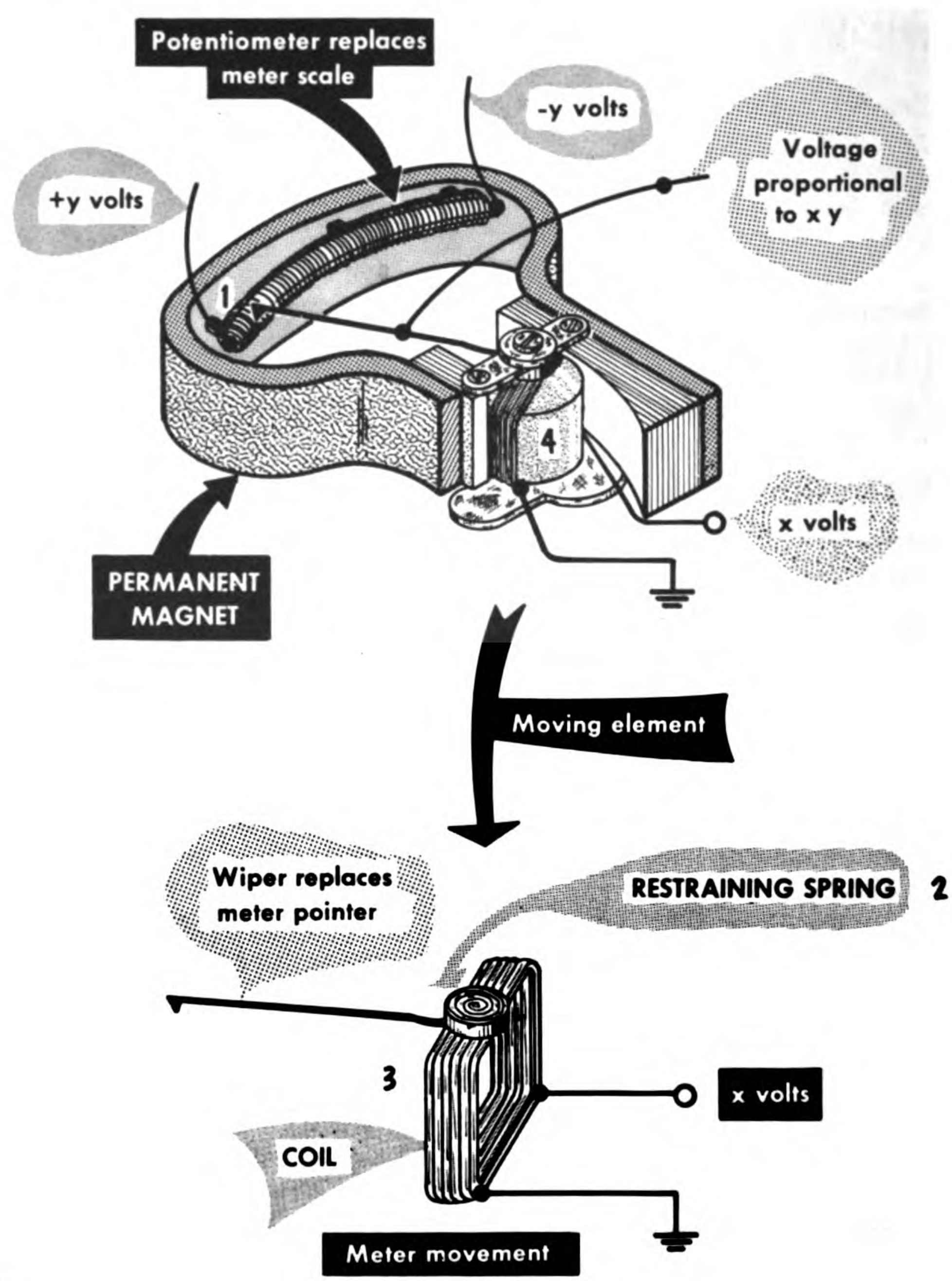


Fig. 3-4

cise, reliable positioning of the wiper of the multiplying potentiometer (Fig. 3-5).

The output quantity in this device is a mechanical position and this must be translated back to a voltage before the negative feedback can be effected. Whenever two quantities are compared or added in an analog computer they *must be of the same nature.* We cannot add together apples and oranges, just apples *or* oranges. To translate the mechanical wiper position

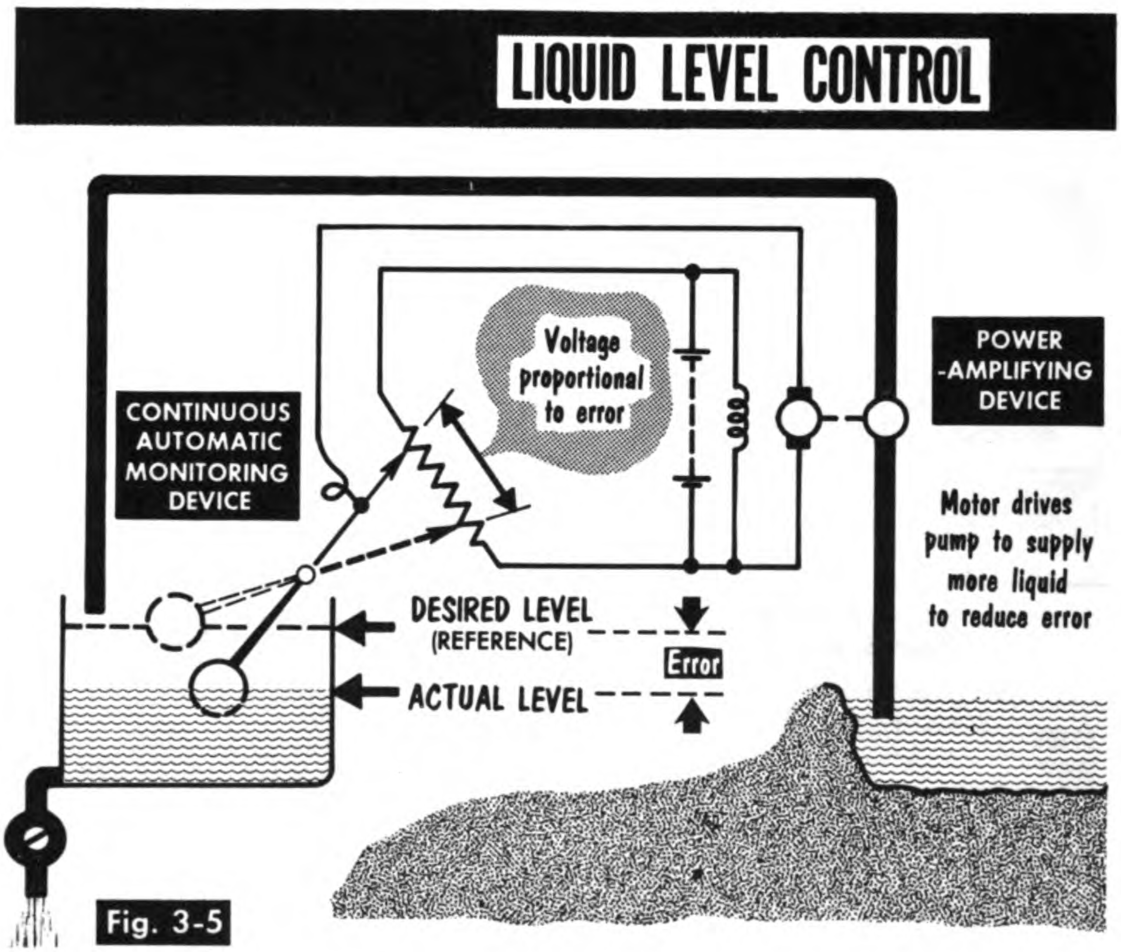


Fig. 3-5

to an equivalent electrical voltage we use a second potentiometer, known as the *follow-up potentiometer.* The wiper is ganged to that of the multiplying potentiometer. Supplied with a constant 100-volts reference voltage the follow-up potentiometer gives a voltage proportional to the mechanical position. With a high gain in the transducer, precise correspondence between input voltage and output position is obtained.

The Position Servomechanism

The follow-up positioning device used in the electromechanical multiplier is one of a large class of position servomechanisms. Making use of the principles of negative feedback they permit an accuracy of operation not other-

AUTOMATIC TENSION CONTROL OF STEEL STRIP

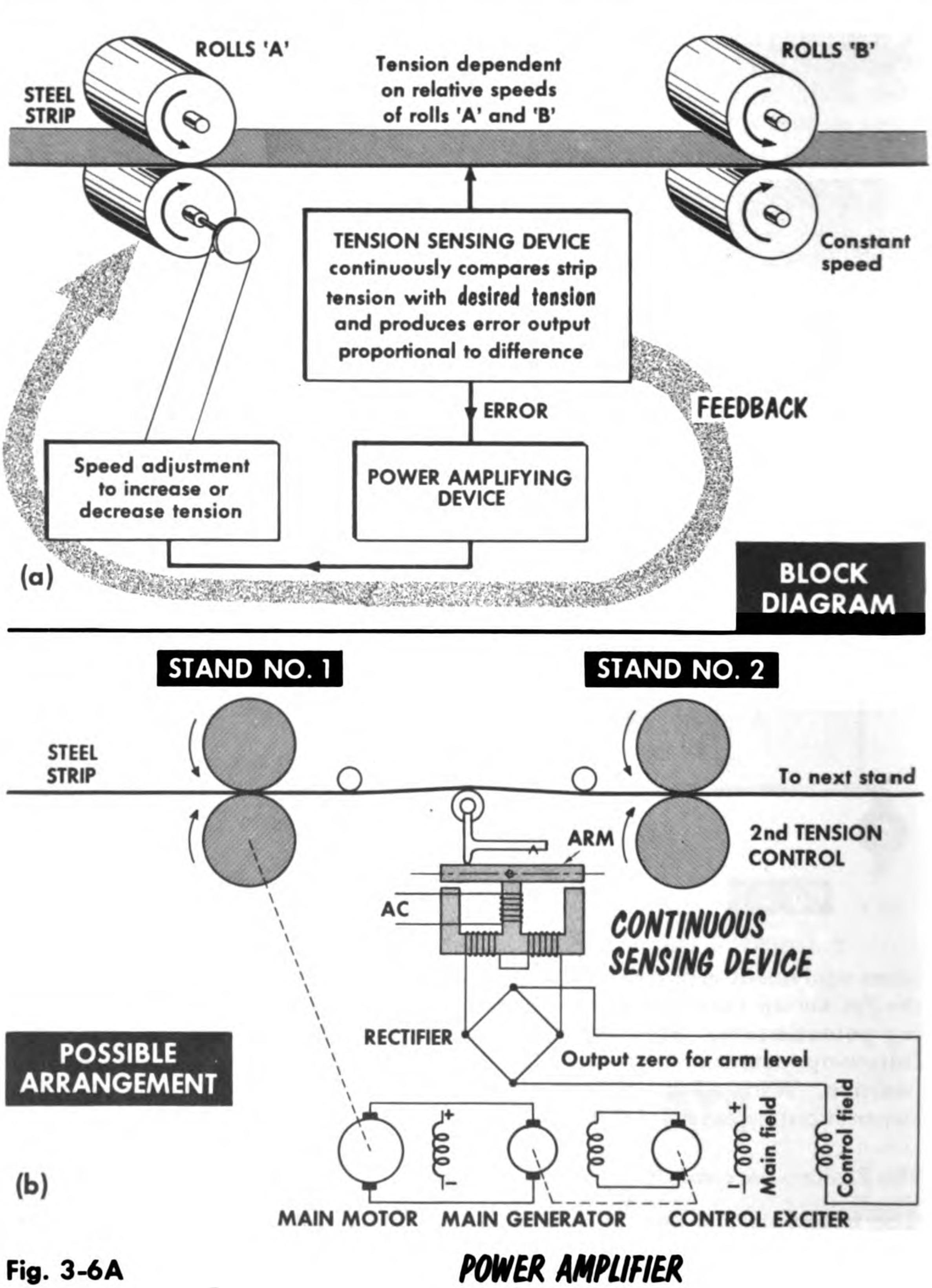


Fig. 3-6A

wise possible. In such a device one might consider that the output quantity being determined — whether it is a mechanical position or an electrical voltage — is continually compared with the value desired, represented by the input driving voltage, and any error is immediately used to change the output quantity to exact correspondence (Fig. 3-6A). The electromechanical transducer or controller amplifies any error between the input and output

The ELEMENTS of the SERVO MULTIPLIER CONTROL SYSTEM

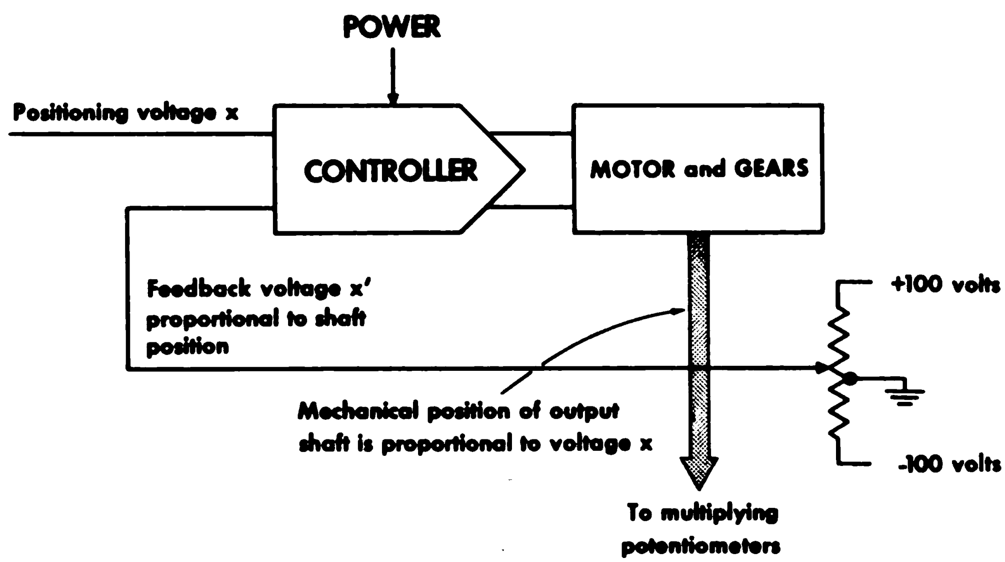


Fig. 3-6B The power applied through the amplifier, and the high gain present, produce correspondence between voltage x and the shaft position

quantities, and the amplified signal is applied to the output driving mechanism, usually an electric motor. The motor rotates in the direction required to change the output quantity so that the error between it and the input quantity will be reduced.

How closely the wiper position of the electromechanical multiplier follows the input-driving voltage depends directly on the gain of the controller amplifier (Fig. 3-6B). However, if this gain is too high, unsatisfactory operation occurs due to unstable oscillations being generated in the system. Thus a compromise must be made between high gain plus accurate following but the possibility of oscillation, and low gain plus poor following but with no oscillation. A satisfactory compromise is possible with the electromechanical multiplier.

The Comparator

One very simple and efficient method of comparing two voltages which are expected to be equal, uses a potential divider. If one voltage, e_1, is applied

A SIMPLE but EFFECTIVE COMPARATOR for DIRECT VOLTAGES

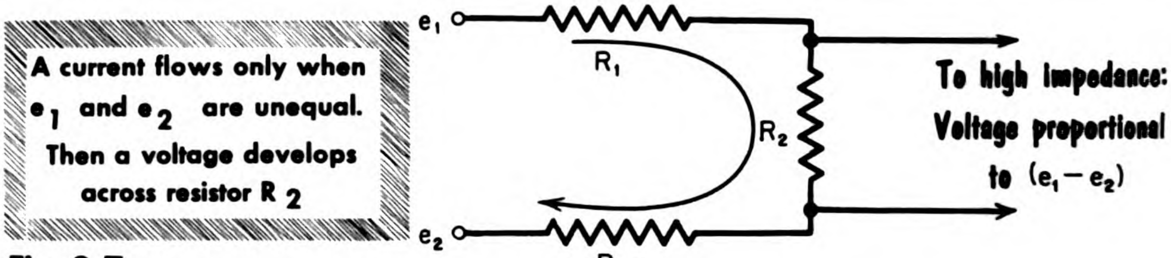


Fig. 3-7

to one end of three resistors in series, and e_2 to the other end, then a current flows through the resistors whenever e_2 is not equal to e_1 (Fig. 3-7). Thus there will be a voltage developed across the middle resistor whenever the two voltages are not equal, and this voltage will be proportional to the difference, $e_1 - e_2$. If e_1 is greater than e_2, the difference voltage will be positive; if e_1 is smaller than e_2, the difference voltage will be negative. Of course, if $e_1 = e_2$ the voltage will be zero. Thus the resistor chain is suitable for producing an error voltage in a position servomechanism.

Alternating or Direct Voltages for Low-Power Servomechanisms?

We learned earlier that direct-coupled amplifiers suffer from the unfortu-

AC or DC for Low-Power Servomechanisms?

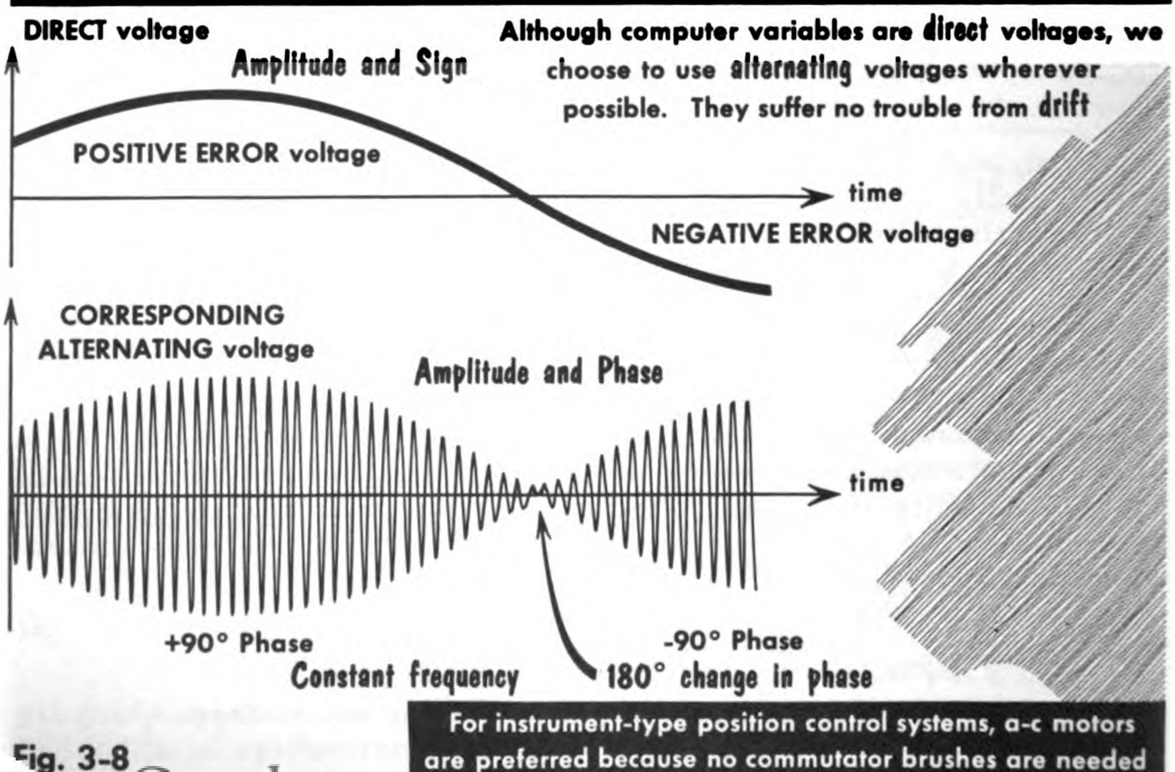


Fig. 3-8

nate malady of drift, and therefore whenever possible we avoid their use, preferring the simpler and more reliable a-c amplifier. In an instrument-type positioning system there is a second reason for choosing to use alternating voltages rather than direct voltages. It is the advantage of the brushless induction motor over the low-power d-c motor, for at these small powers — normally a few watts — the work done by a d-c motor against the brush frictional forces is frequently comparable with the usable output, making it an inefficient device. Having no slip-ring or commutator brushes the a-c induction motor is much more desirable.

Thus the first action taken with the error voltage is to modulate a suitable carrier frequency to be able to treat it as an alternating voltage. As shown in Fig. 3-8, to the direct voltage there is a corresponding alternating voltage. The amplitude and sign of the direct error voltage are given by the amplitude and phase of the alternating voltage. If the phase of the alternating voltage is 90° ahead of the 115-volt, 60-cps line voltage, then it results from a positive servo error. If it is 90° behind the line voltage, it results from a negative servo error.

The Modulator

Modulation of the error voltage can be achieved quite simply and effectively with an electromechanical vibrator. Just as in the drift compensator of the operational amplifier, a metal reed is made to vibrate at a convenient frequency, in this case at 60 cps (Fig. 3-9). The reed alternately connects the high and the low end of the primary of a transformer, to one end of the re-

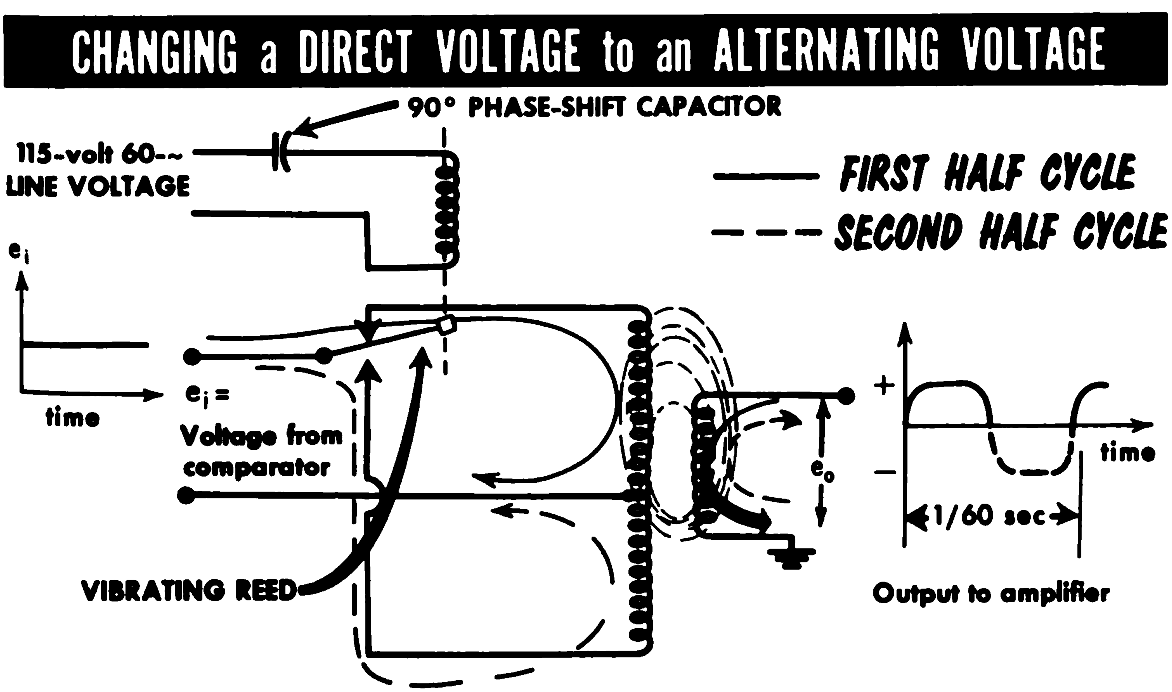


Fig. 3-9 The comparator voltage is placed alternately across each half of the transformer primary - an alternating secondary voltage results

sistor across which the error voltage is developed. The other end of the resistor is connected to the centertap of the primary winding. In this manner the error voltage is alternately placed across one or the other half of the primary winding so that current flows for one half cycle in one direction, and for the other half cycle in the opposite direction.

The secondary winding of the transformer is hardly conscious of how the effects are achieved in the primary. It simply is excited by an alternating 60-cps square wave whose amplitude is dependent on the amplitude of the error voltage, and whose relative phase angle depends on the sign of the error voltage.

Note that the voltage supply to the coil that moves the reed passes through a series capacitor. This effects the 90° phase change required between the error voltage and the line voltage supplied to the reference winding of the two-phase motor used by the servo. The relative phases of the voltages supplied to the two windings of the motor determine its direction of rotation.

The R-C Coupled Amplifier

The amplification of the alternating error voltage is carried out by a conventional carrier-type amplifier. Though possessing a much lower fre-

CONVENTIONAL ALTERNATING-VOLTAGE AMPLIFIER

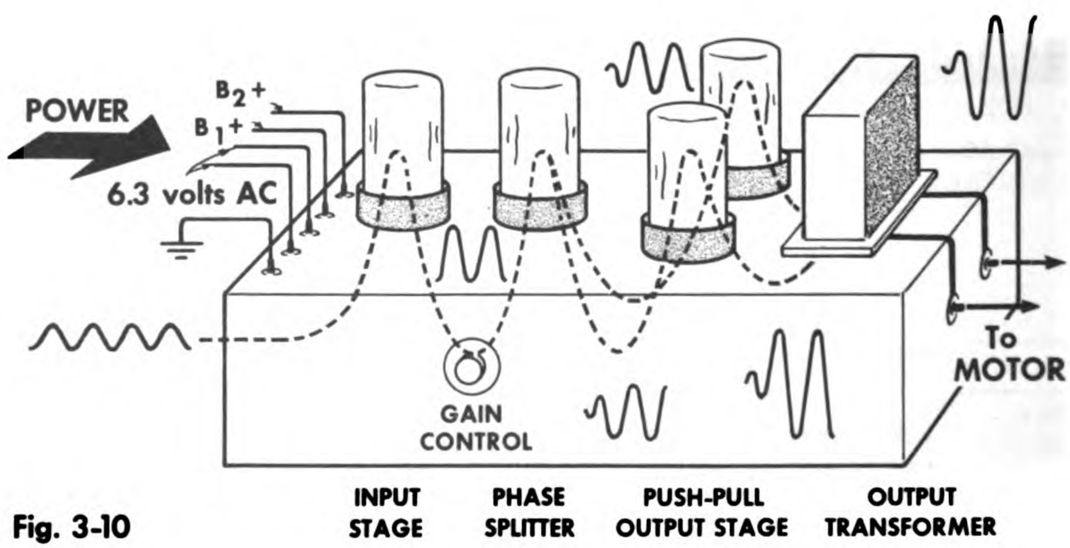


Fig. 3-10

quency response, it can be compared with the audio amplifier in any commercial radio. The output stage is a push-pull circuit using power tetrodes or pentodes, and a matching transformer is required to ensure good power transfer to the control winding of the two-phase motor (Fig. 3-10).

The amplifier has a simple gain control for adjustment of the servomechanism performance, and a fuse to protect the output tube against prolonged overload which might occur due to misuse of the servomultiplier.

The Two-Phase Servo Motor

The output from the a-c amplifier is used to control the operation of a two-phase servo motor. This motor has certain characteristics that make its use

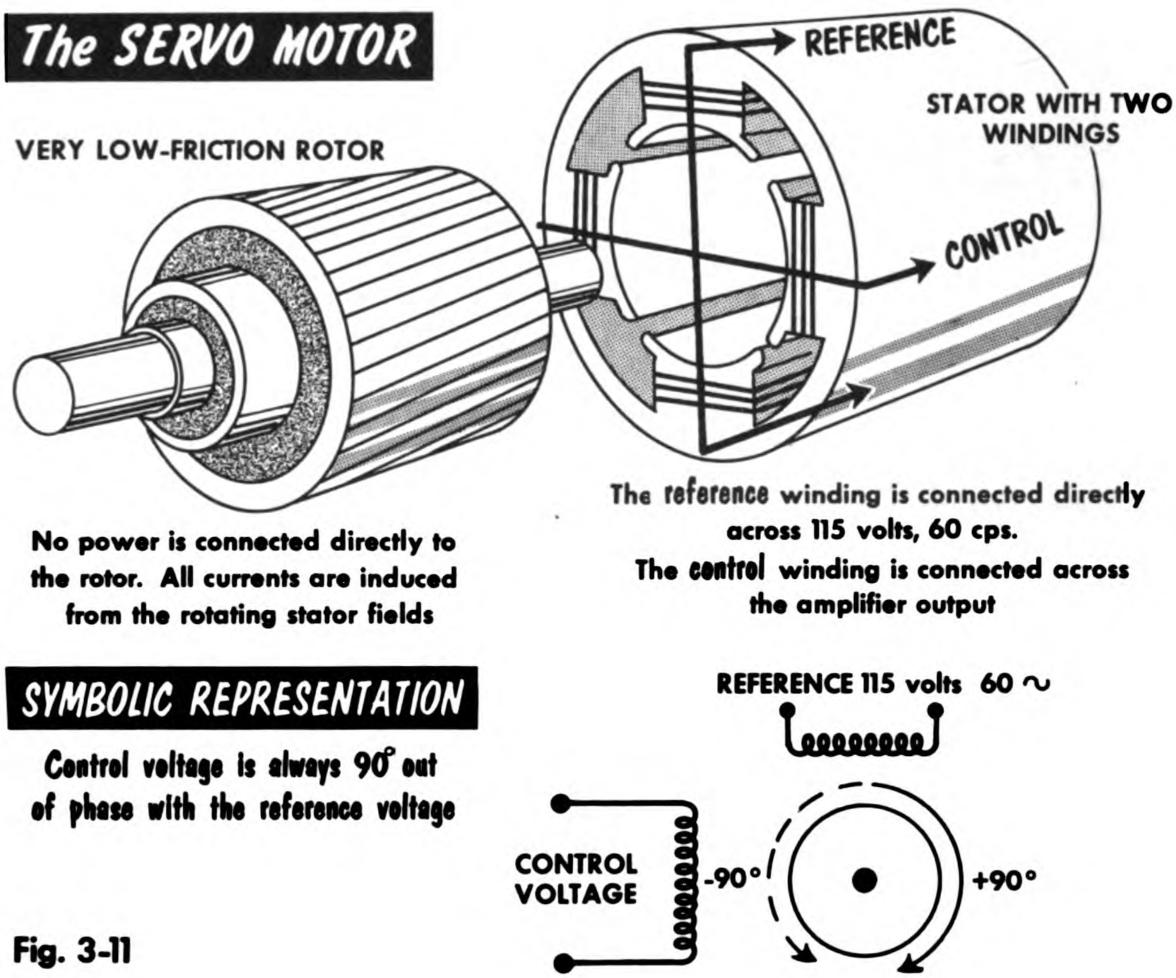


Fig. 3-11

in a position servomechanism very desirable. As its name implies, it contains two windings (or phases) — a reference winding and a control winding. The reference winding is permanently supplied with 115 volts, 60 cps, from the primary power supply. The control winding is energized by the a-c amplifier, and thus its voltage depends on the value of the servo error voltage (Fig. 3-11). The two-phase servo motor rotates whenever there is a voltage across the control winding, the speed of rotation depending on the value of this voltage. The direction of rotation, clockwise or anticlockwise, is determined by the relative phase angle between the voltages applied to the control winding and the reference winding. If this angle is −90° the rotation is in one direction; if it is +90°, the rotation is in the opposite direc-

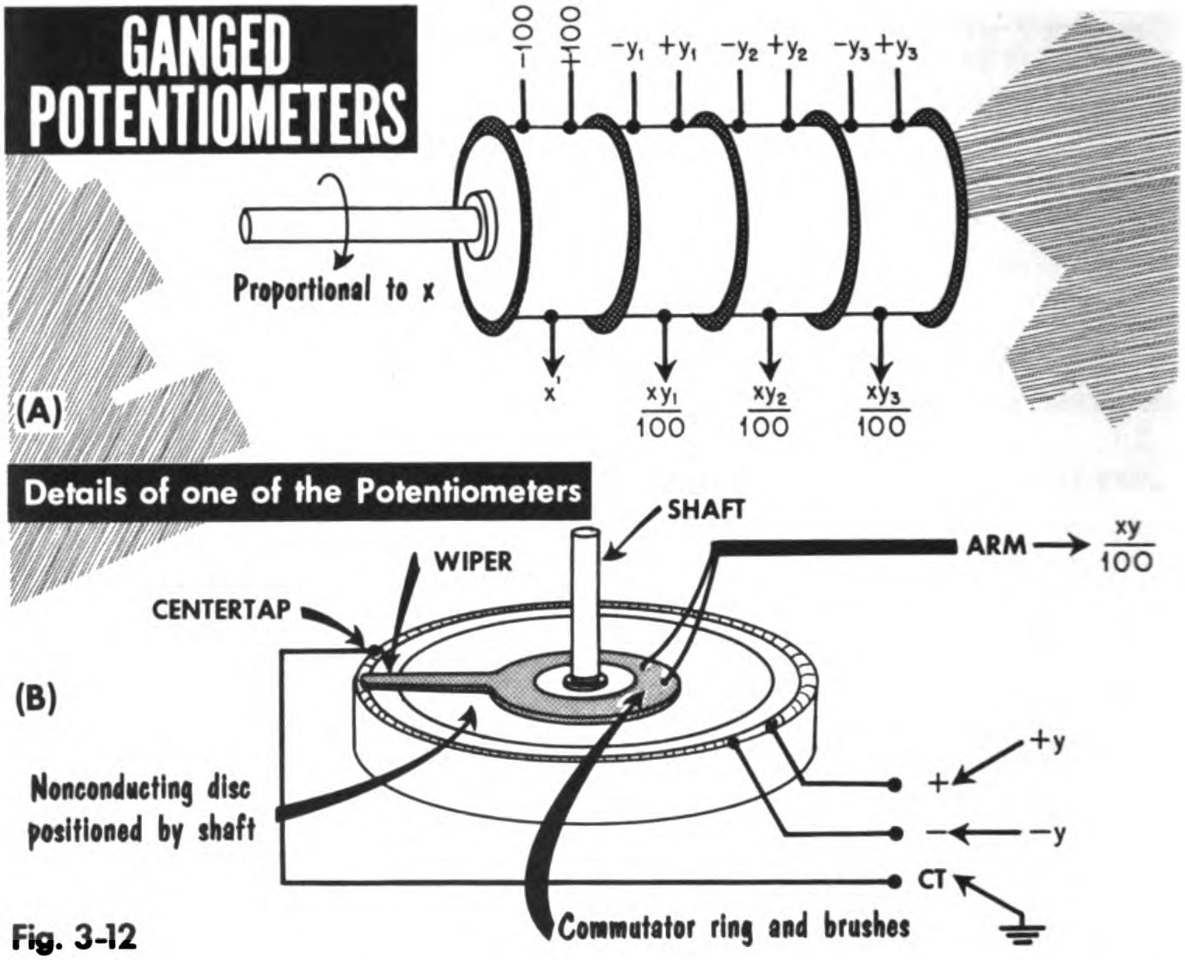


Fig. 3-12

tion. Thus, when used in a position servomechanism, the motor's direction of rotation depends on the sign of the servo error voltage, and is such as to drive the follow-up potentiometer wiper to a position where the fed-back voltage equals the servo input voltage. In this position the error voltage is zero, the servo is balanced, and the motor stops turning — a null is achieved.

The Potentiometer Assembly

The purpose of the servomechanism is to position the wiper of a multiplying potentiometer so that it corresponds proportionately to one of the voltages to be multiplied. In practice, it is usual to gang together a number of potentiometers so that more than one product can be formed, each product, however, having a common multiplier represented by the voltage applied to the servo [Fig. 3-12, (A) and (B)]. One of the ganged potentiometers is used as the follow-up potentiometer, and from this potentiometer a voltage is obtained to feed back to and compare with the input-positioning voltage. The follow-up potentiometer is supplied with constant +100-volt and −100-volt reference voltages, this being the range over which the input voltage is expected to vary and thus to which there must correspond potentiometer assembly wiper positions.

For protection against serious current overloads that would destroy the very fine wirewound potentiometers, each wiper connection is in series with a fuse.

Multiplying Potentiometers

We have seen how the wiper of the multiplying potentiometer is positioned to correspond to one of the voltages in the product required. Let this voltage be represented by the symbol x, where x can have any value between -100 volts and $+100$ volts. Let the other voltage be y, where we apply $+y$ volts to that end of the multiplying potentiometer which corresponds to the $+100$-volt end of the follow-up potentiometer, and $-y$ volts to the other end. All the potentiometers are centertapped and all the centertaps are grounded (Figs. 3-13A and 3-13B). Now it is obvious that the voltage output

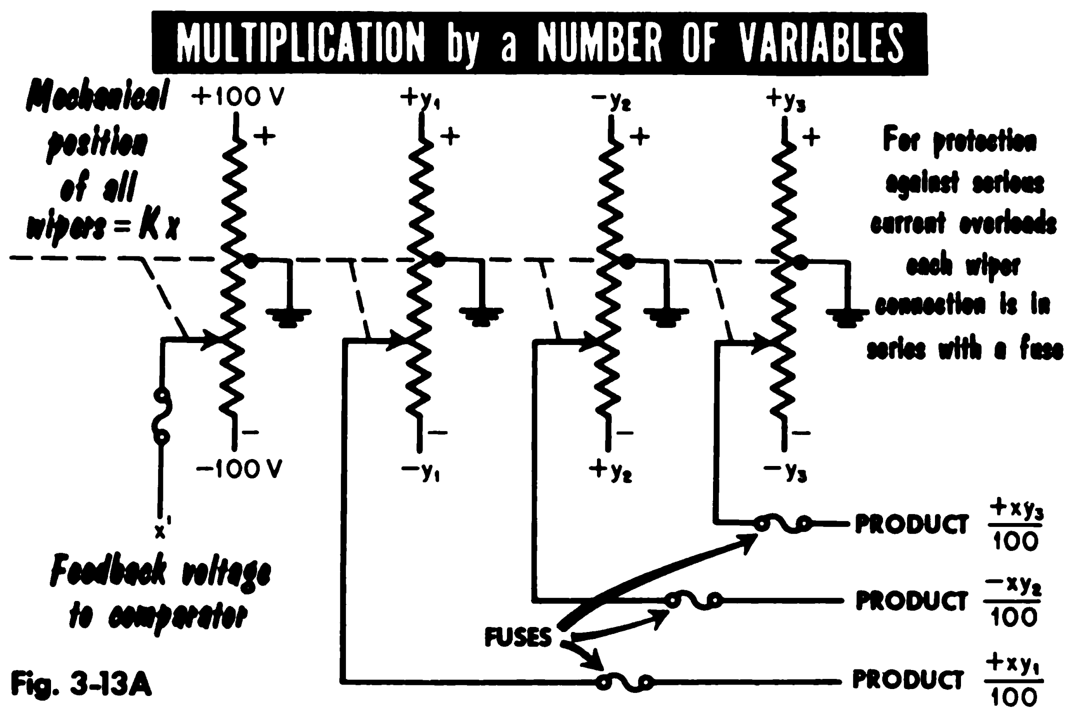


Fig. 3-13A

from the wiper of the multiplying potentiometer will be proportional to the product xy. As x increases in value the wiper will be moved along the potentiometer and the voltage output increases; as y increases, the voltage output also increases. However, the voltage output does not equal xy, but is $xy/100$. To see this clearly one has merely to recognize that the output voltage becomes equal to $+y$ when x is 100 volts driving the wipers to the extreme ends of the potentiometers.

Loading Error

As described thus far, the servomechanical multiplier would give accurate multiplication only if the output were not used in a practical way. Any con-

Symbolic Representation of a Servomultiplier (No. 2 giving three products by using the A, B, and C potentiometers)

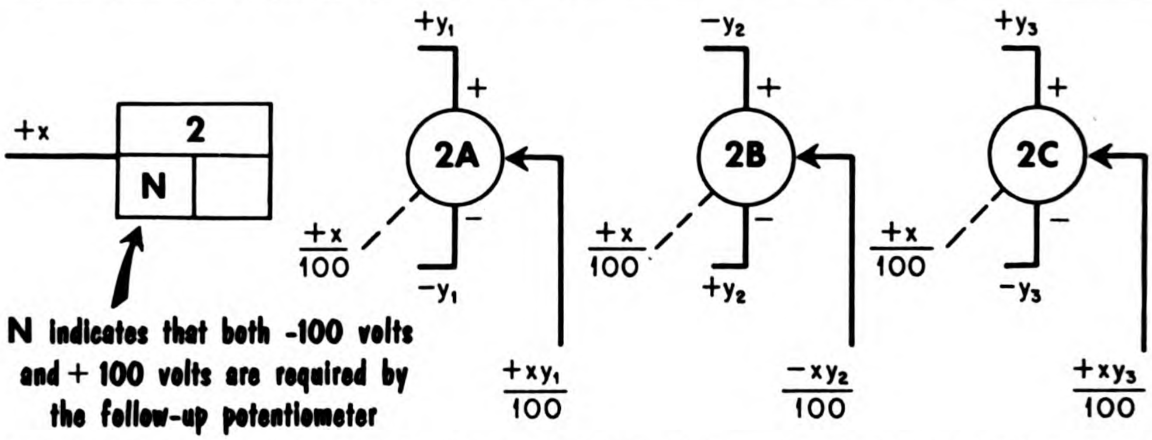


Fig. 3-13B

nection of the multiplier output to another computing unit (Fig. 3-14) would load the potentiometer by drawing current from it, and lead to the same inaccuracies discussed previously (Chapter 2) in the pages on attenuators. There the difficulty was overcome by electrically adjusting the setting of the potentiometer to counterbalance loading effects. The solution to this problem in the multiplier is quite different, but nevertheless easily achieved. Loading errors arise due to the current drawn by the loading circuit causing

LOADING ERROR is caused by the circuits to which the multiplying potentiometers are connected drawing different currents from that drawn from the follow-up potentiometer

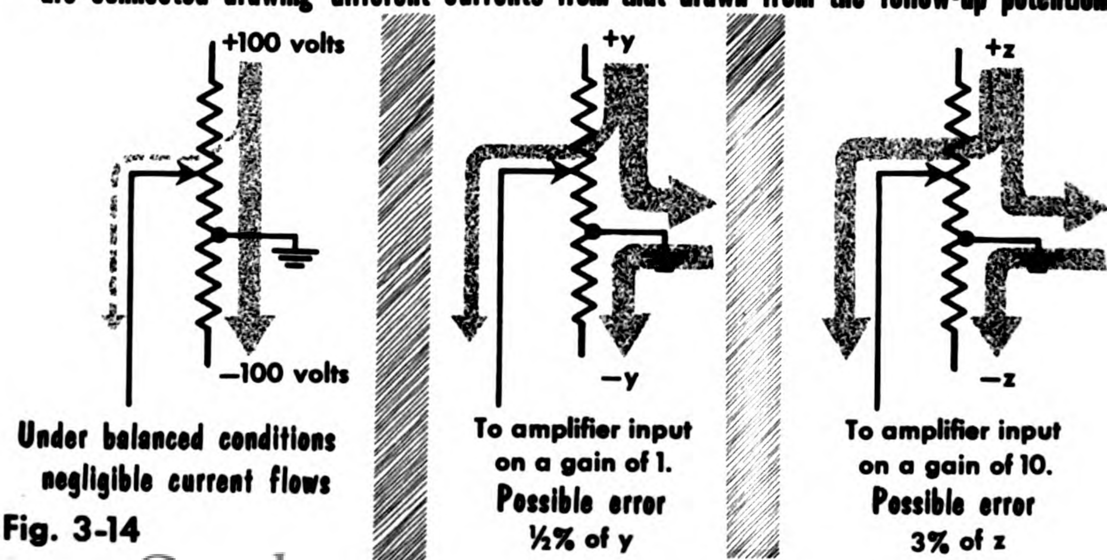


Fig. 3-14

the voltage at the wiper to fall. For correction, the wiper must be moved further along the potentiometer than a direct correspondence between the wiper position and input-positioning voltage calls for. The amount by which the position must be changed depends both on the loading circuit and on the original position. It is certainly not a simple, constant adjustment.

Loading Error Correction

Consider the follow-up potentiometer. Under normal accurate following conditions the voltage at the wiper of the follow-up potentiometer equals

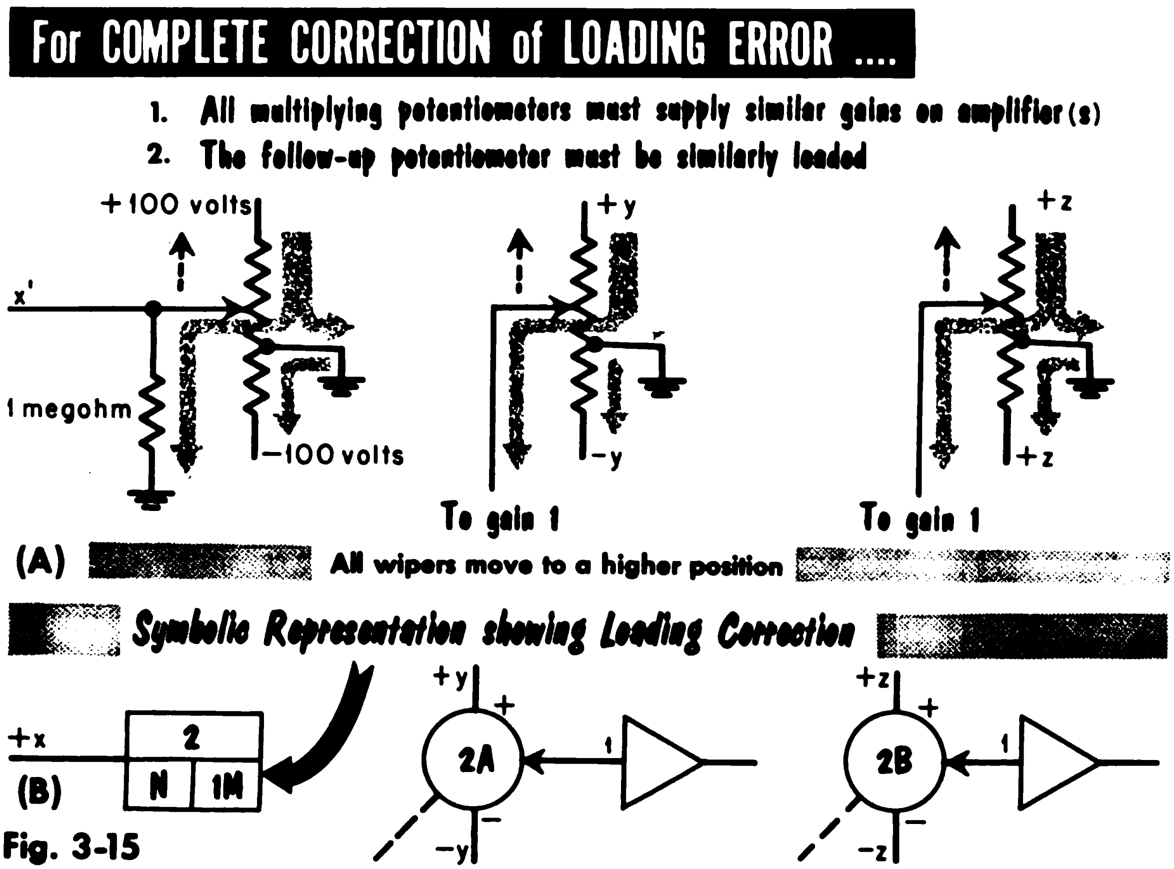


Fig. 3-15

that positioning the multiplier wipers. The servo is at a null and the voltage at every point of the comparator is the same. Thus no current flows through the comparator resistor chain — no current is drawn from the follow-up potentiometer — the follow-up potentiometer is NOT loaded. It is the difference in the loading conditions between this potentiometer and the multiplying potentiometers that causes the errors in multiplication. Realizing this fact the solution is simple [Fig. 3-15, (A) and (B)].

Firstly, all multiplying potentiometers on the same unit must be connected to similar loads, all must go to gains of 1 on operational amplifiers (1-megohm load) or all must go to gains of 10 on operational amplifiers (100,000-ohm load). Then the follow-up potentiometer is similarly loaded with a

1-megohm resistor or a 100,000-ohm resistor to ground. Arrangements are usually made to facilitate this kind of loading correction on the multiplier unit. The follow-up potentiometer wiper will now be moved by the servo to a higher position than was previously so, to achieve an electrical null, and all the other potentiometers, being aligned, will also move to higher positions, thus giving corrected outputs. It should be clear from the similarity between the follow-up and multiplying-potentiometer connections that each wiper will detect the same fraction of total voltage applied to its potentiometer, and since the follow-up voltage must equal the servo input voltage, the multiplication is free from loading error.

Following Lag

One slight disadvantage of the servomultiplier when compared with all-electronic devices is that it does have a "following-lag". Being a mechanical device with motor, gear train, and potentiometers, the response of the unit

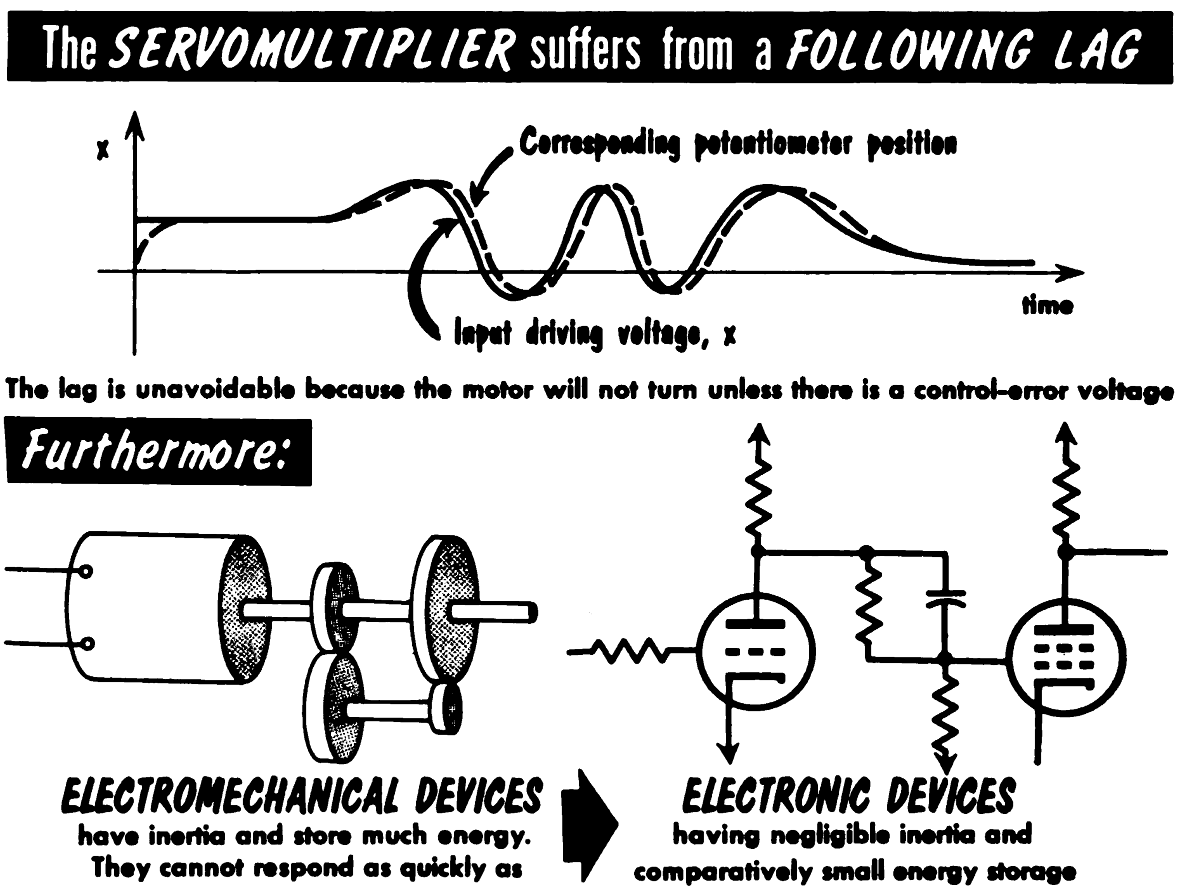


Fig. 3-16

to changes in the positioning voltage is not instantaneous. Time is required for the parts to accelerate, for indeed an error is necessary, although it be a small one, to drive the output to its new position (Fig. 3-16). If accuracy of computing is to be maintained the x-voltage, driving the servo, must

change only slowly with time — with a rate not greater than say, 200 volts per second.

Of course, there is no such restriction on the *y* voltages that go directly to the multiplying potentiometers. They can change as rapidly as required by the other computing devices and no error will be introduced. Thus where

INACCURATE FOLLOWING is INDICATED by an ALARM

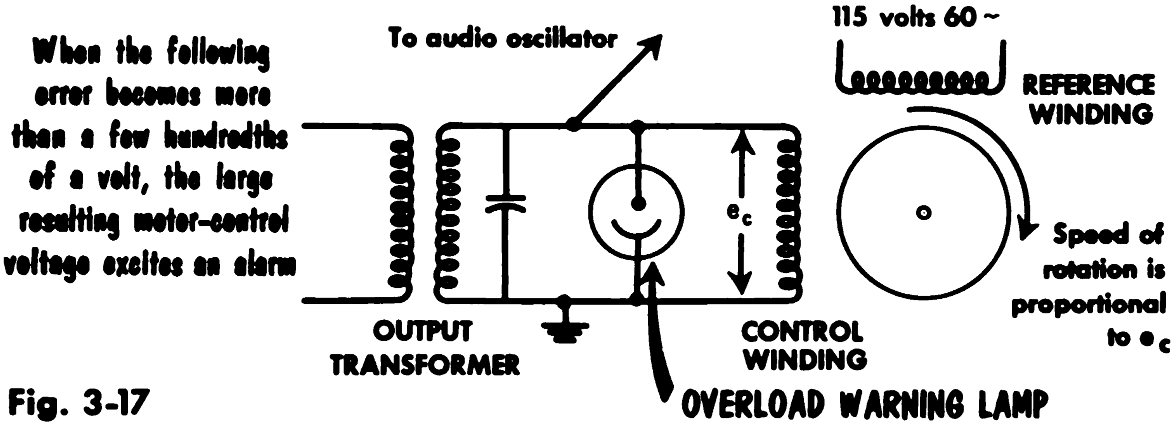


Fig. 3-17

two voltages are to be multiplied it is wise to apply the more slowly varying one to the positioning servo to obtain the more accurate results.

Overload Alarm

The errors described on the opposite page do not go undetected in the well-designed unit. When they grow to serious magnitudes, that is when the servo is trying to follow a too-rapidly changing voltage, the motor will be turning very fast, and the voltages in the a-c amplifier will be large, corresponding to a large error voltage. At the output of the amplifier, across the control winding of the two-phase servo motor, it is usual to place an overload alarm. This indicates visually or audibly whenever conditions in the servo are such that an unacceptable positioning error is present. Though termed an overload alarm, it would more accurately be called an error alarm (Fig. 3-17).

Squaring and Cubing

There is of course, no reason at all why the voltage driving the servo could not also be applied to a multiplying potentiometer. This permits the square of a quantity to be obtained [Fig. 3-18, (A) and (B)].

Repeating the process, taking the output of one potentiometer and applying it to a second, allows the cube of a variable quantity to be obtained. When using the output of a servomultiplier as the supply voltage for another multiplying potentiometer, it is essential to include an operational amplifier

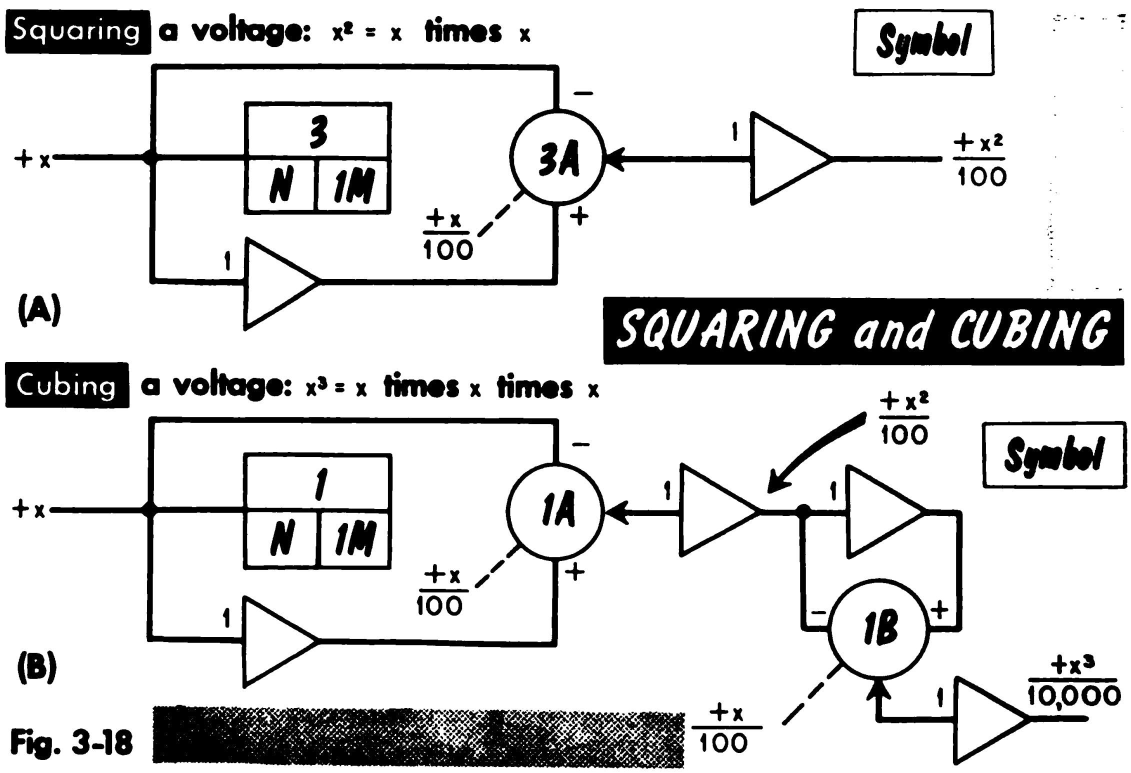


Fig. 3-18

between the two potentiometers. Without it, the load on the first potentiometer would be both variable and excessive.

Four-Quadrant Operation

It is important to note that the multiplier used in the way so far described gives what is known as *four-quadrant operation*. By this we mean that the factors in the product can be either positive or negative and the correct result will be obtained. Multiplying −20 volts by +80 volts will give −16 volts. Multiplying −50 volts by −60 volts will give +30 volts. The sign of the product is automatically taken care of (Fig. 3-19A).

In some applications a variable may be known to be always positive. For example, the speed of an aircraft is unlikely to be negative for aircraft are not in the habit of flying backwards. Then a simplification in connections and an improvement in accuracy is possible in any product of this variable with any other. The resulting two-quadrant operation could be achieved as shown (Figs. 3-19B and 3-19C).

ELECTRONIC MULTIPLIERS

Introduction

A number of different electronic multipliers (Fig. 3-20) have been designed to permit the solution of problems at higher speeds than is possible with the use of servomultipliers. The electromechanical positioning operation pres-

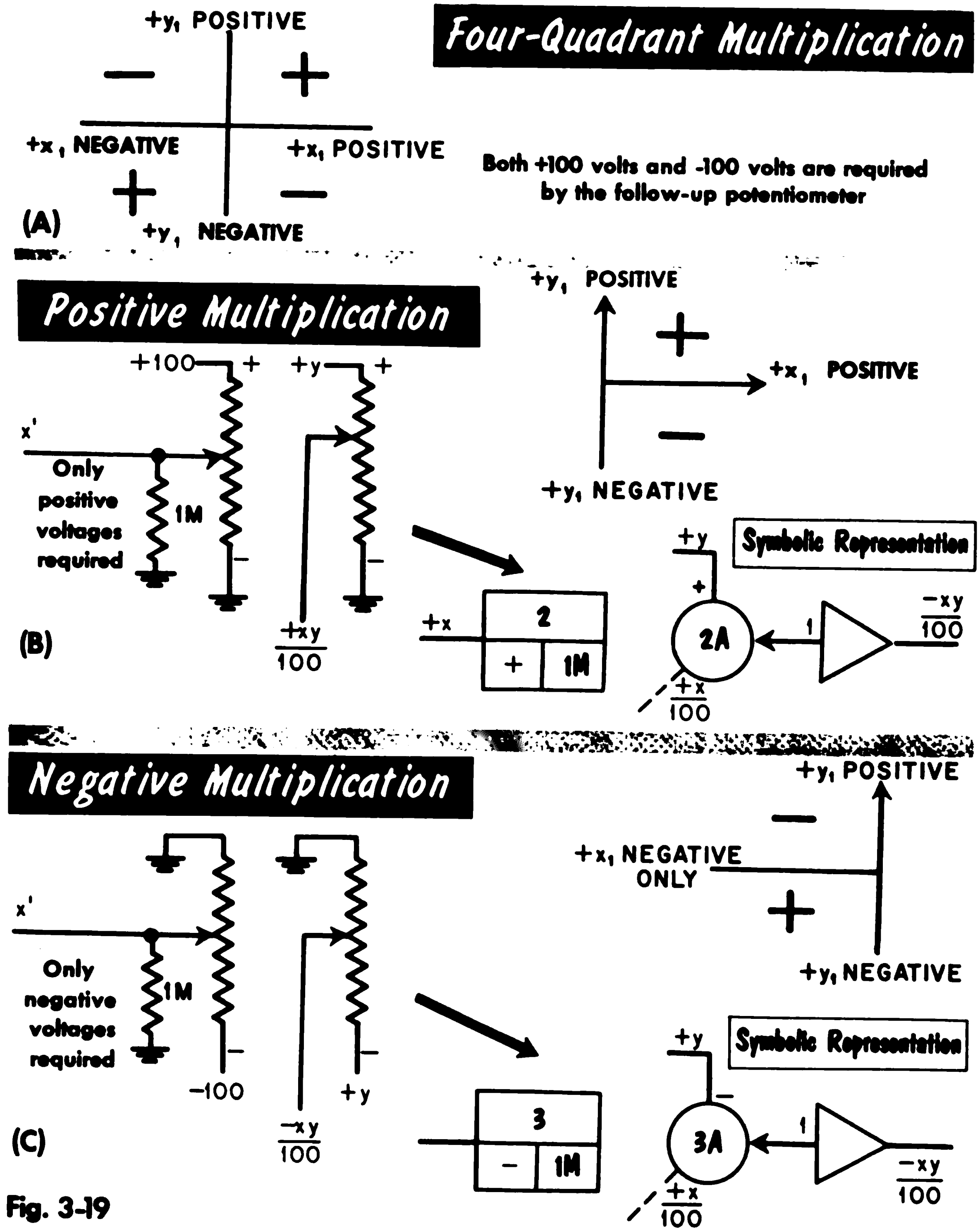


Fig. 3-19

ent in the servomultiplier requires that one of the variables to be multiplied be slowly changing. When both variables change rapidly then an electronic multiplier is necessary. The greater use of repetitive-operation computers in recent years has called for more and more electronic multipliers. Two types are used far more than any other, and they will be described here. They are quite different in their operation and must be considered sepa-

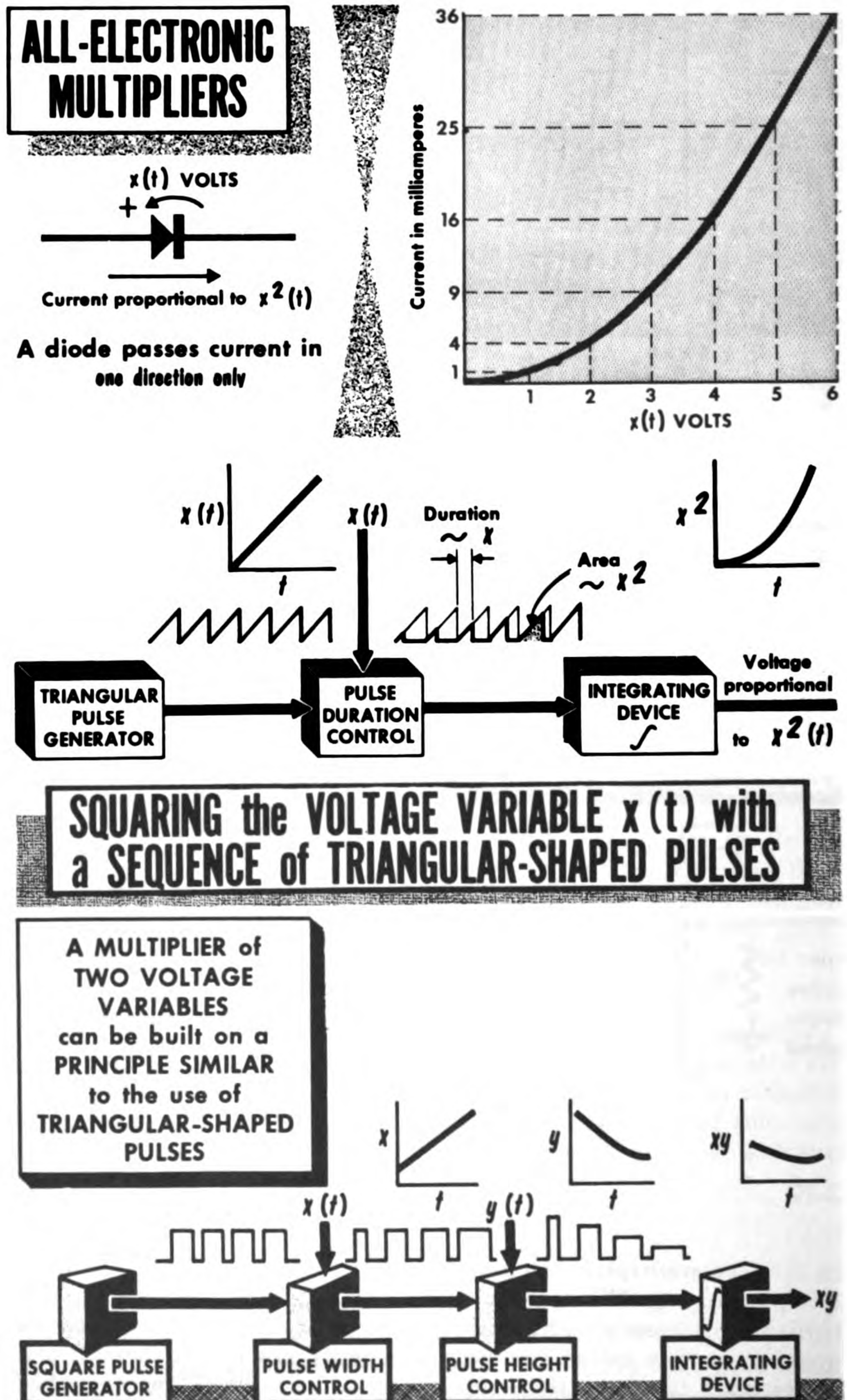

ALL-ELECTRONIC MULTIPLIERS
x(t) VOLTS
+
Current proportional to x²(t)
A diode passes current in one direction only
Current in milliamperes
36
25
16
9
4
1
1 2 3 4 5 6
x(t) VOLTS
x(t)
t
x(t)
Duration ~ x
Area ~ x²
x²
t
TRIANGULAR PULSE GENERATOR
PULSE DURATION CONTROL
INTEGRATING DEVICE
Voltage proportional to x²(t)
SQUARING the VOLTAGE VARIABLE x(t) with a SEQUENCE of TRIANGULAR-SHAPED PULSES
A MULTIPLIER of TWO VOLTAGE VARIABLES can be built on a PRINCIPLE SIMILAR to the use of TRIANGULAR-SHAPED PULSES
x
t
y
t
xy
t
x(t)
y(t)
xy
SQUARE PULSE GENERATOR
PULSE WIDTH CONTROL
PULSE HEIGHT CONTROL
INTEGRATING DEVICE

Fig. 3-20

rately. They are known as the time-division multiplier and the quarter-square multiplier.

Time-Division Multiplication

The area of a rectangle is given by the product of the length of its sides. The time-division multiplier, in a very loose way, uses this fact. Consider the rectangular waveform in the figure where the repetition frequency is quite high compared with the normally computed frequencies on a computer. The duration of the pulses is controllable and can be made dependent on variable x. The amplitude of the pulses is controllable and proportional to variable y (Fig. 3-21). If we can obtain a measure of the area under the rectangular wave train we have a measure of the product xy. This is achieved by passing the waveform through a smoothing, low-pass filter.

Time-Division Multiplication

Time-division multipliers depend on the area of a rectangle being equal to the product of the two sides

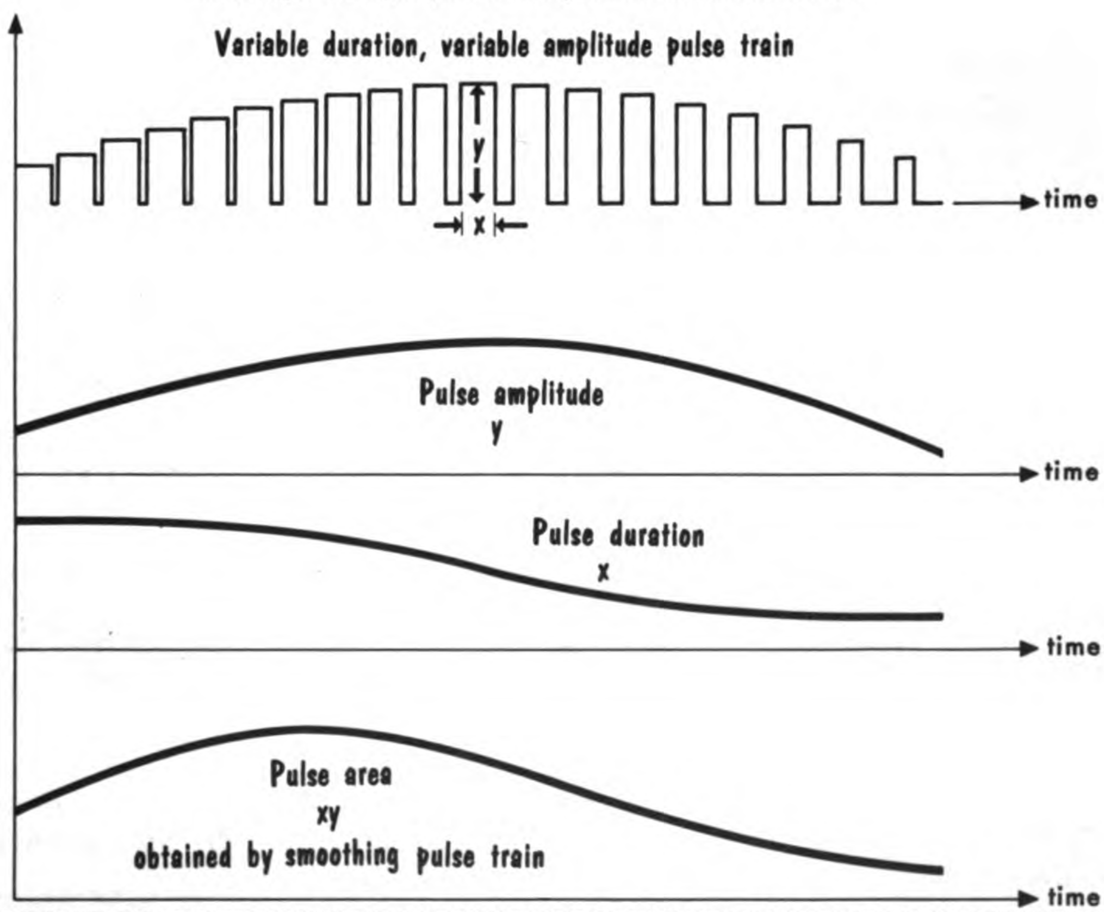


Fig. 3-21 Pulse amplitude is controlled by y. Pulse duration is controlled by x

As x and y change (at frequencies small compared with the pulse-repetition frequency) the shapes of the pulses change, and the low-pass filter output changes to correspond to xy. The components required to produce one scheme of time-division multiplication are,

1. Multivibrator
2. Phantastron gate control
3. Two gating tubes
4. An integrator
5. A constant-current generator
6. An output amplifier and filter

The Eccles-Jordan Multivibrator

As a source of high-frequency (10 kc, say) rectangular pulses one can use an Eccles-Jordan multivibrator. The circuit is simply a two-stage R-C coupled amplifier with positive feedback more than enough to cause oscil-

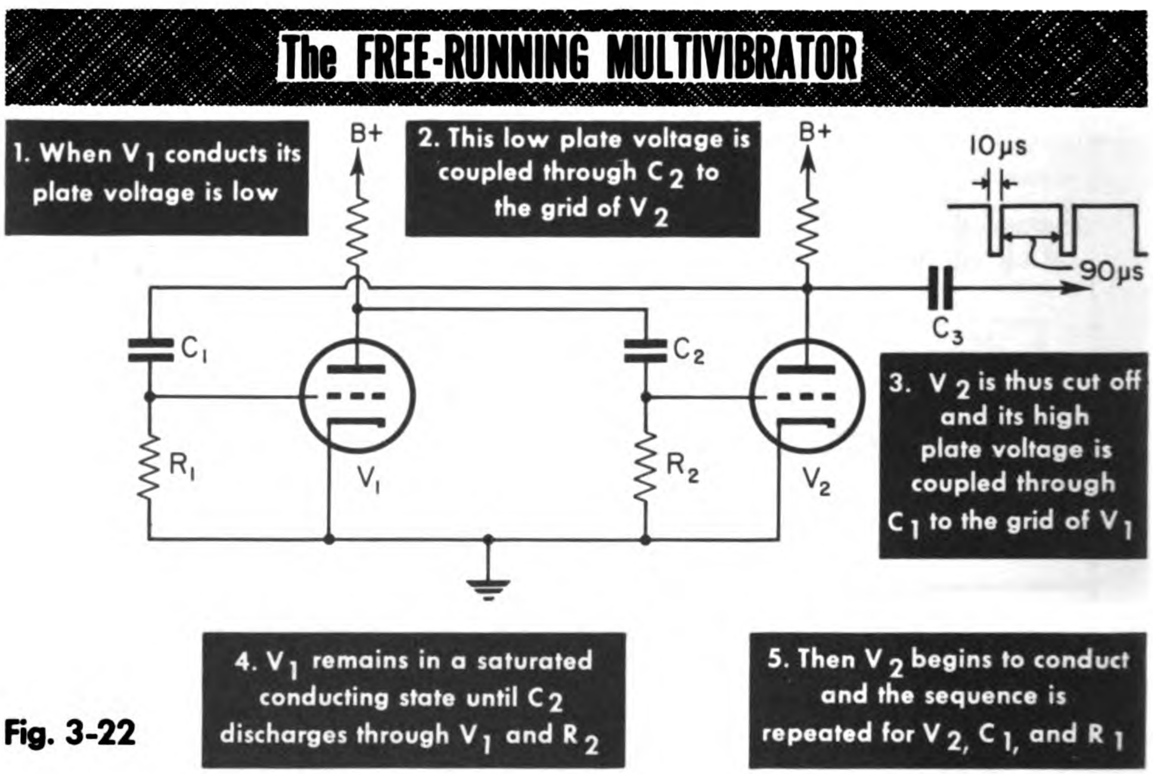


Fig. 3-22

lation. One tube can be considered to be an amplifier and the other a simple phase inverter. Each tube is alternately driven hard into saturation or cut-off, the frequency of the oscillation being determined by the time constants of the discharge paths for capacitors C_1 and C_2 (Fig. 3-22).

Consider the tube V_1 to begin conducting. Then its plate potential will fall, and this fall in potential will be transmitted through C_2 to the grid of V_2 causing the second tube to conduct a smaller current than previously. This results in an increase in the plate potential of V_2, and by coupling through

C_1 an increase in the grid potential of V_1. Thus more current flows through V_1, and the cumulative result is that V_1 is rapidly driven hard into saturation and V_2 is driven beyond cutoff.

With V_1 conducting and V_2 cut off, capacitor C_2 discharges through R_2 and V_1, returning the grid of V_2 to cutoff level and beyond. As soon as V_2 begins to conduct, changes occur in the condition of the circuit which drive it hard into saturation and cause V_1 to be cut off. The recovery time will now be determined by the discharge of capacitor C_1 through R_1 and V_2. In the time-division multiplier the pulse-repetition frequency is required to be about 10 kc, and the recovery times of the two saturation states are arranged to be different by a factor of 9. Thus the output from the multivibrator circuit through capacitor C_3 is a rectangular waveform having 90-microsecond positive pulses, and 10-microsecond negative pulses.

The Phantastron or Time-Delay Generator

To generate a voltage pulse with width proportional to some other voltage, requires a special timing circuit. The charging or discharging of a simple resistor and capacitor circuit are quite frequently used for timing [Fig. 3-23 (A)]. However, the charging rate, or current, in such a circuit is not linear, that is, the time to charge and the voltage produced across the capacitor are not proportional, only approximately so over a small range. Consequently, an improved R-C circuit employing a vacuum-tube amplifier is used. The actual R-C timing circuit resembles an analog-computer integrator with a constant-voltage input and a linear-ramp output [Fig. 3-23 (B)]. Such a decreasing "ramp" signal is ideal for timing the width of the desired pulse, because the duration of the ramp varies directly with the voltage itself, which is easily compared with the input signal e_i.

In the circuit shown, Fig. 3-23 (C), e_i (always positive) determines the maximum positive value at the plate of the phantastron tube. Thus before the tube conducts, C is charged up to a voltage approximately equal to e_i. The control grid is at a potential just above that of the cathode (ground) and is drawing current. In the absence of other grids in the tube a large plate current would flow, lowering the plate voltage, discharging C, and saturating the tube. The time to reach saturation would be proportional to the voltage e_i times A/RC, which is a constant. To form a rectangular pulse of this duration a screen grid is inserted [Fig. 3-23 (D)]. Since the rate of change of the capacitor voltage is constant, and R_L is large, the plate current is approximately constant during discharge of the capacitor, and furthermore, the screen-grid current and hence the screen-grid voltage (output) is also constant. The output pulse terminates when the capacitor is discharged, the plate current saturates, and the screen grid current increases rapidly due to the inability of the plate to collect the large electron current from the cathode.

To initiate a pulse, a second control grid is inserted to bring the tube out of saturation. This grid is normally positive, but is driven negative by short pulses. Initiating pulses arrive at regular intervals from the multivibrator

The PHANTASTRON produces VARIABLE-WIDTH PULSES

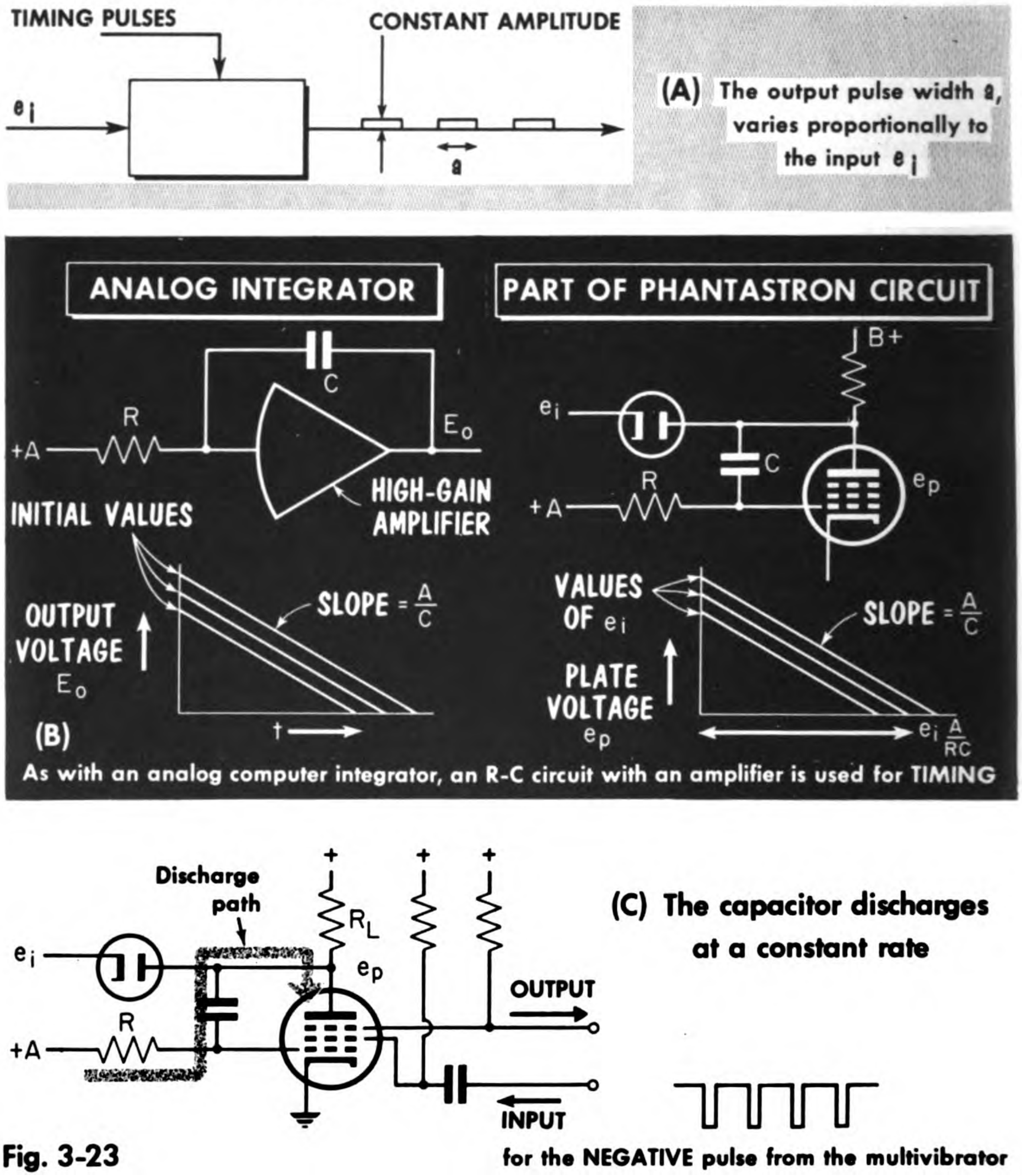
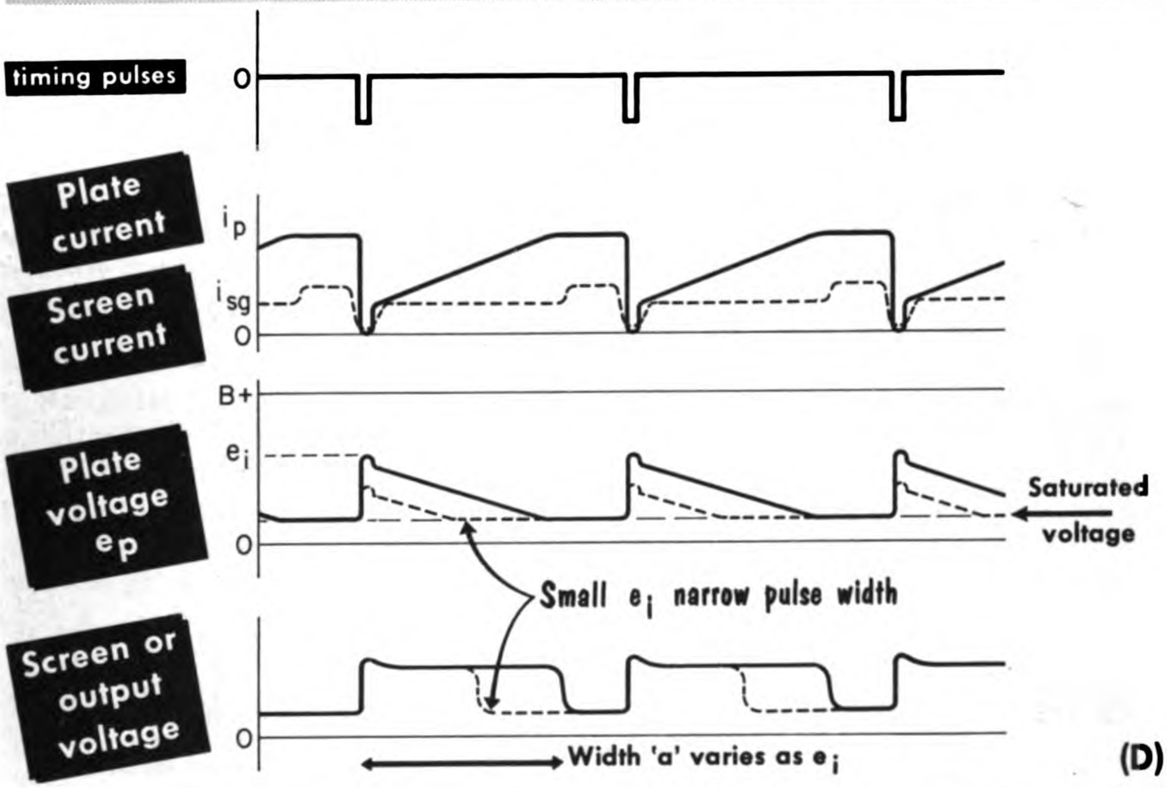


Fig. 3-23

just discussed. A pulse must cut off the tube conduction so that screen grid and plate voltages can return to their maximum values. It must last long enough to permit the charging of C through R_L to the voltage e_i. To speed up the charging of C, a cathode follower is inserted as shown [Fig. 3-23 (E)]. The triode does not change the operation of the circuit except to cause C to charge through the smaller resistance of V_1. In other applications (oscilloscope, for instance), the plate voltage is used as an accurately-timed

The PHANTASTRON produces VARIABLE-WIDTH PULSES



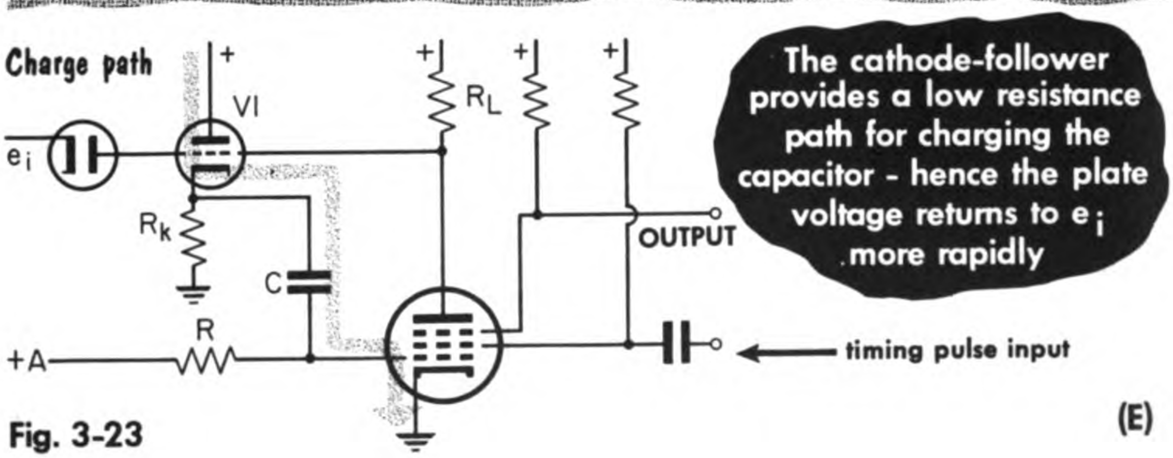


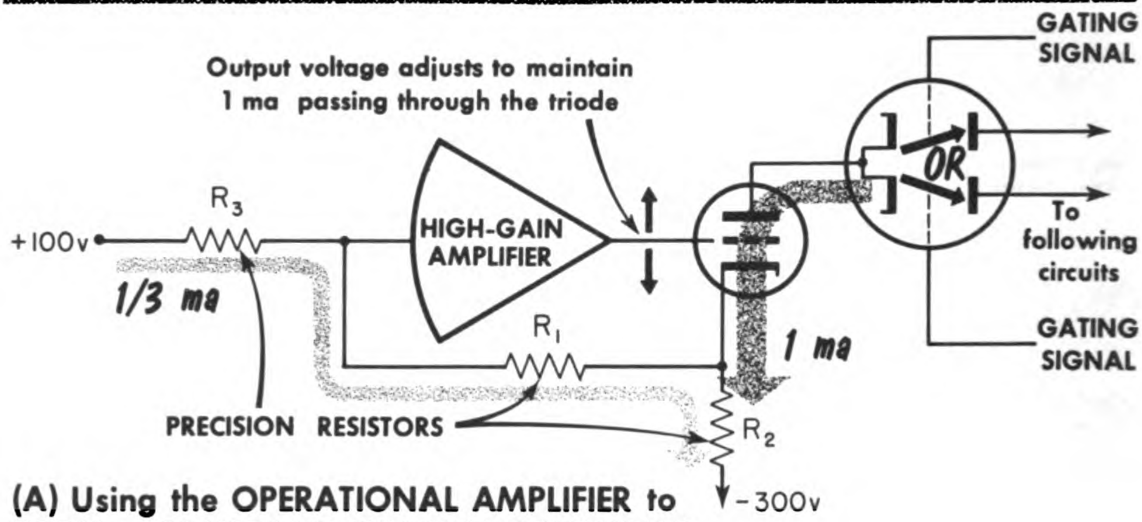
Fig. 3-23

linear sweep or ramp voltage. For the electronic multiplier only the accurately-timed rectangular pulse at the screen grid is required.

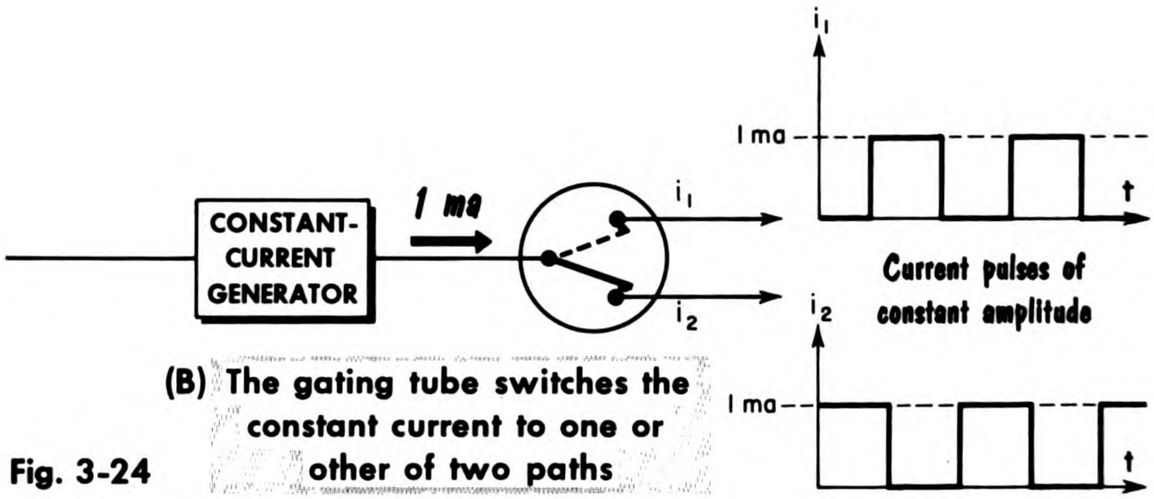
The Constant-Current Generator

A rather neat circuit is used to ensure that no matter what the change in impedance seen looking into a gating tube as this tube is switched, a con-

The CONSTANT-CURRENT Generator



(A) Using the OPERATIONAL AMPLIFIER to FORM a CONSTANT-CURRENT GENERATOR



(B) The gating tube switches the constant current to one or other of two paths

Fig. 3-24

stant current of 1 ma is delivered to the gate (Fig. 3-24). The output of a conventional high-gain operational amplifier is connected to the control grid of a triode. The plate of the triode is connected directly to the cathode of the gating tube. Thus the current flowing through the triode passes through the gate. To the cathode of the triode are connected two resistors: R_1, of 500,000 ohms, goes to the grid of the operational amplifier and forms the feedback connection around the amplifier; R_2 has a resistance of 100,000 ohms, and is connected to the -300-volt power supply. The amplifier has an input resistor R_3 of 300,000 ohms. All three resistors are precision components and are located in a controlled environment to preserve their values.

If the $+100$-volt reference supply is now connected to the input resistor R_3, a current of 1/3 ma will flow through it, for the amplifier grid is at ground potential. This current forced to flow through R_1 places a potential of -166.66 volts at the cathode of the constant-current tube, and thus a cur-

rent of 4/3 ma must flow through R_2. The difference current of 1 ma must flow through the triode no matter what the condition of the gating tube. Thus, provided there is an accurate adjustment of the −300-volt power supply against the +100-volt reference supply, the combination of the triode and the operational amplifier forms a very precise current generator.

The Pulse-Width Control Circuitry

Having described the major components of the pulse-width control circuitry, we can now appreciate how gate-control pulses are produced to correspond precisely to a signal voltage x. These pulses are required to open and close a multiplying gate through which passes a current proportional to the voltage y. The average current through the multiplying gate will then be proportional to the product xy.

Consider the circuit illustrated in Fig. 3-25A. A constant amplitude, rectangular pulse, voltage waveform is supplied by the multivibrator to the phantastron. The output from the phantastron is a rectangular pulse, voltage waveform whose width is determined by a control voltage supplied to the phantastron. This variable-width pulse waveform is boosted by a power amplifier having a double-ended, push-pull output, and used to drive the control grids of parallel triodes in a gating tube. At any instant one triode

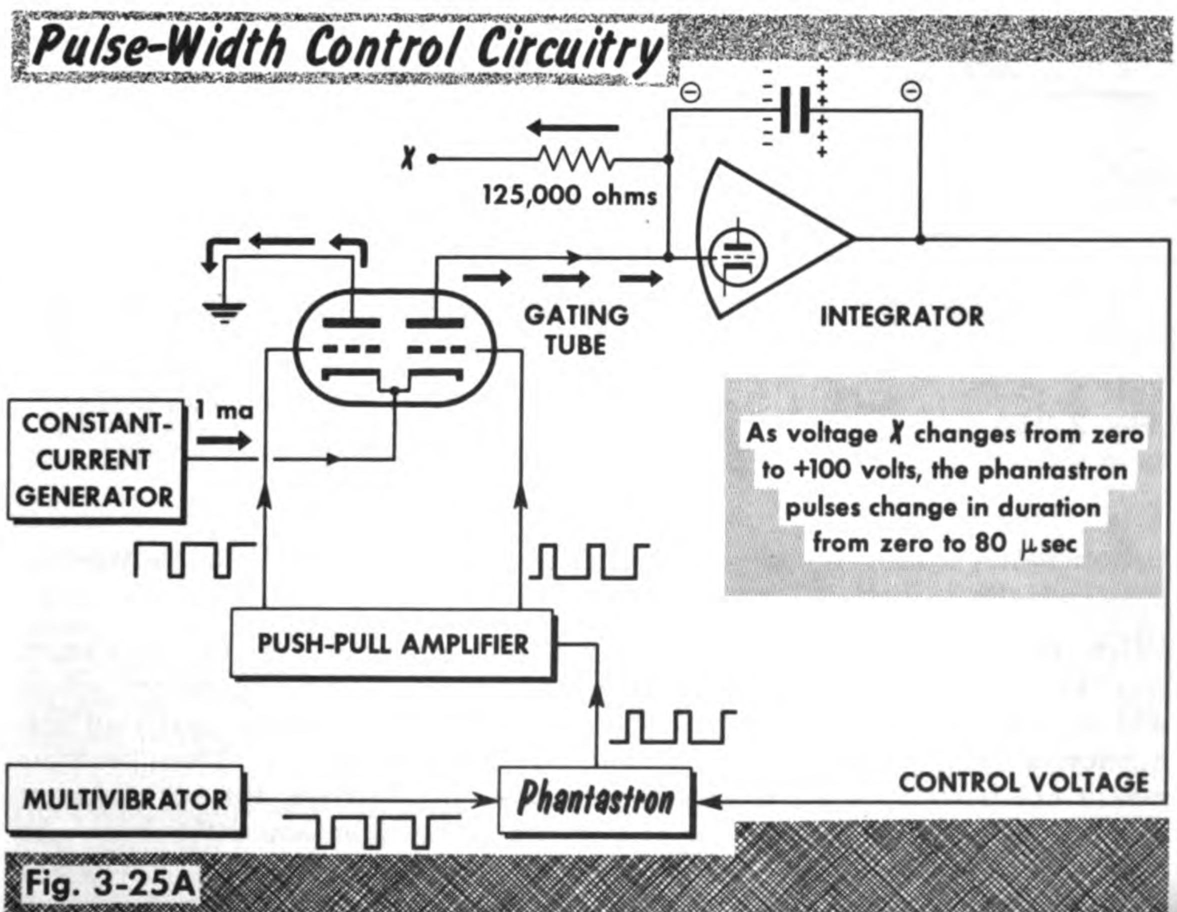
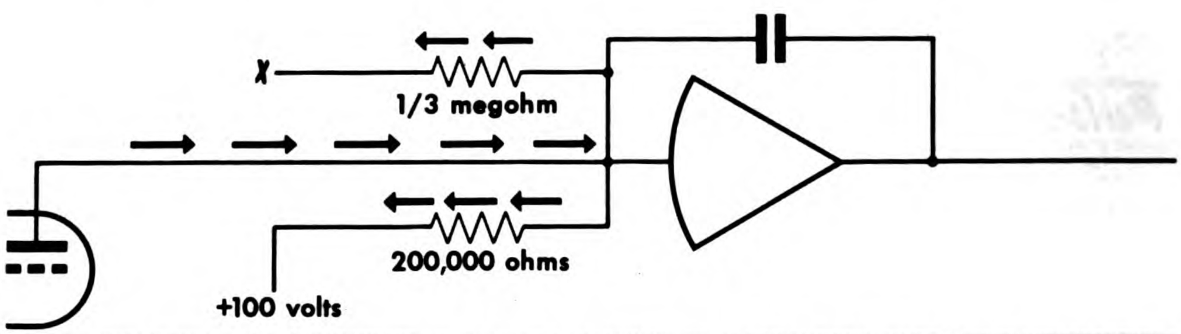


Fig. 3-25A

is cut off while the other is hard-conducting or saturated, permitting the constant current from the generator to be delivered through one plate or the other. The plate of the triode which conducts during the time of the positive pulses from the phantastron, is connected directly to the summing point of an integrating amplifier. Thus pulses of current are applied to the integrating capacitor and the average charge flowing into the capacitor is proportional to the width of the positive-going phantastron pulses. To the same integrator, through a suitable input resistor, the voltage x is applied. It also causes the capacitor to charge. With the capacitor charging, the integrator's output voltage will be changing. However, should the average charge supplied by the pulsed currents be balanced by that due to the voltage x, the output voltage would remain sensibly constant.

The output voltage of the integrator is used as the control voltage of the phantastron. When the charge supplied to the integrating capacitor due to the voltage x is balanced by that from the constant-current generator through the gate, the width of the pulses controlling the duty cycle of the gating tube corresponds to the voltage x. If the two sources of charge do not balance, the pulse width does not correspond to the voltage x. The changing charge on the integrator capacitor, with the resulting change in



BY ADDING A BIAS VOLTAGE TO THE INTEGRATOR an improvement is achieved. Now X can have negative values, and as X changes from —100 to +100 volts, the pulse duration changes from 20 to 80 μsecs

Fig. 3-25B

output voltage, acts to change the phantastron sweep until the gating-tube duty cycle is correct.

With this arrangement, the voltage x can range from zero to +100 volts, and the duty cycle will correspondingly have positive-going pulses whose widths range from zero to 80 μsec (assuming a 10-kc pulse-repetition frequency, a 1-ma generator current, and $R_1 = 125{,}000$ ohms). The operation would not permit negative values of x, and furthermore, the very narrow pulses required for small values of x would create practical difficulties. An alternative arrangement is far preferable where, as shown in Fig. 3-25B, a bias current of 1/2 ma is introduced, which for balance required 50 μsec

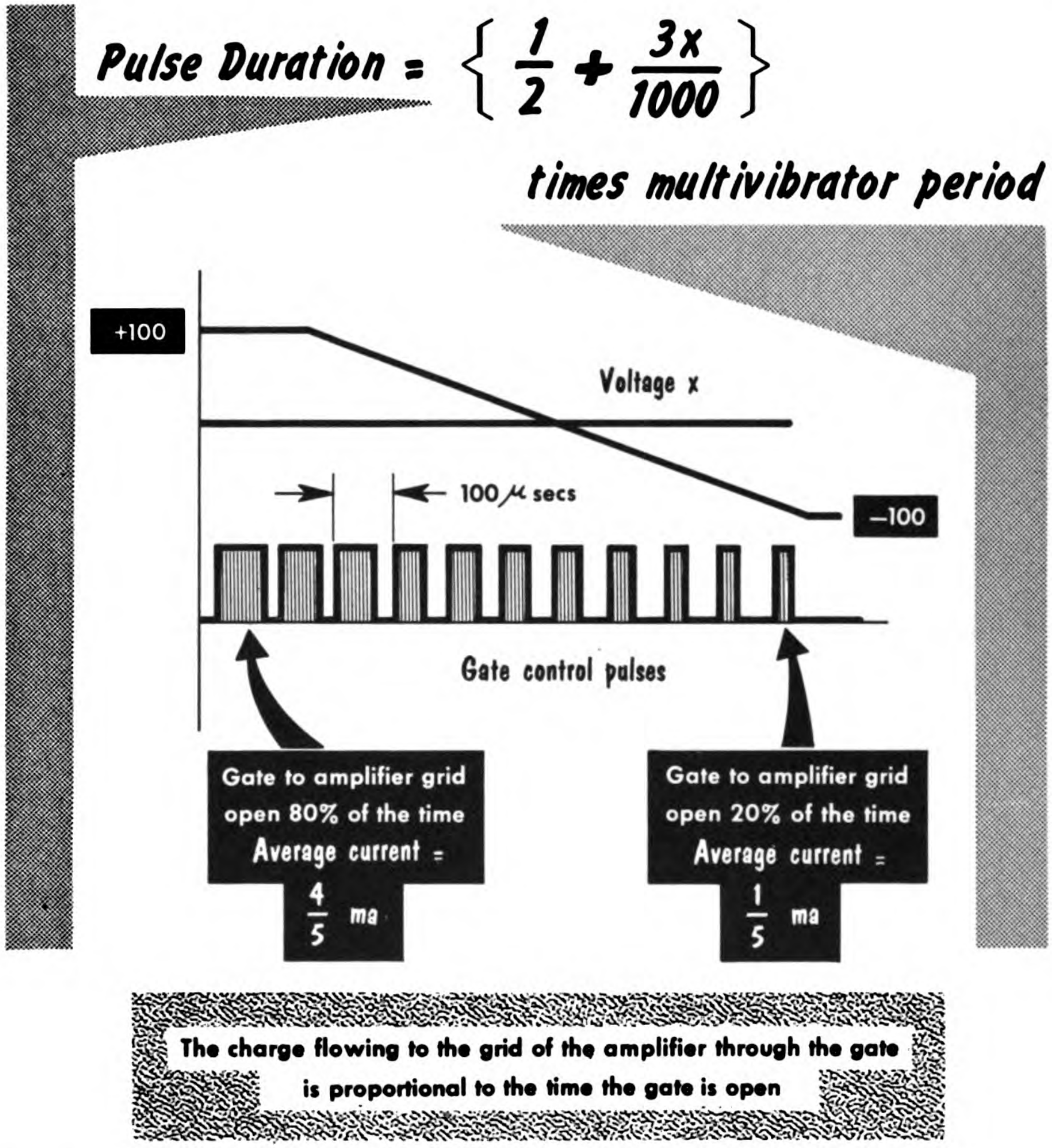


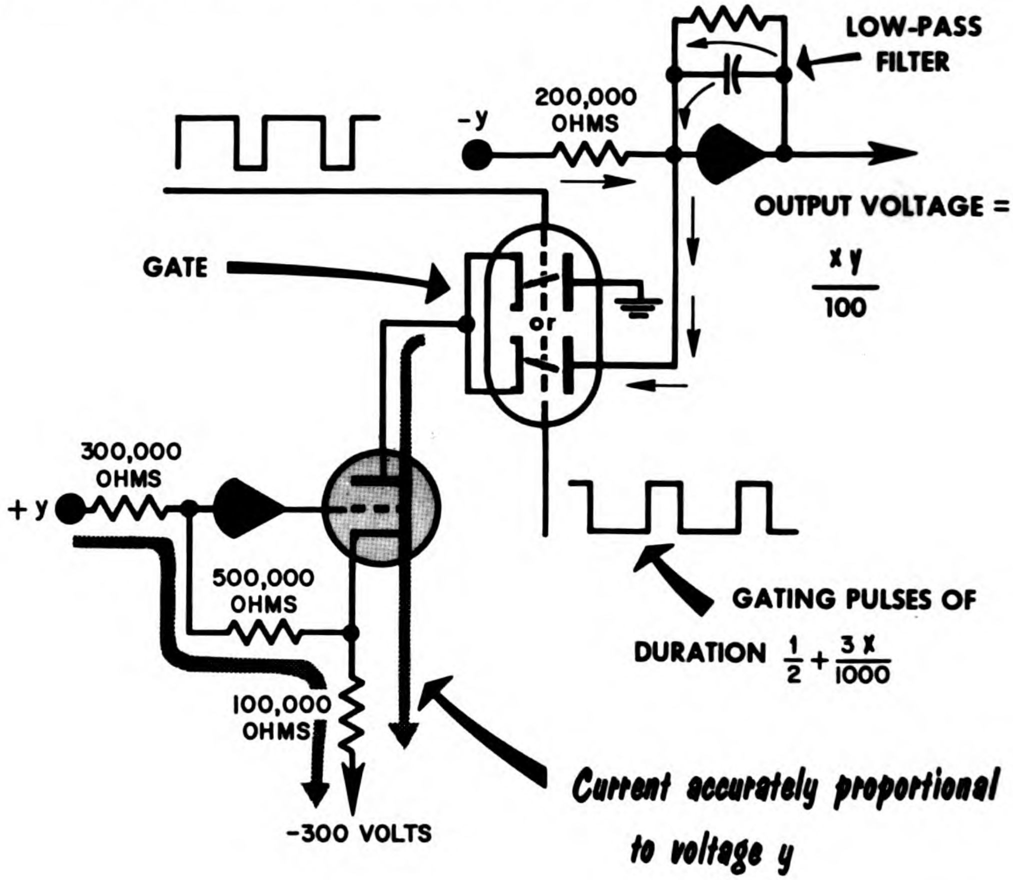
Fig. 3-25C

pulses to exist even when $x = 0$. Now by making $\dot{R}_1 = 1/3$ megohm, as x varies from -100 volts to $+100$ volts, the required pulses have widths ranging from 20 μsec to 80 μsec. Still there is a proportional correspondence between x and the pulse widths, but to allow negative values of x and also to avoid very narrow pulses, the width is a fraction $(3x/1000) + (1/2)$ of the periodic time. A zero shift has been introduced which is taken into account in the multiplying-gate circuitry (Fig. 3-25C).

The Multiplying-Gate Circuitry

The gate-control pulses accurately produced by the phantastron tube and its control circuitry open a second gating tube, to which is applied a current

The Multiplying Gate



By passing a current accurately proportional to y through a second gating tube controlled by the phantastron, a voltage proportional to the product xy is obtained

Fig. 3-26

accurately proportional to the voltage y. Thus the charge passing through the gate during one cycle is proportional to the product $y\,[(1/2) + (3x/100)]$. From this is subtracted a charge $y/2$, and the remaining charge, proportional to the product xy, is applied to the capacitor of the low-pass filter at the multiplier output. The voltage developed across the capacitor by the charge is $xy/100$ volts (Fig. 3-26).

A discharge path is placed around the capacitor so that succeeding pulses of charge instead of accumulating without limit, simply replenish the charge that has leaked away. The time necessary to discharge the capacitor in the filter, though short with respect to the time taken for any reasonable change in voltages x or y, is long compared with the maximum pulse dura-

tion of 80 μsec. The output waveforms for different possible behaviors of voltages x and y are shown in the figure. Note that it necessarily has a slight ripple at 10 kc which is virtually eliminated by the output filter.

The Quarter-Square Multiplier

An alternative scheme for electronic multiplication uses a well-known equality and depends upon being able to square a voltage electronically. If

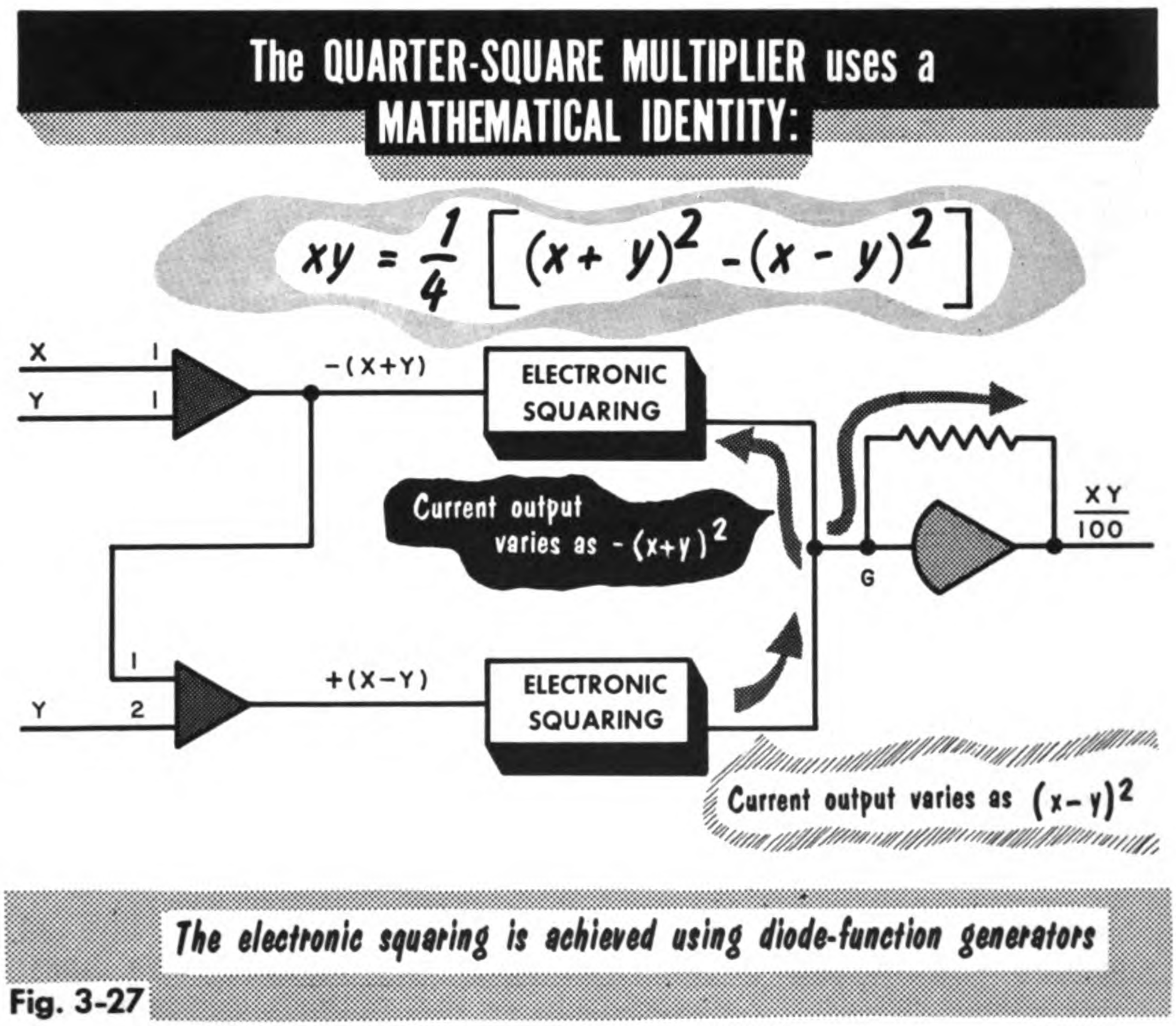


Fig. 3-27

one squares the sum of two voltages, x and y, the result contains not only the squares of both x and y but also the cross-product $2xy$ [from the algebraic identity: $(x + y)^2 = x^2 + 2xy + y^2$]. The square of the difference of two voltages also contains the square of x and the square of y. However, in this case the cross-product is $-2xy$, namely $(x - y)^2 = x^2 - 2xy + y^2$. Thus by subtracting the square of $(x - y)$ from the square of $(x + y)$ we have only a cross-product term $4xy$ (Fig. 3-27). Or:

$$(x + y)^2 - (x - y)^2 = (x^2 + 2xy + y^2) - (x^2 - 2xy + y^2) = 4xy$$

Therefore:

$$xy = \frac{1}{4}\,[(x + y)^2 - (x - y)^2]$$

To square a variable voltage we use a fixed diode-function generator (see later), an all electronic device. The diodes are biased to contribute currents proportional to the input driving voltage only after it reaches a suitable value. When these currents are summed, the total current varies as a straight-line approximation to the square of the input voltage, and by suitably forcing this current through the feedback resistor of an operational amplifier a corresponding voltage is produced. The two currents, for $(x + y)^2$ and $-(x - y)^2$, are summed by the output amplifier.

A similar technique to that used in the quarter-square multiplier — also employing an electronic fixed-function generator — would use the relation:

$$xy = exp\,[\log_e (a + x) + \log_e (a + y)] - a^2 - a(x + y)$$

The value of a would need to be greater than the maximum negative value of x or y, so that the logarithm of a negative number would not be called for. The device requires two logarithmic function generators and one antilog or exponential-function generator.

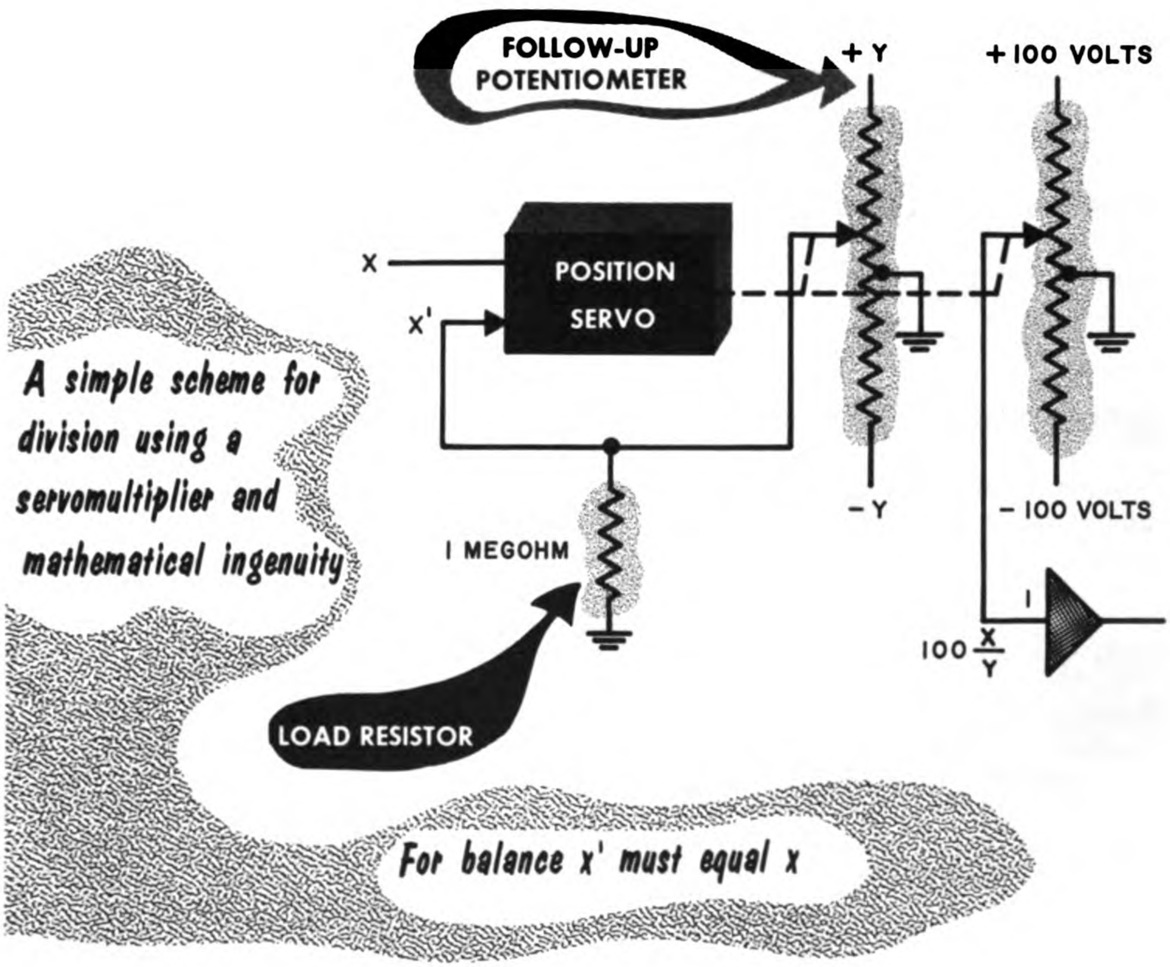
IMPLICIT USES OF MULTIPLIERS

Division

In the description of physical systems the need to divide one variable by another occurs almost as frequently as does that of multiplying two variables. Despite this, the analog computer contains no unit whose specific purpose is that of division. Instead, one makes use of a multiplier and a little mathematical ingenuity.

One might consider that the simplest method for effective division with the computer is to rearrange the connections on a servomultiplier (Fig. 3-28). If the variable voltages $\pm y$ are applied to the ends of the follow-up potentiometer, and the constant ± 100 volts are applied to the multiplying potentiometer, then the output of the unit when driven by x will be $100x/y$. The mechanical position of the wipers will be x/y of the full travel in order that the follow-up potentiometer wiper shall have a voltage x for a null. This scheme for division, though appealing in its simplicity, is not used, for with a variable voltage applied to the follow-up potentiometer, a variable gain around the servo loop exists and satisfactory operation of the position servomechanism is not possible. For small values of y the gain is low and the servo behavior sluggish; for large values of y the gain is high and the servo behavior may be oscillatory.

The ANALOG COMPUTER contains NO UNIT whose SPECIFIC PURPOSE is that of DIVISION

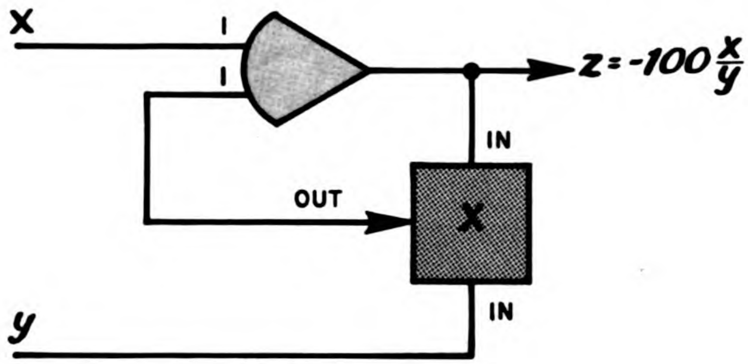


Mechanical position of wiper must be x/y, and the voltage output from the multiplying potentiometer is then +100x/y

The position-servo gain is proportional to y, and thus its performance cannot be good for both small and large values of y

Fig. 3-28

IMPLICIT DIVISION
To Divide, Multiply in the Feedback Path of a High-Gain Amplifier



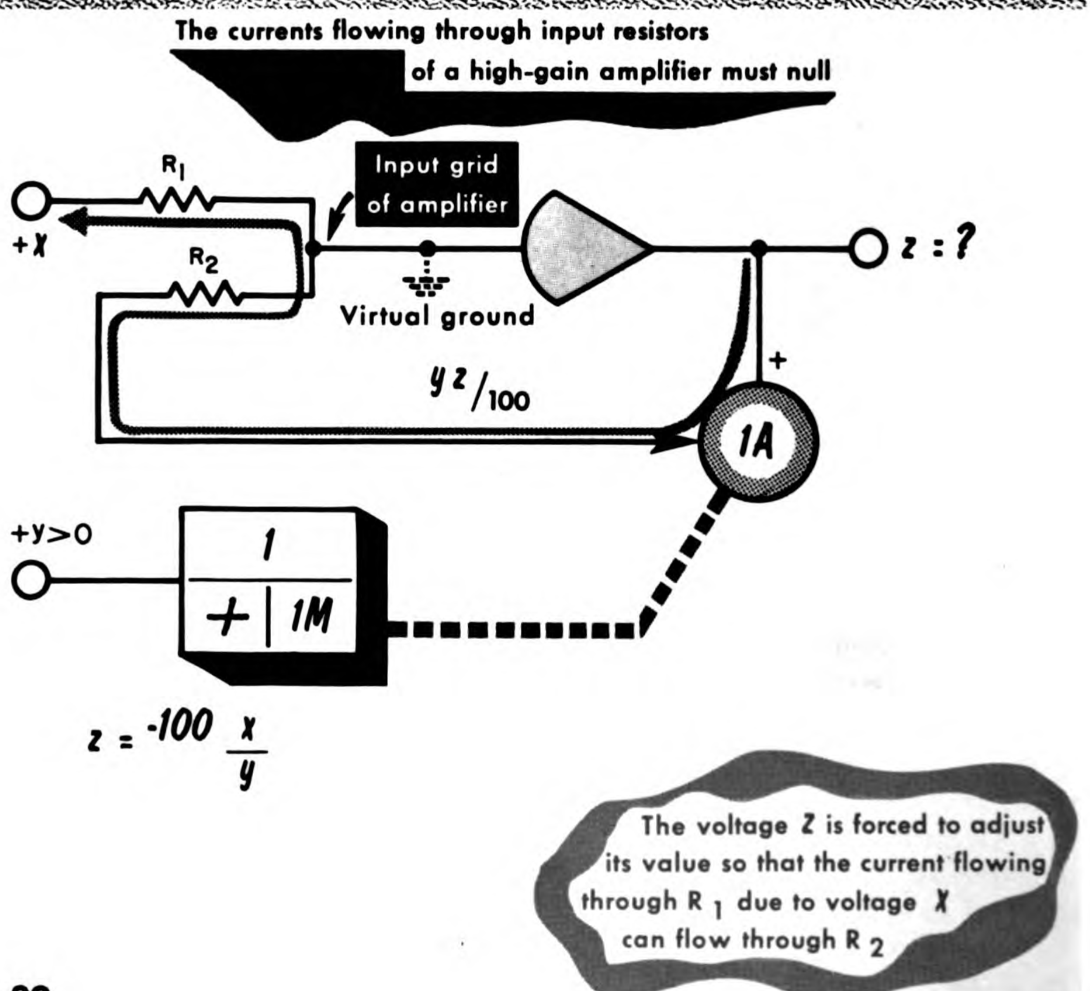


Fig. 3-29

Implicit Division

Division is performed on an analog computer using a multiplier and a high-gain amplifier as a nulling device. Suppose that we wish to divide a variable x by another variable y to form a quotient z. Consider the circuit arrangement shown in Fig. 3-29. The high-gain amplifier is assumed to draw no grid current. Thus summing the currents arriving at the grid we have:

$$\frac{x}{R_1} + \frac{1}{R_2}\,\frac{yz}{100} = 0$$

Making $R_1 = R_2 = 1$ megohm, and solving for z gives:

$$z = -100\,\frac{x}{y}$$

We see therefore that using the properties of a high-gain operational amplifier we can perform division by feedback multiplication. The high-gain amplifier forces the output voltage z to change until the sum of the currents arriving at the input grid is zero. Then z is proportional to the required quotient.

Limitations of Division Circuit

1. When dividing variable x by variable y, the latter must not be zero. If the divisor y should go to zero, then the product yz will be zero, and there will be zero output from the multiplier (Fig. 3-30A). This implies that the

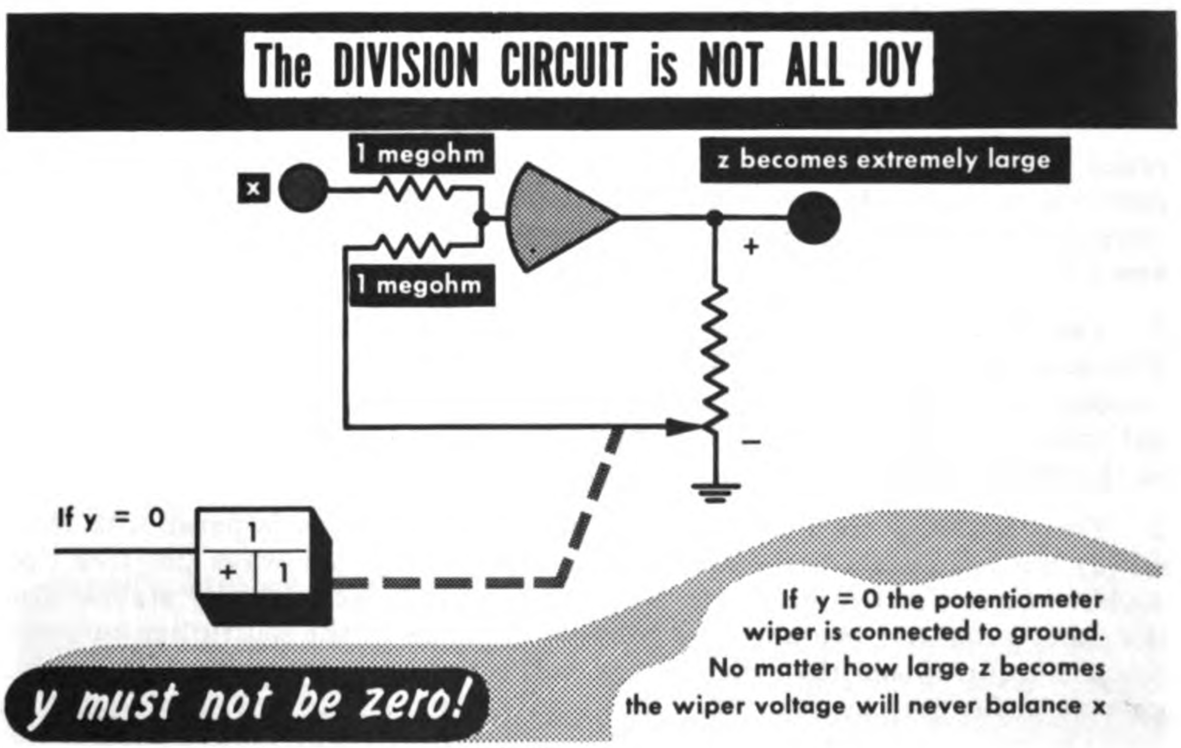


Fig. 3-30A

high-gain amplifier is operating without any feedback connection. The slightest input voltage will cause a large voltage to appear at the output of the amplifier. A very unsatisfactory and inaccurate computation exists under these conditions.

Of course, if when y is zero x is not zero, then the correct value for z, the quotient, is infinite. Infinite values for quotients of variables in solutions of physical problems are rarely, if ever, required. Thus, the more usual occur-

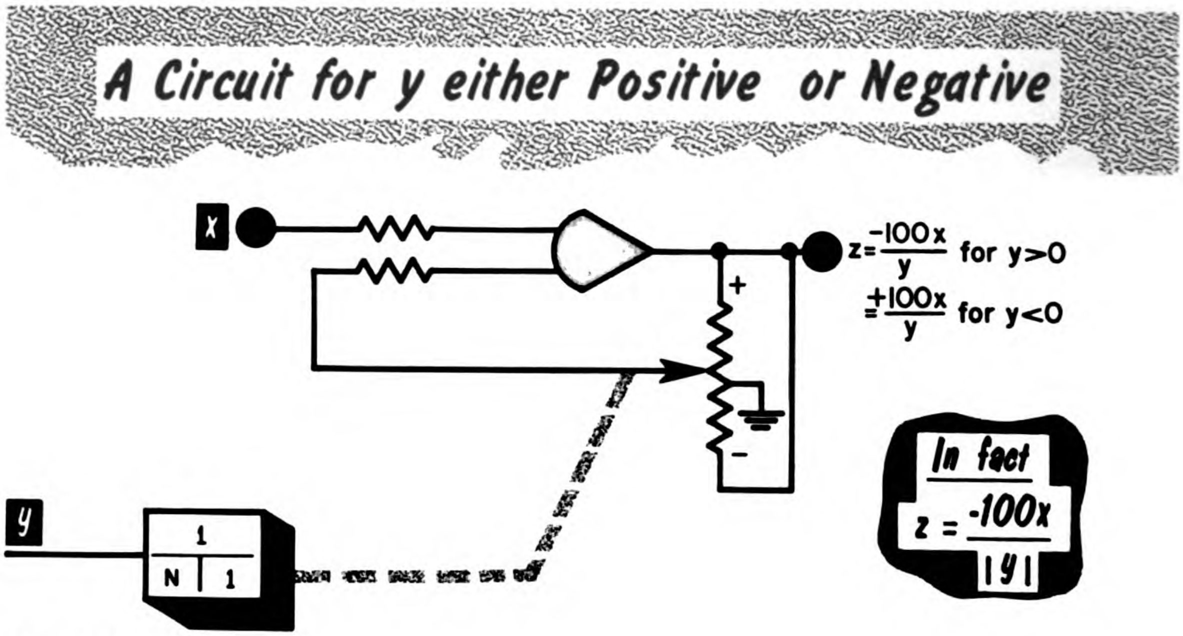


Fig. 3-30B

rence is for y to approach zero as x approaches zero, and then z will most probably be finite. However, even so, the behavior of the circuit producing the quotient in such cases is unsatisfactory, and attempts should be made to avoid the condition if possible.

2. The variable y must always be greater in amplitude than the variable x. The quotient z is equal to $-100\ x/y$ (Fig. 3-30B). Thus if y is less than x, the magnitude of z will exceed 100 volts, the desired maximum for any signal voltage in the computer. It is desirable that y be always greater than x, so that the magnitude of z will be less than 100.

3. The variable y must not change sign. This condition is parallel to that of (1) but requires a little further discussion. If y is always positive the division circuit shown gives correct results, and is electronically stable. On the other hand, if y is a negative voltage, a change in the multiplier connections is required and then the current null equation at the grid of the high-gain amplifier is:

$$\frac{x}{R_1} - \frac{1}{R_2}\frac{yz}{100} = 0$$

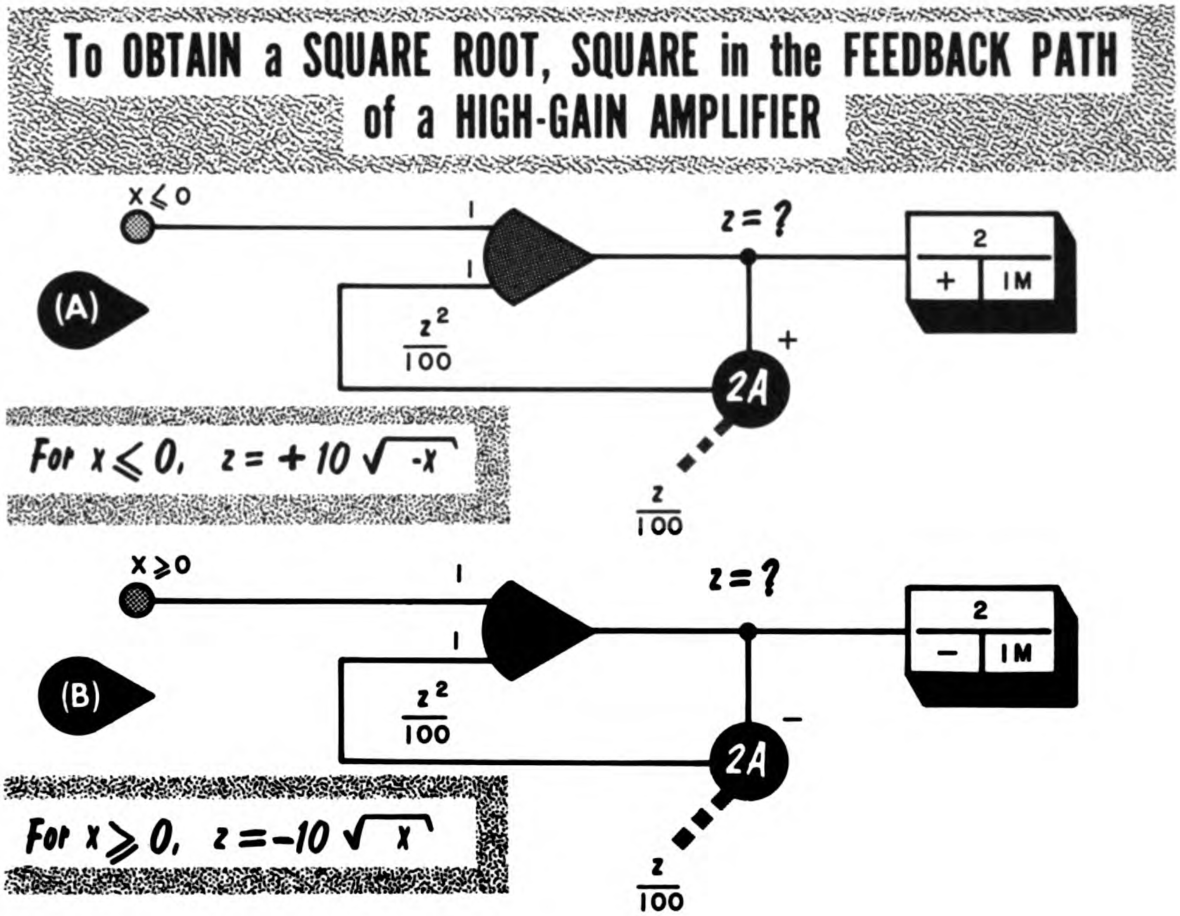


Fig. 3-31

Thus with $R_1 = R_2$,

$$z = +\ 100\ \frac{x}{y}$$

Again, with the rearrangement, the circuit is electronically stable, but we have a change in the sign of the quotient. This circuit can thus be made to provide a quotient of two variables for either a positive or a negative divisor, but not for a divisor changing sign. Rarely in the simulation of physical systems does the divisor of a quotient change sign. If it does, more elaborate circuitry is required to produce the quotient.

Square-Root Circuit

The same technique used to produce a quotient can be used to produce the square root of the variable. Suppose one wishes to find y, the square root of a given variable voltage x. Then, following the technique used in the division circuit we try feedback squaring [Fig. 3-31 (A)]. Thus the null at the grid of the high-gain amplifier leads to:

$$\frac{y^2}{100} + x = 0$$

$$y = +10\sqrt{-x}$$

For satisfactory operation x must always be a negative voltage. Then y will always be the positive square root.

If x is a positive voltage, a rearrangement of the circuit similar to that necessary in a division circuit for a change in the sign of the divisor will produce a negative voltage [Fig. 3-31 (B)]

$$y = -10\sqrt{x}$$

An alternative circuit would use two fixed-function generators placed around a high-gain amplifier. With each function generator producing a voltage proportional to the square of the voltage applied to it, the high-gain amplifier produces the root of the magnitude of the input variable. The function generators do not change the sign of a voltage, and thus the overall circuit is stable for both signs of input voltage. However, the sign of the output is opposite to that of the input.

QUESTIONS

1. Why do we use a closed-loop position controller in a servo multiplier? Contrast its operation with an open-loop controller.
2. What is the purpose of the modulator in the servomultiplier? Explain its operation and show how it enables a two-phase induction motor to be used in the controller. Why do we prefer to use an induction motor?
3. Explain the loading-error correction of a servomultiplier.
4. What are the limitations of using a servomultiplier?
5. Explain the principle of the time-division multiplier and show a block diagram for this unit.
6. Determine a circuit for generating a current of 1.5 milliamperes, no matter what the load.
7. Explain how one produces a quotient of two variable voltages. Give the limitations on this circuit.
8. Draw circuits for producing the following functions: $x^{1/3}$, $x^{3/4}$, $x^{5/6}$.

Chapter 4

D-C ANALOG COMPUTER: FUNCTION GENERATION

Introduction

Some variables which appear in a problem for solution on an analog computer are not known mathematically in terms of other variables. That is to say, no expression involving the standard manipulations (summation, multiplication, etc.,) of the variables is available (Fig. 4-1). Instead, experimental data or other considerations permit the variable to be related to another known variable by a graph or table of corresponding values. To permit a voltage corresponding to this variable to be produced in the computer requires the use of a function generator. A number of different schemes exist for generating functions of a voltage. Some are more effective for generating those functions which are commonly required, e.g., the trigonometric functions; others are more flexible and are used to generate functions peculiar to any one study, being capable of quick and simple changes in their settings. Some are electromechanical in their operation, while others are completely electronic.

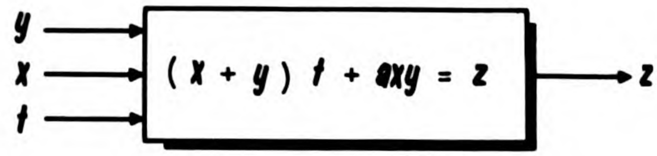
FIXED-FUNCTION GENERATORS

The Resolver

One kind of mathematical function that occurs commonly in the description of physical systems is the trigonometric sine function. The sine of an angle, Θ, is the ratio of the lengths of the side opposite the angle, Θ, and the hypotenuse, of a right-angled triangle. It has values between -1 and $+1$, and for a complete description of the function one needs to consider only those values corresponding to angles between -180° and $+180^\circ$.

Because of its common occurrence in problem descriptions we choose to have available for its generation a fixed-function generator. Called a *resolver*, this unit accepts voltages representing the angle Θ and produces

Some variables are known in the form of an analytical expression of other variables:

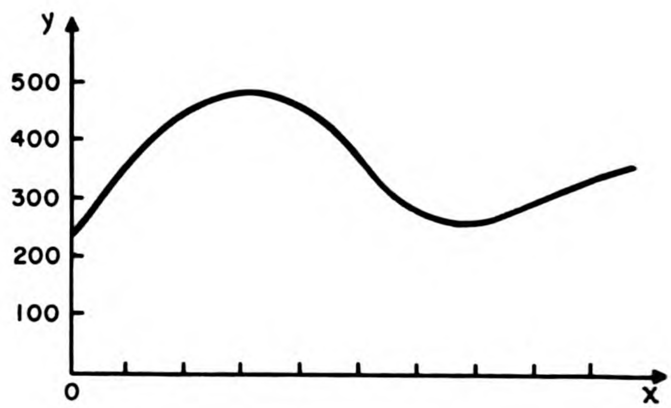


(A)

Other variables are known to be related to a second variable only by a curve or table of values

x	z
1	4
2	7
3	13
4	15
5	12
6	10
etc	etc

OR



These variables are generated on the computer by a function generator

Fig. 4-1 (B)

voltages proportional to sin Θ (Fig. 4-2). It can be looked upon as a reference table to which the computer continually refers for the values of sin Θ in its computations involving this function. The name *resolver* is due to the fact that this function is frequently required in the resolution of polar coordinates into rectangular coordinates and *vice versa*.

Variable-Resistance Potentiometers

The potentiometers used in a servomultiplier are linear, i.e., the voltage at the wiper is proportional to the position of the wiper along the potentiometer. The wiper voltage depends on the current flowing through the

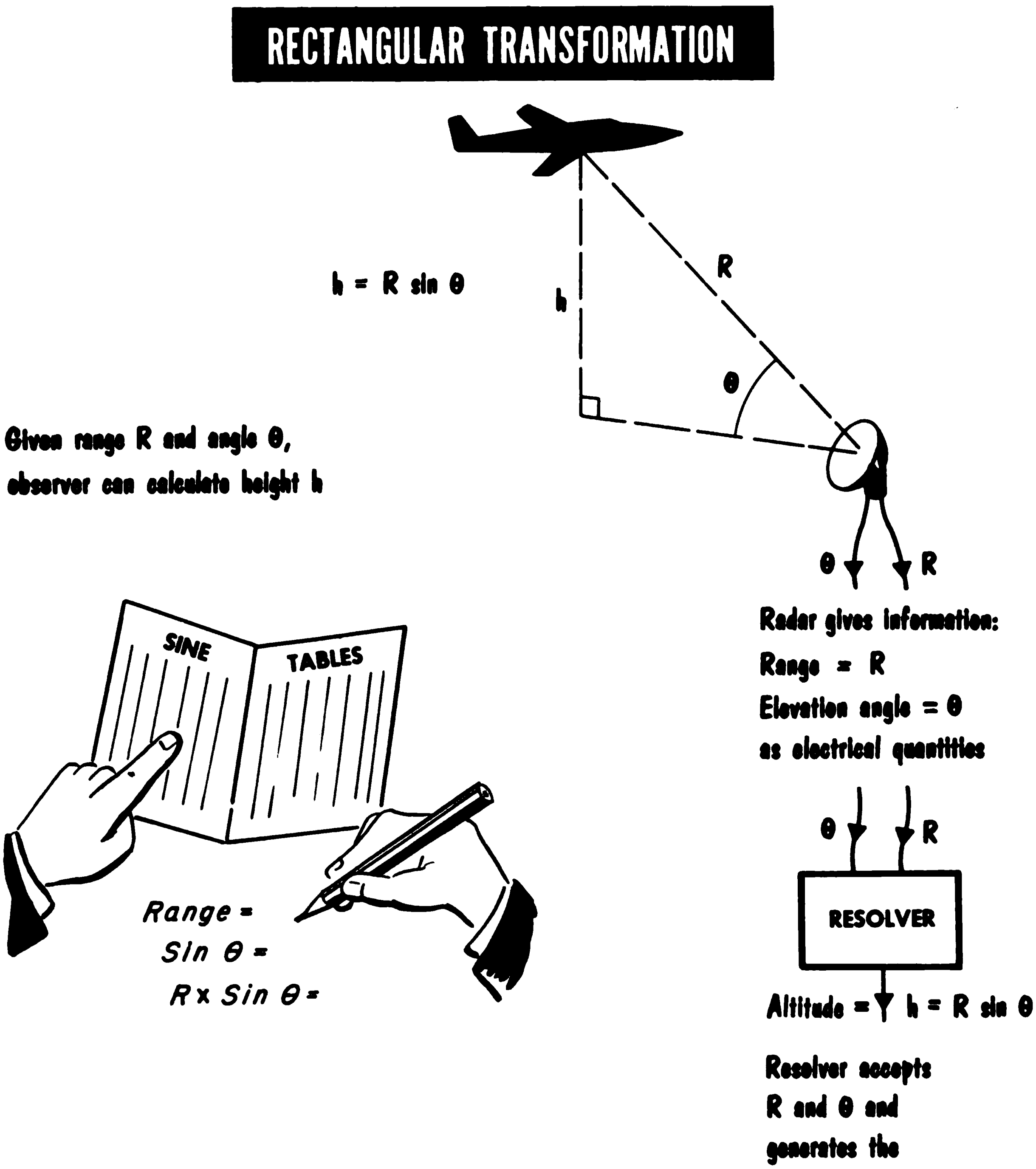


Fig. 4-2

potentiometer and the resistance between the wiper and the ground potential connection. For a constant applied voltage, neglecting loading effects, the current will be constant. Thus for a potentiometer having a constant resistance per unit length, the wiper voltage changes linearly with wiper position. By adjusting the resistance per unit length, a nonlinear output voltage/wiper position characteristic can be obtained.

Specifically in the resolver the resistance wire is wound on a tapered card.

Variable-Resistance Potentiometers

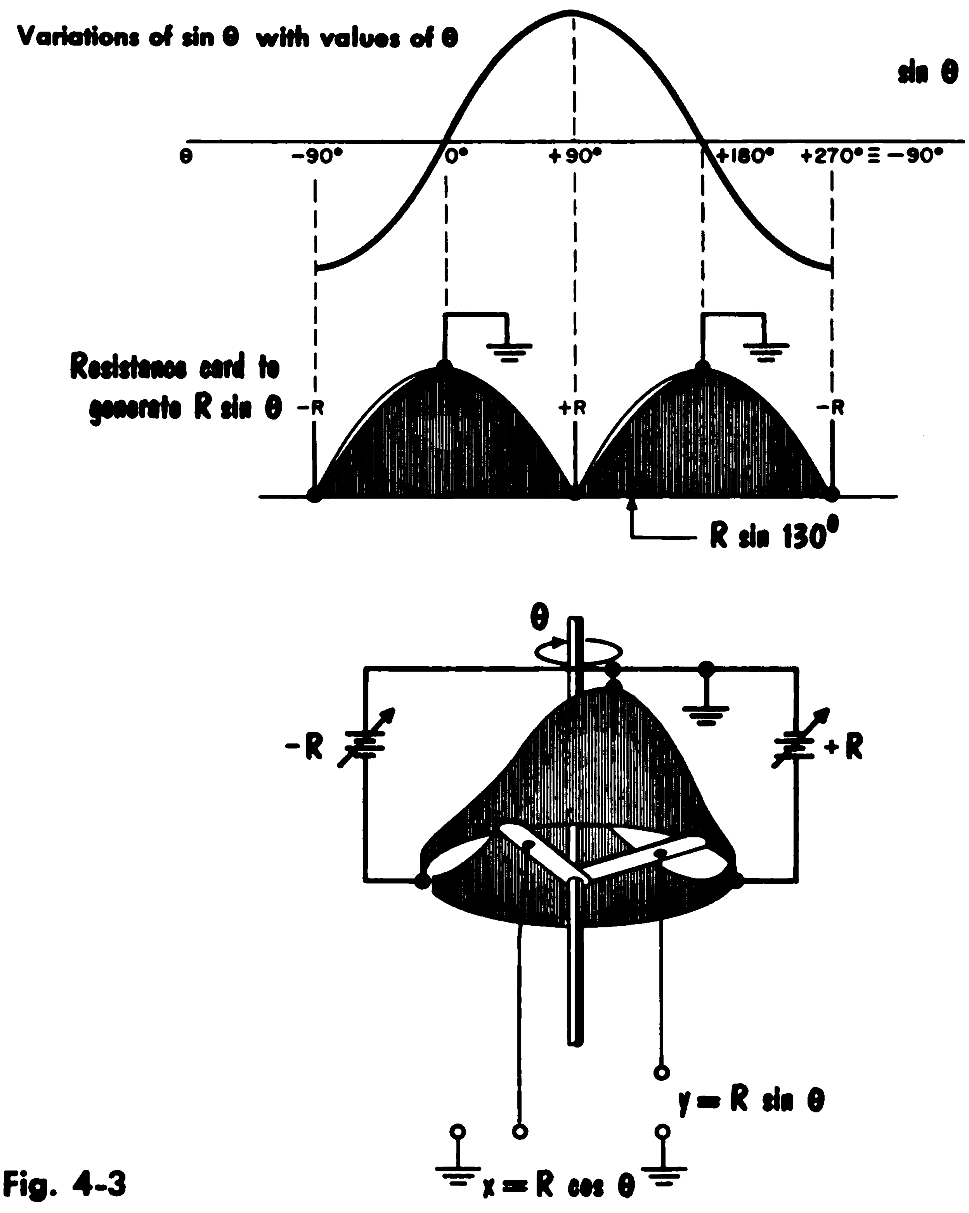


Fig. 4-3

Where the card is shallow there is a small resistance per unit length, and thus a small change in voltage with wiper position. Where the card is deep the resistance per unit length is large, and large changes in voltage with wiper motion occur. This method for producing a nonlinear relationship between wiper position and output voltage could be used in generating any function. For only the most common functions, however, is the cost of winding the special cards worthwhile.

The diagram (Fig. 4-3) shows a sine card. The card is placed around a drum

and the wiper, riding on the edge of the card, is on an arm rotating about the center of the drum. The two ends of the card are joined, and a continuous output voltage function is possible for any value of wiper position Θ. A second wiper displaced from the first by 90° produces sin (Θ + 90°) which equals the function, cos Θ. At the appropriate points of the continuous potentiometer, +R, and −R volts, and ground potential are applied, where R can have any value in the range $-100 < R < +100$. Then the two output voltages from the resolver will be R sin Θ and R cos Θ — the rectangular coordinates of a point whose polar coordinates are R, Θ. Observe that a resolver not only produces the trigonometric functions of the input variable, Θ, but also multiplies them by a second input variable, R.

Polar Resolution

At times, it is necessary to compute from the rectangular coordinates x, y of a point, the polar coordinates R, Θ (Fig. 4-4A). The geometry provides the following relations:

$$R = x \cos \Theta + y \sin \Theta$$

$$\Theta = x \sin \Theta - y \cos \Theta$$

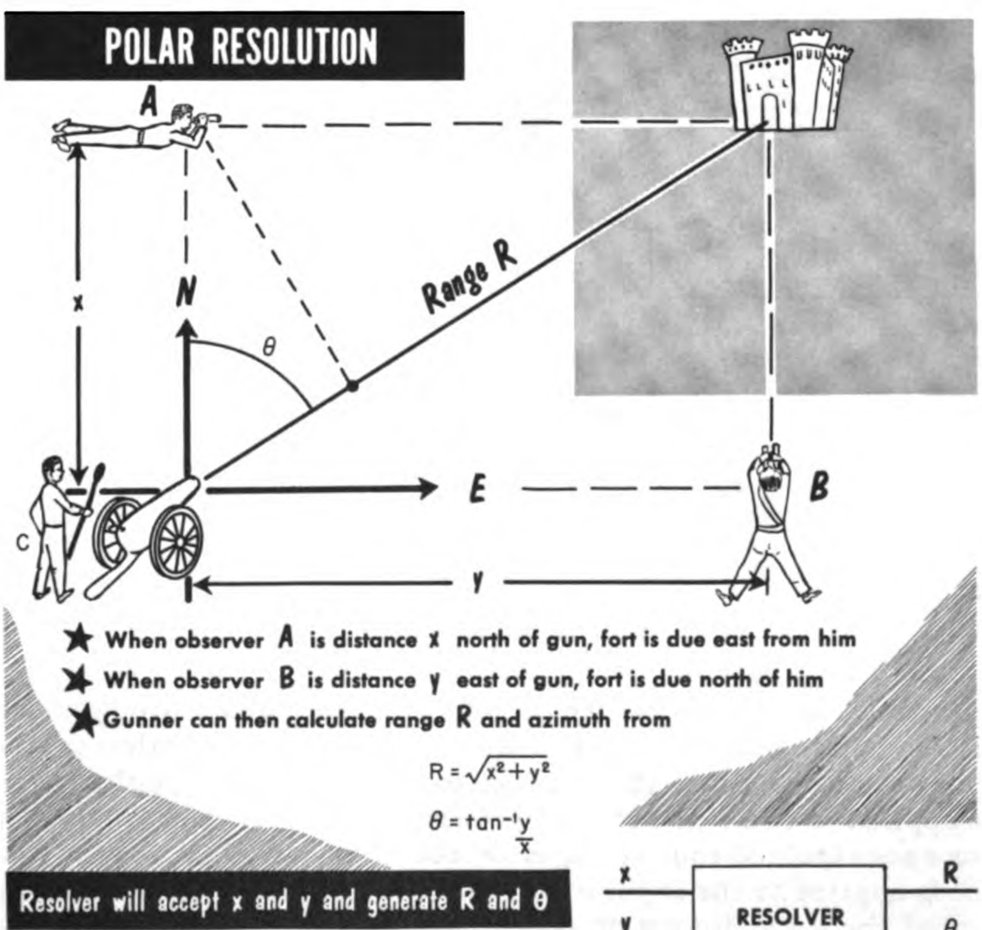


Fig. 4-4A

VOLTAGES $\pm x$ and $\pm y$ are APPLIED to TWO SINE POTENTIOMETERS of a RESOLVER to obtain VOLTAGES R and θ

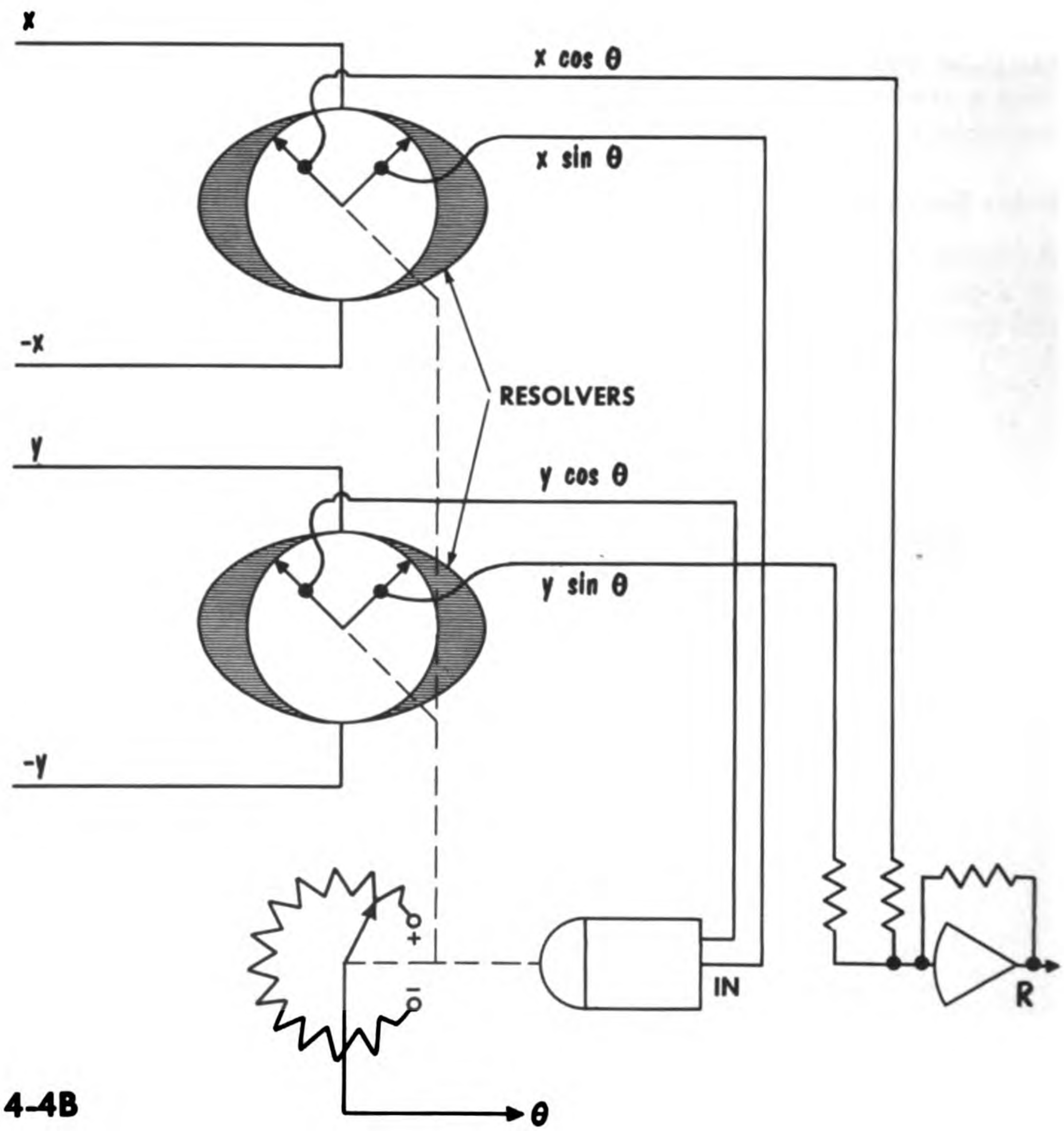


Fig. 4-4B

Unfortunately, in this particular computation no voltage proportional to Θ is available to position a resolver. However, the second relation can be used as a null relation to drive the resolver to the position Θ. Voltages $+x$ and $+y$ are applied to two sine potentiometers of a resolver, and from the potentiometers are obtained four voltages: $x \cos \Theta'$, $x \sin \Theta'$, $y \cos \Theta'$ y *sin* Θ'. $x \sin \Theta'$ is applied to the input of the resolver servo, and y *cos* Θ' is applied in place of the usual follow-up potentiometer connection. These two voltages will not null unless $\Theta' = \Theta$ (Fig. 4-4B). The servo therefore drives to

position the resolver to the correct angle Θ, at which condition a voltage equal to R is obtained by summing $x \cos \Theta$ and $y \sin \Theta$. A voltage proportional to Θ is obtained from the follow-up potentiometer which is no longer required by the position servo.

Automatic Gain Control

In the computation of the polar coordinates from rectangular coordinates, the servo error voltage is given by:

$$\epsilon = -x \sin \Theta' + y \cos \Theta'$$
$$\epsilon = -R \cos \Theta \sin \Theta' + R \sin \Theta \cos \Theta'$$
$$\epsilon = R \sin (\Theta - \Theta')$$

For small values of $(\Theta - \Theta')$, and we hope that this is always very small for otherwise we would obtain poor computation, $\sin (\Theta - \Theta')$ can be approximated very closely by $(\Theta - \Theta')$. Thus the servo error voltage is proportional to $R (\Theta - \Theta')$. Ideally, for good servo operation, this error voltage should be independent of R and simply proportional to $(\Theta - \Theta')$ (Fig. 4-5A). Dependence on the magnitude of R causes the servo to be sluggish

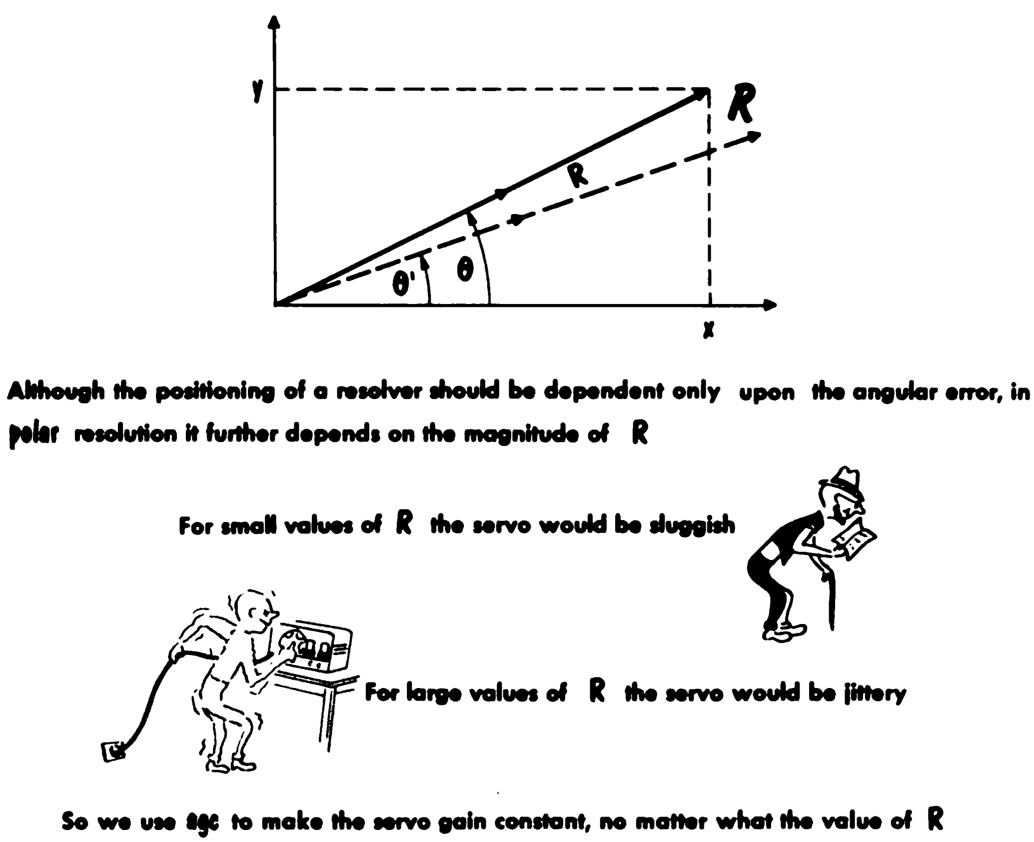


Fig. 4-5A

for small values of R, oscillatory for large values. To overcome the dependence of R in this particular use of the resolver, the gain of the servo amplifier is made inversely proportional to R. An agc circuit is incorporated which decreases the available gain when R exceeds an appropriate level, so that the servo will not become oscillatory. When not used in this manner the resolver servo has a constant gain.

A typical agc circuit is shown in Fig. 4-5B. Using a variable reactance tube in the coupling circuitry between the first and second stages of amplification, the gain of the amplifier will be controlled by the effective plate resistance of this tube. This in turn depends upon the screen potential of the tube, which is made dependent on the magnitude of R. As R increases, the

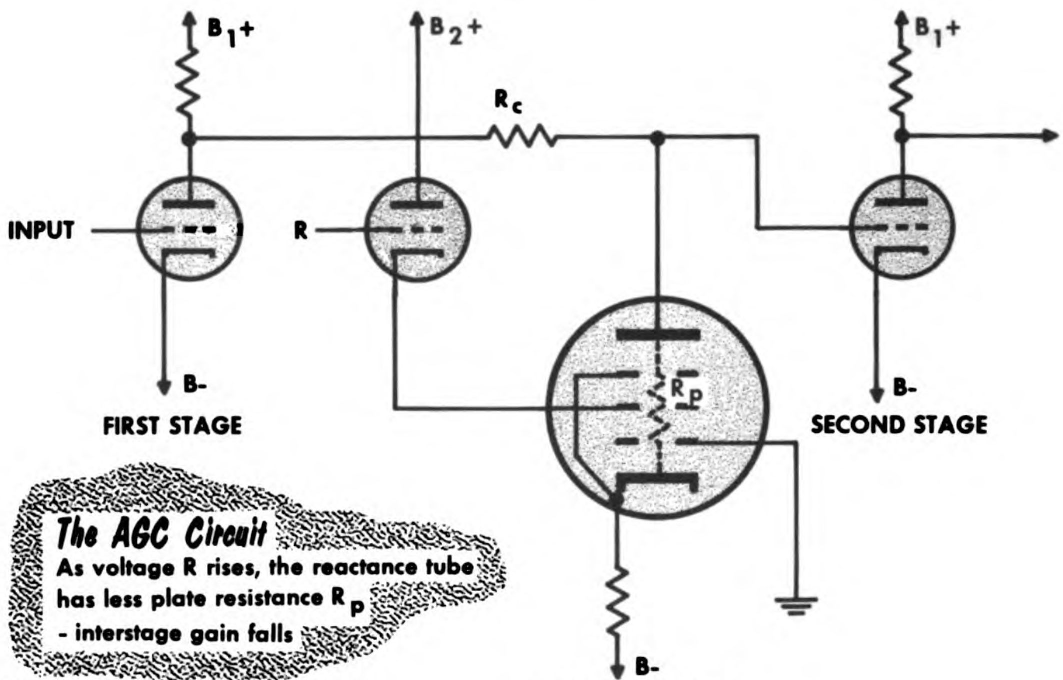


Fig. 4-5B

screen potential rises, reducing the effective plate resistance of the tube, and thereby reducing the gain of the amplifier. As R decreases, the screen potential falls, and the amplifier gain increases correspondingly.

Loading Error

Being a potentiometer device, the resolver is susceptible to the same kinds of loading errors that have been discussed previously. Current is drawn from the wiper disturbing the voltage ratio of the potentiometer. However, it is no longer possible to compensate for the loading error by applying an equal load to the follow-up potentiometer (Fig. 4-6A). The sine potentiometer is a nonlinear potentiometer, the follow-up potentiometer is linear, and correct compensation is no longer possible by the method discussed earlier.

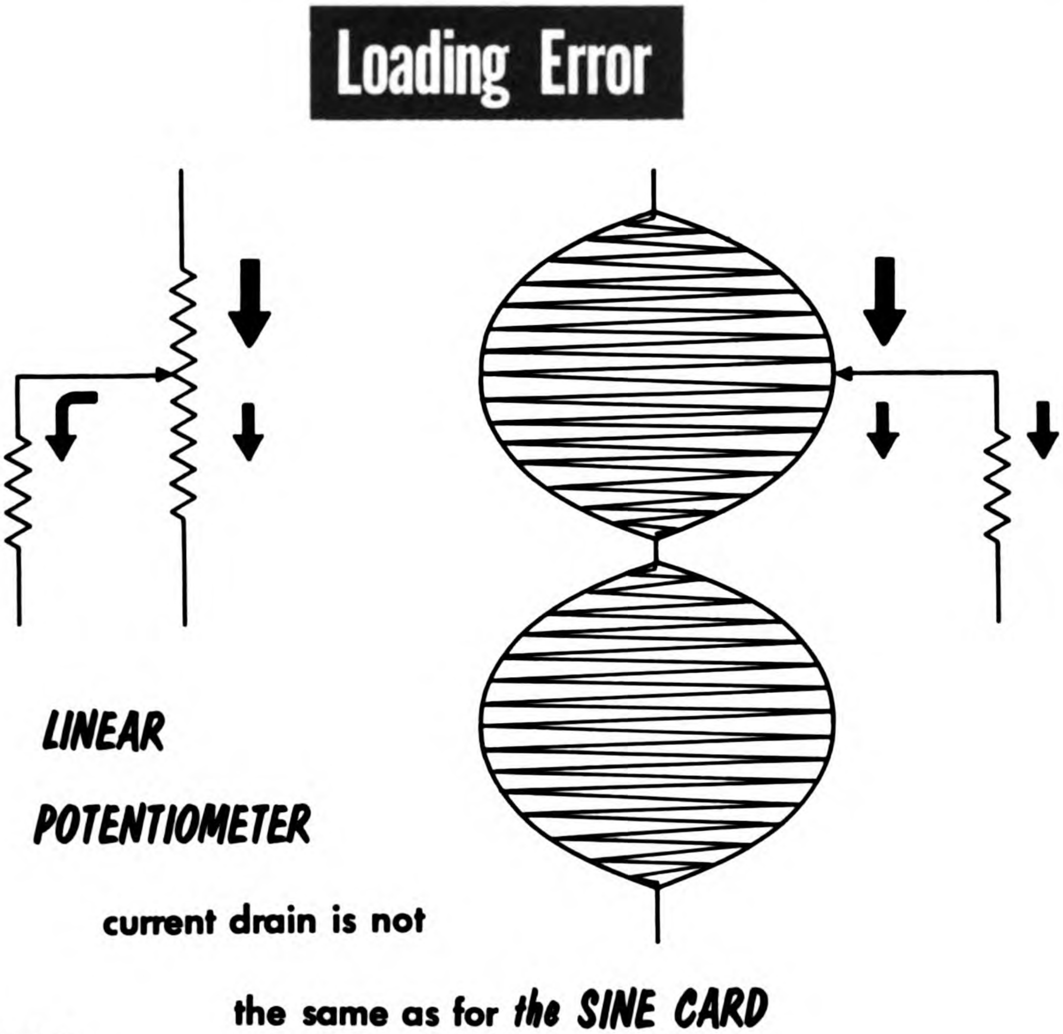


Fig. 4-6A

Instead, the sine potentiometer is wound to produce a correct output voltage when feeding a given load, usually 1 megohm, to ground potential, for this is the load presented by the input resistor of an amplifier with a unity gain. No resistor is connected to the follow-up potentiometer, and for accurate computations the output connections of the sine potentiometers must go to the loads prescribed by the designer of the device.

In using a resolver for polar resolution the voltage representing Θ is obtained from the follow-up potentiometer. An unloading circuit is thus required if the voltage for Θ is to be accurate, as the wiper voltage is only proportional to Θ when no current flows through it. An unloading circuit is shown in Fig. 4-6B.

The current flowing through the 1-megohm input resistor of amplifier A is supplied mainly from the low-output impedance of amplifier B, rather than from the follow-up potentiometer. This kind of circuit is frequently useful.

UNLOADING CIRCUIT used to OBTAIN VOLTAGE θ in POLAR RESOLUTION without loading follow-up potentiometer

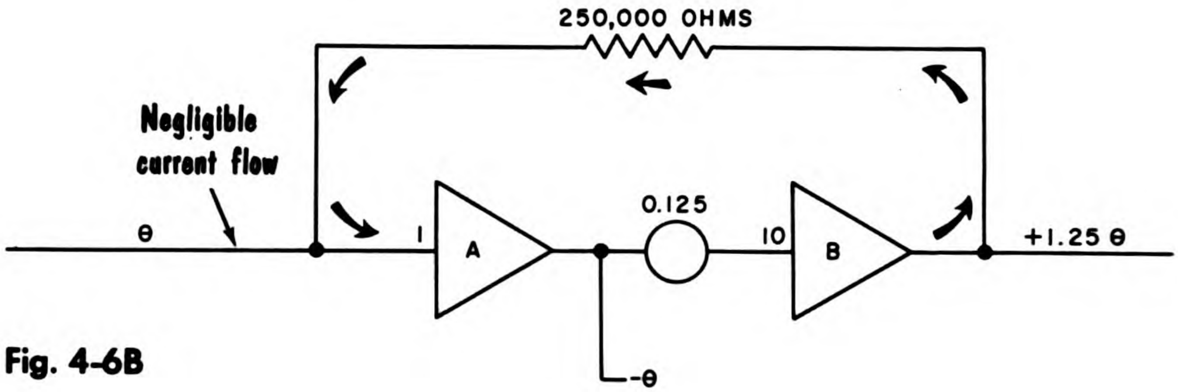


Fig. 4-6B

VARIABLE FUNCTION GENERATORS

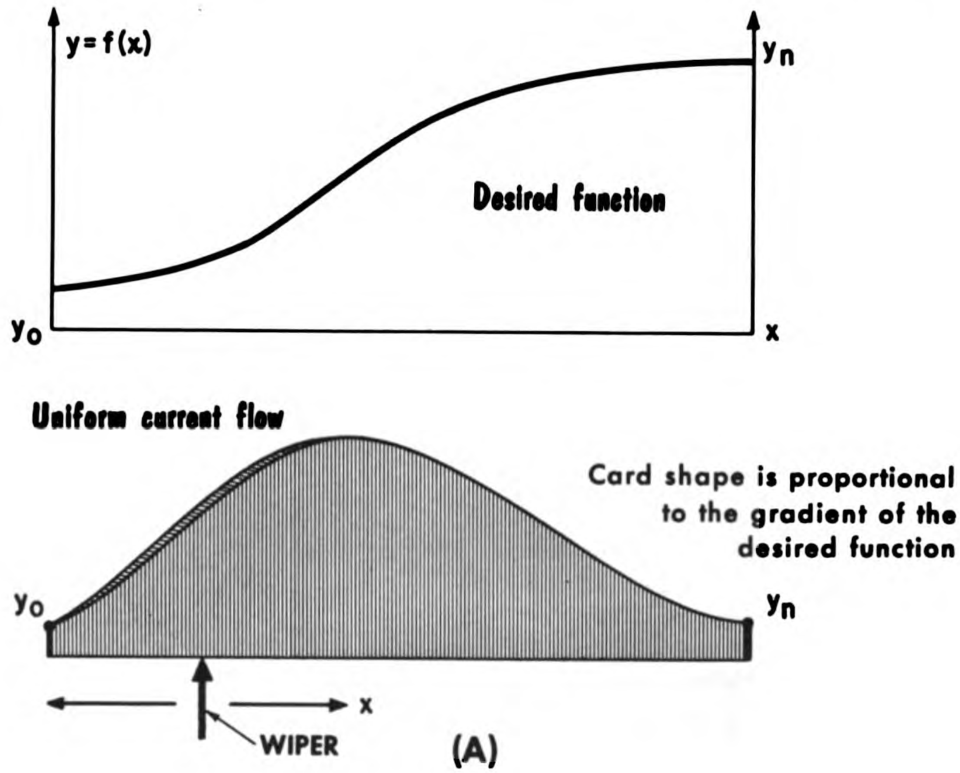
Tapped-Potentiometer Function Generator

In the resolver, a fixed sinusoidal function of the input driving voltage is produced by using a potentiometer whose resistance per unit length is not constant, but changes appropriately from point to point. The voltage sensed by the wiper, being dependent on both the current flowing through the potentiometer and the resistance between the point of contact and the end where a fixed voltage is applied, is thus a nonlinear function of the driving voltage [Fig. 4-7 (A)]. The unit requires the construction of a special precision-wound potentiometer, and consequently such a technique for nonlinear function generation is desirable only for commonly-occurring functions such as sine and cosine. It is not appropriate in producing a voltage required to satisfy some peculiar experimentally-derived relationship to some other varying quantity, but if we are not to vary the resistance of the potentiometer we have to arrange a variation in the current so that the output voltage will be a nonlinear function of the wiper position. This type of variation is made possible in the tapped-potentiometer function generator [Fig. 4-7 (B)].

To each of a number of equally-spaced points along a linear potentiometer, connections are tapped. These permit the segments into which the potentiometer is divided to carry different currents as determined by the voltages externally applied to the taps. As the potentiometer wiper is positioned linearly to correspond to an input voltage, it senses a voltage which is that function of the wiper position, and thus the input voltage, determined by the arranged currents. In this way an approximation to any reasonable, smooth function can be generated.

Tapped-Potentiometer Function Generator

A potentiometer with varying resistance could be wound for each arbitrary function



ALTERNATIVELY, and MORE DESIRABLY, the current flowing in the potentiometer can be made to change from point to point

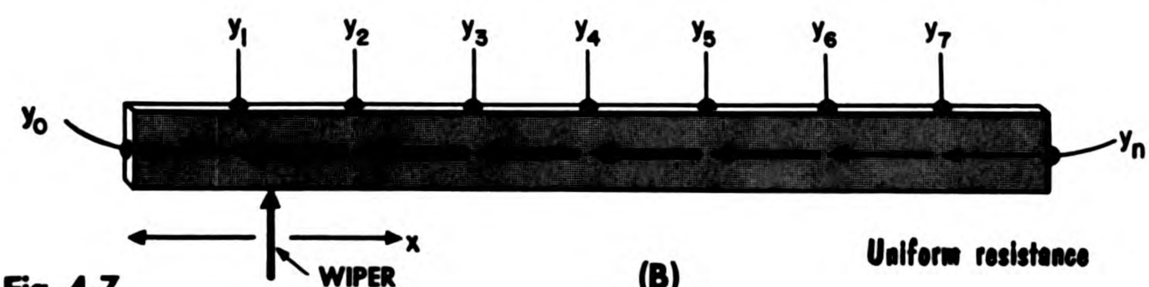


Fig. 4-7

Straight-Line Approximation

Considered from a somewhat different viewpoint, the tapped potentiometer permits for certain equally spaced values of the input variable, the production of quite accurate values of the output variable. These values are those impressed onto the potentiometer through the tapped connections, from the

The TAPPED-POTENTIOMETER FUNCTION GENERATOR produces a STRAIGHT-LINE APPROXIMATION to the FUNCTION

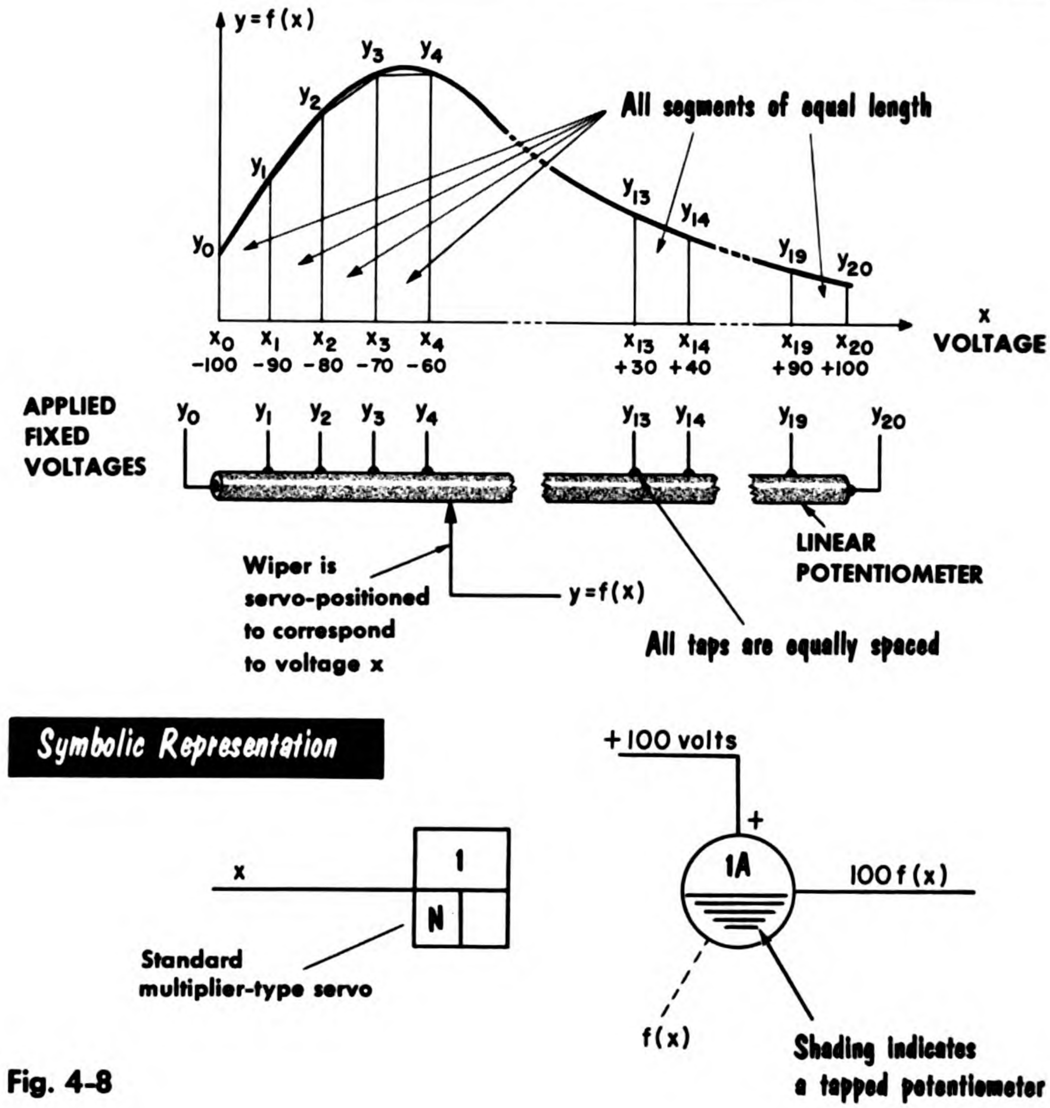


Fig. 4-8

external constant voltage sources (Fig. 4-8). Between the taps the output voltage will change linearly from the value at one tap, to that at the next. As the input voltage changes over its entire range of values, the output voltage changes along a sequence of straight lines. We obtain a straight-line approximation to the desired nonlinear function which will be sufficiently accurate for the representation of most experimental data, provided the functional changes are not too rapid. It is usual to have more than a dozen taps on a function-generating potentiometer, allowing a generous number of changes in slope in the straight-line approximation.

The potentiometer is usually one of those on a servomultiplier, and the wiper will be positioned by the servo. If there are 21 taps on the potentio-

meter (20 segments) and the wiper moves from one tap to the next, the input voltage has to change by 10 volts. Each tap corresponds to an input voltage lying between −100 volts and +100 volts, and equal to some multiple of 10. The loading error that could exist in the use of this device cannot be compensated by repositioning the wiper as was possible in the use of the servo-multiplier. The nonlinear nature of the output voltage requires any loading corrections to be achieved in the setting of the tap voltages.

Adjusting the Generated Function

To adjust a tapped-potentiometer function generator so that the output voltage from the wiper corresponds to the voltage positioning the wiper

Adjusting the Generated Function

In adjusting a tap voltage the neighboring voltages must be correct

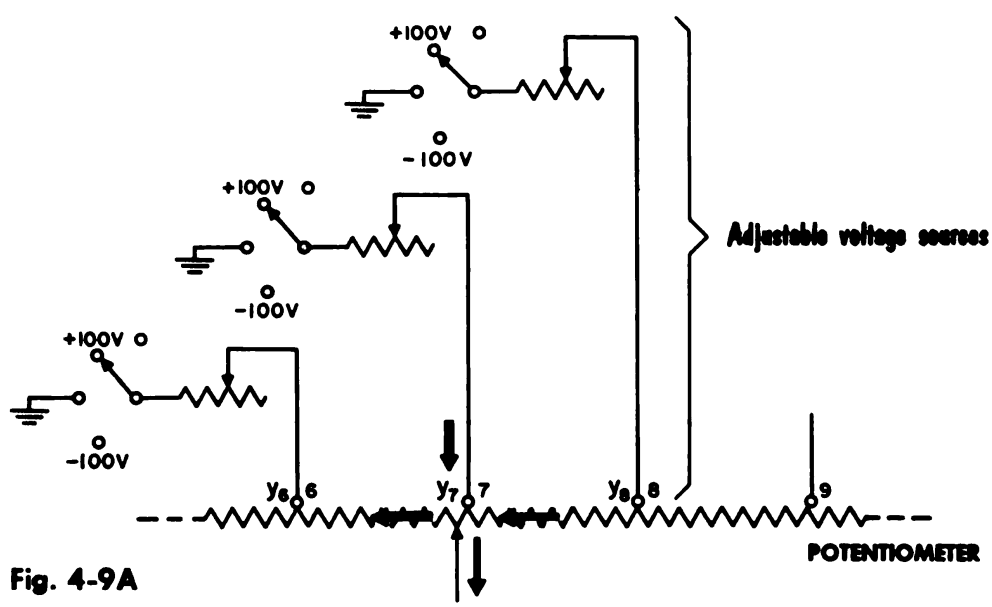


Fig. 4-9A

according to the required nonlinear function, one needs to change the applied tap voltages. Their variations are easily effected by rheostats, one to each tap which provides a variable resistance in the line from the reference-voltage source. To allow for a need for both positive and negative output voltages, each rheostat can be connected to either +100 volts, −100 volts, or ground potential.

The adjustment of each tap voltage must be performed with care, and according to an appropriate procedure, if the accuracy of the function generation is to be maintained. It is necessary to recognize that with the form of voltage adjustment provided, the currents flowing in the unit and thus the voltage produced at a tap, depend on factors other than the setting of the rheostat (Fig. 4-9A). For example, the setting of the voltage at tap 7 de-

pends on, (1) the adjustment of rheostat 7, (2) the voltages at taps 6 and 8 due to their influence on the currents flowing in the segments 6-7 and 7-8, and (3) any loading current drawn through the wiper when it is positioned at tap 7. (The accuracy of the voltage at tap 7 is only of concern when the wiper is positioned there.) When adjusting rheostat 7 to provide the correct voltage at tap 7, the correct operating voltages must exist at taps 6 and 8, and a correct load current must be drawn from tap 7 through the wiper or an equivalent path. For the unit to be useful, the procedure for making the voltage adjustments and checking them must be simple. The circuitry necessary is exemplified in Fig. 4-9B.

Voltage adjustments begin at tap 1 and proceed in succession along the potentiometer, each tap being set to the voltage appropriate to its correspondence to that value of input voltage determined by its position (+69

CIRCUIT ARRANGEMENT for simple adjustment of tap voltages

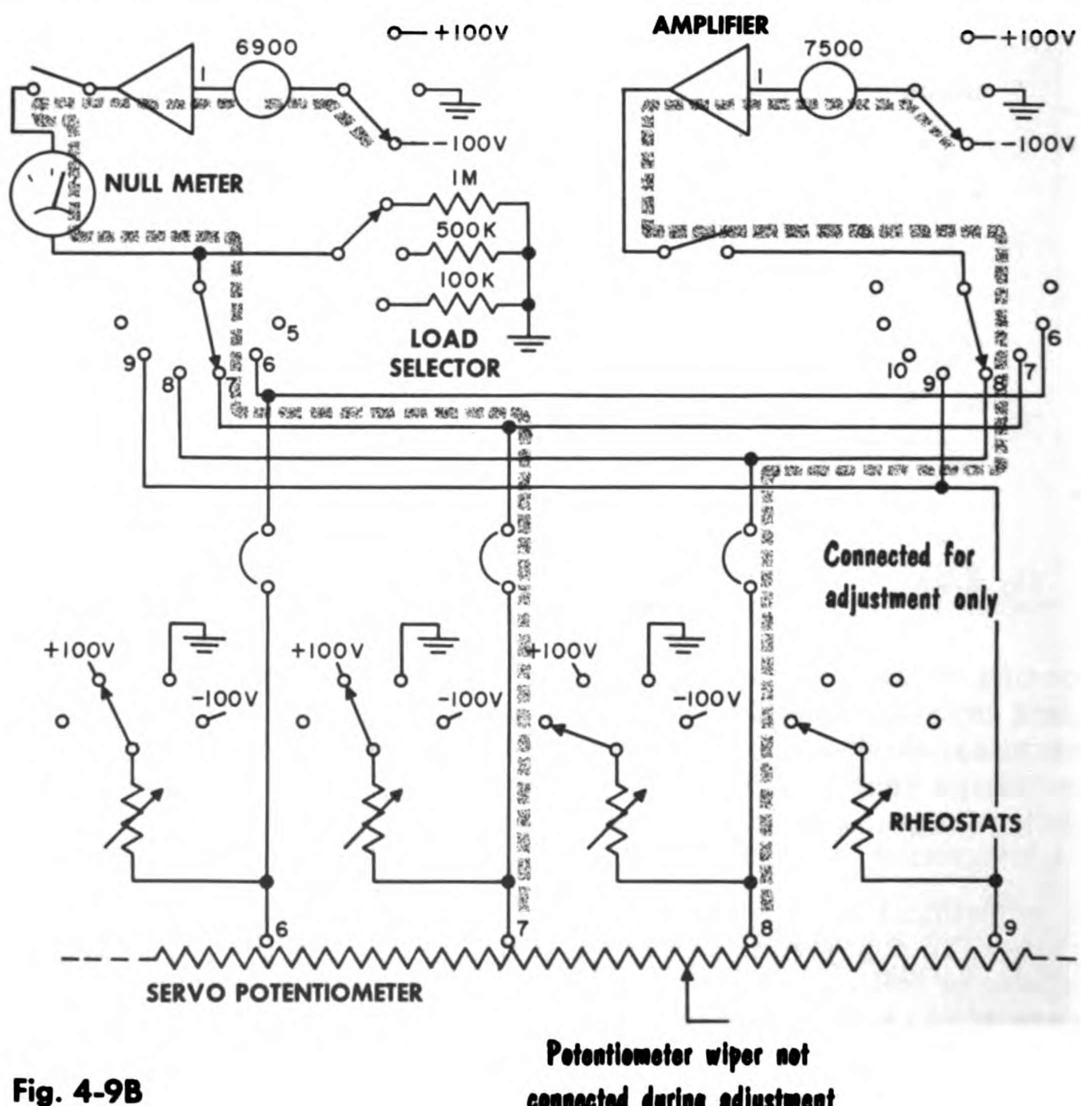


Fig. 4-9B

volts to tap 7 for $y = +69$ when $x = -40$). When setting tap 7, tap 6 has already been correctly set, tap 8 is held to the required voltage by the output of an amplifier, and an appropriate loading current is drawn from tap 7. Thus rheostat 7 is adjusted until the voltage at tap 7 reaches 69 volts. At this condition all the currents flowing towards or away from 7 are as they will be in problem operation. Thus the adjustments are correct. The amplifier can now be moved to tap 9, the loading connection and monitoring device to tap 8 to adjust rheostat 8.

The Computing Features of a Tapped-Potentiometer Function Generator

The tapped potentiometer is a reliable, relatively cheap, and easy to use function generator. It is quite commonly used to generate the more simple

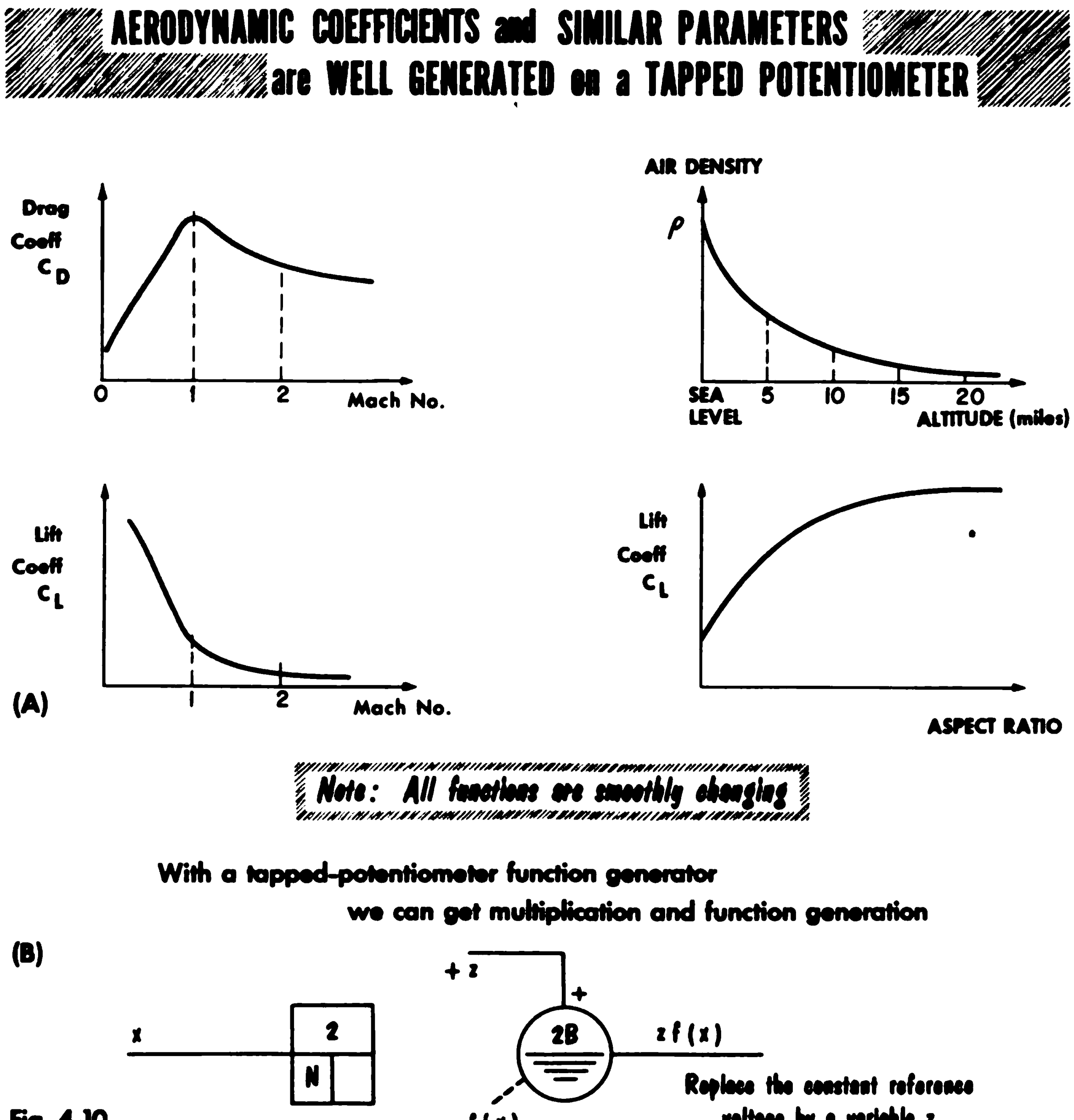


Fig. 4-10

forms of nonlinearities present in many models that require the use of experimental results. The aerodynamic coefficients that are determined experimentally and contribute significantly to the study of high-speed aircraft or missile stability are well generated with this type of device [Fig. 4-10 (A)]. By replacing the reference voltage connected to the rheostats by a varying voltage, $+z$ say, one can both generate the function $f(x)$ and with the same unit multiply by z to obtain the product, $zf(x)$ [Fig. 4-10 (B)].

ROTATING-DRUM FUNCTION GENERATOR

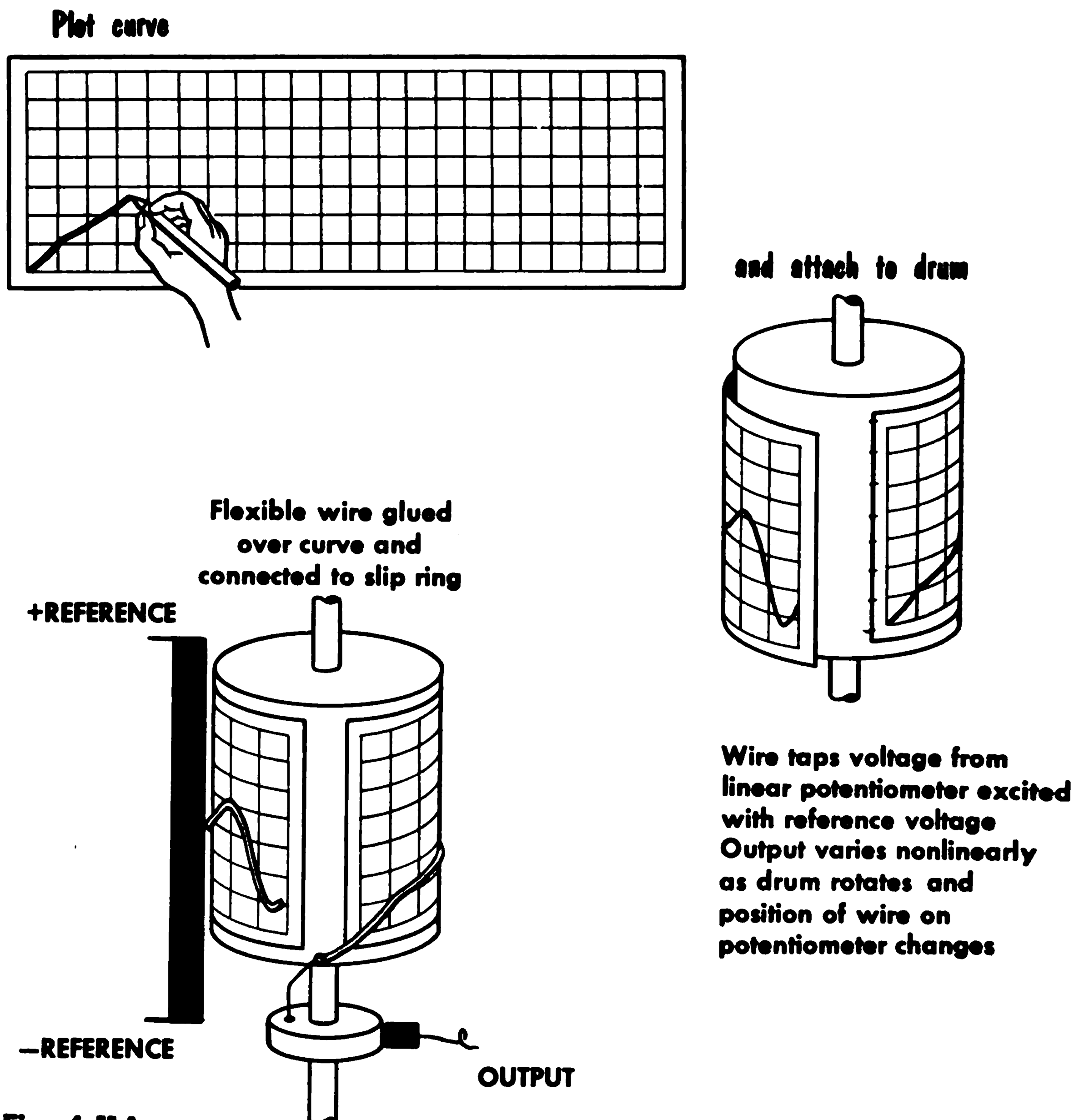


Fig. 4-11A

Any undesirable feature of the tapped potentiometer's operation arises from its electromechanical nature. The variable driving the servo cannot be allowed to change too rapidly, and thus there is a limit to the speed of solution possible when this kind of function generator is present. The fixed, equal-segment nature of the tapping, places a restriction on the irregularity of function that can be generated. Further, the current-carrying capabilities of the fine wire in the potentiometer sets a limit upon the difference of voltages to be applied to adjacent taps, and consequently upon the slope of the function between adjacent *break-points*.

Curve-Follower Function Generator

The resolver and tapped-potentiometer function generator produce a nonlinear output–input relation by operating on the potentiometer. The wiper is still positioned linearly with respect to the input voltage variable. It is obvious that an alternative arrangement would be to position the wiper nonlinearly and retain a linear potentiometer. Two practical examples have been used sufficiently to make their description worthwhile.

1. A curve of the required relationship between the input and output voltages is plotted to a suitable scaling on the surface of a nonconducting cylinder (Fig. 4-11A). Along the curve is glued a flexible wire and this forms the wiper of the linear potentiometer which is fixed parallel to the axis of the cylinder. The cylinder can be rotated relative to the potentiometer, and positioned to correspond to the input variable voltage x. The output is taken from the wire on the cylinder, the voltage $f(x)$ varying nonlinearly as the contact with the potentiometer changes.

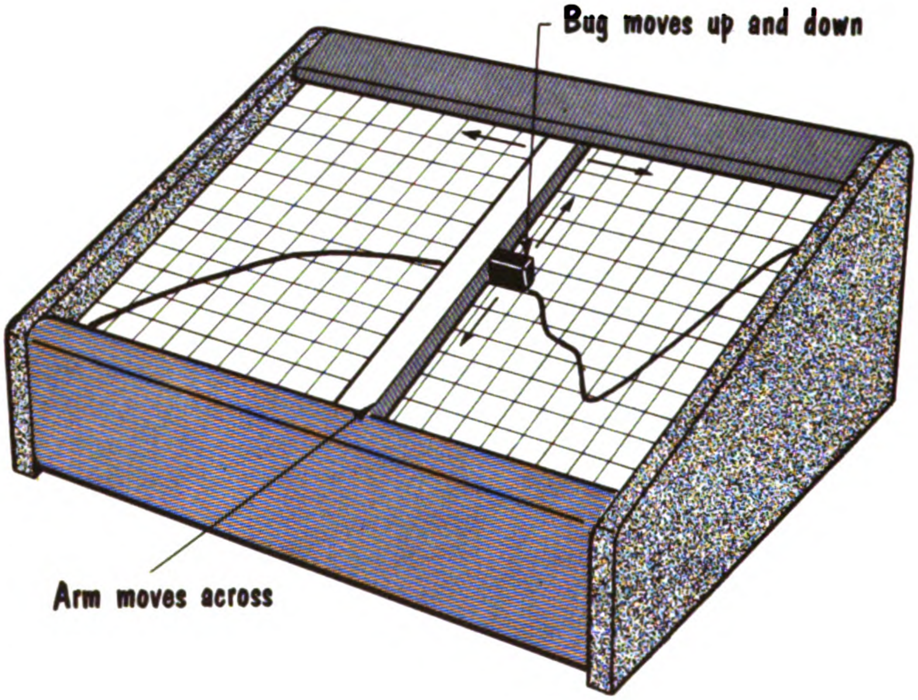
2. In the second device the curve is drawn on the flat surface of a plotting table (Fig. 4-11B). Either by the use of conducting ink or a wire, the curve is made electrically conducting and energized with a high-frequency current, thus causing the line to act as an antenna. A servo system positions a sensing "bug" over this curve, and as the arm of the device is positioned to correspond to the input variable voltage, the bug moves along the arm, following the position of the strongest high-frequency signal. Attached to the bug is the wiper of a linear potentiometer, and from this wiper the generator's output is taken. In both devices any correction of the loading error must be applied before drawing the curves.

The Diode Bridge

Consider the simple circuit shown in Fig. 4-12, part (A), and let us investigate the behavior of the potential difference $(e_A - e_B)$ as the input voltage e_i increases from a value of -100 volts. Initially, the diode will be cut off, for the plate is more negative than the cathode. Under this condition the potential at any part of the circuit below the diode will be equal to e_i, for no current can flow in the circuit. $(e_A - e_B)$ will be zero. When e_i reaches a value slightly more positive than E_o, the diode will begin to conduct, and then the current flowing through the circuit will cause e_A and e_B to be

PLOTTING-TABLE type FUNCTION GENERATOR

Again, draw a curve. Place the curve on flat plotting table and glue wire in position on curve



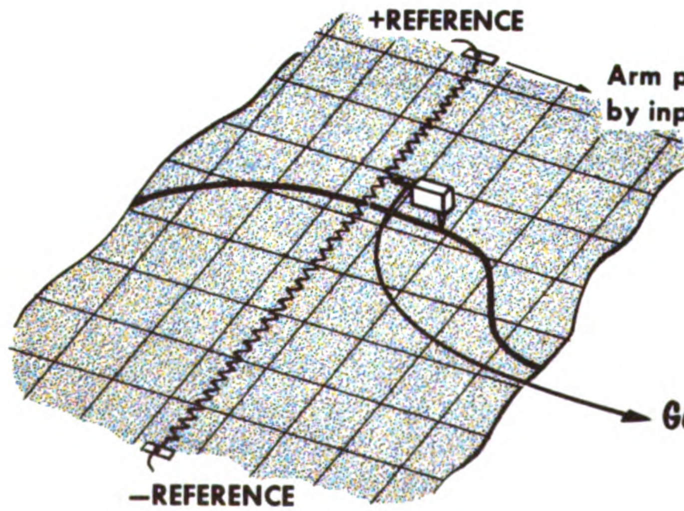
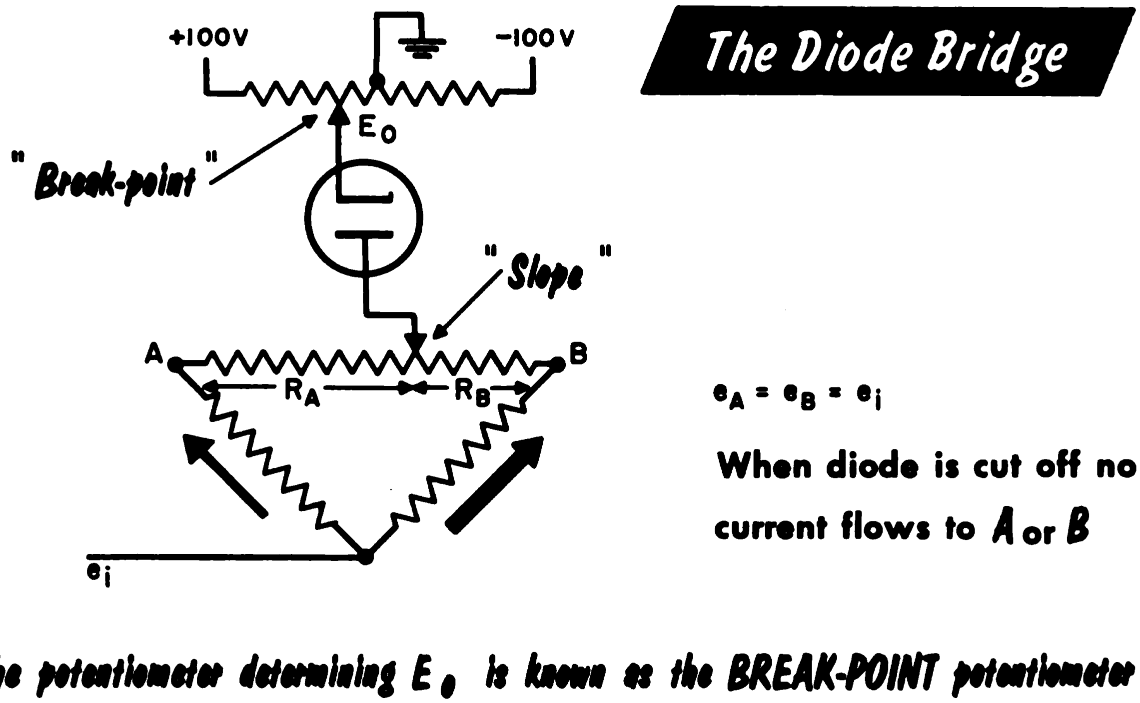


Fig. 4-11B ***PORTION OF CURVE WITH ARM AND BUG POSITIONED***



(A)

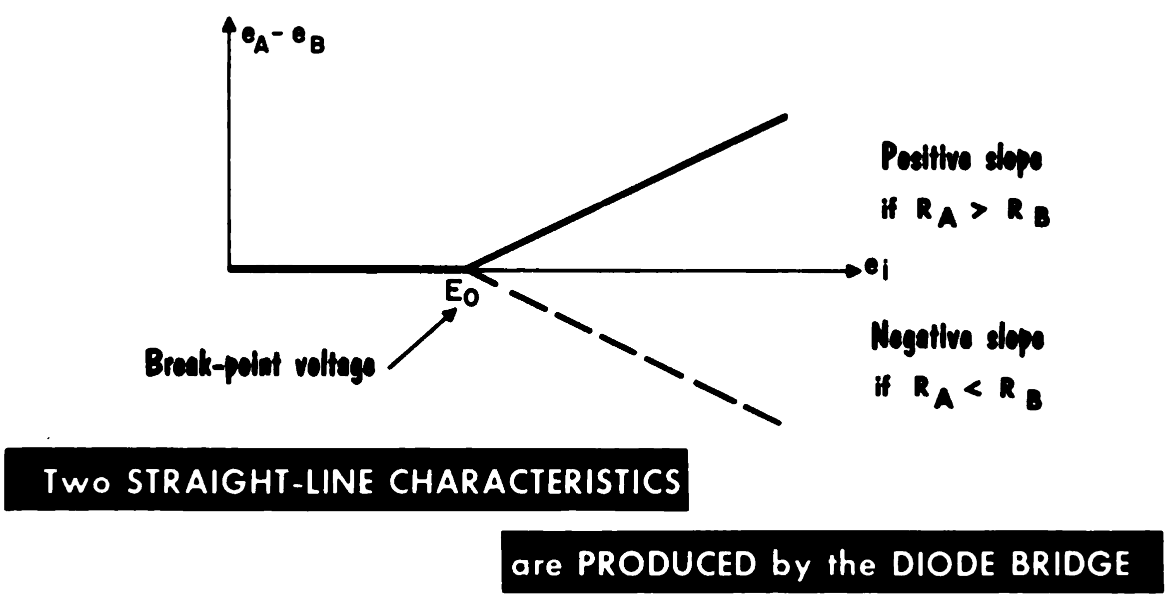
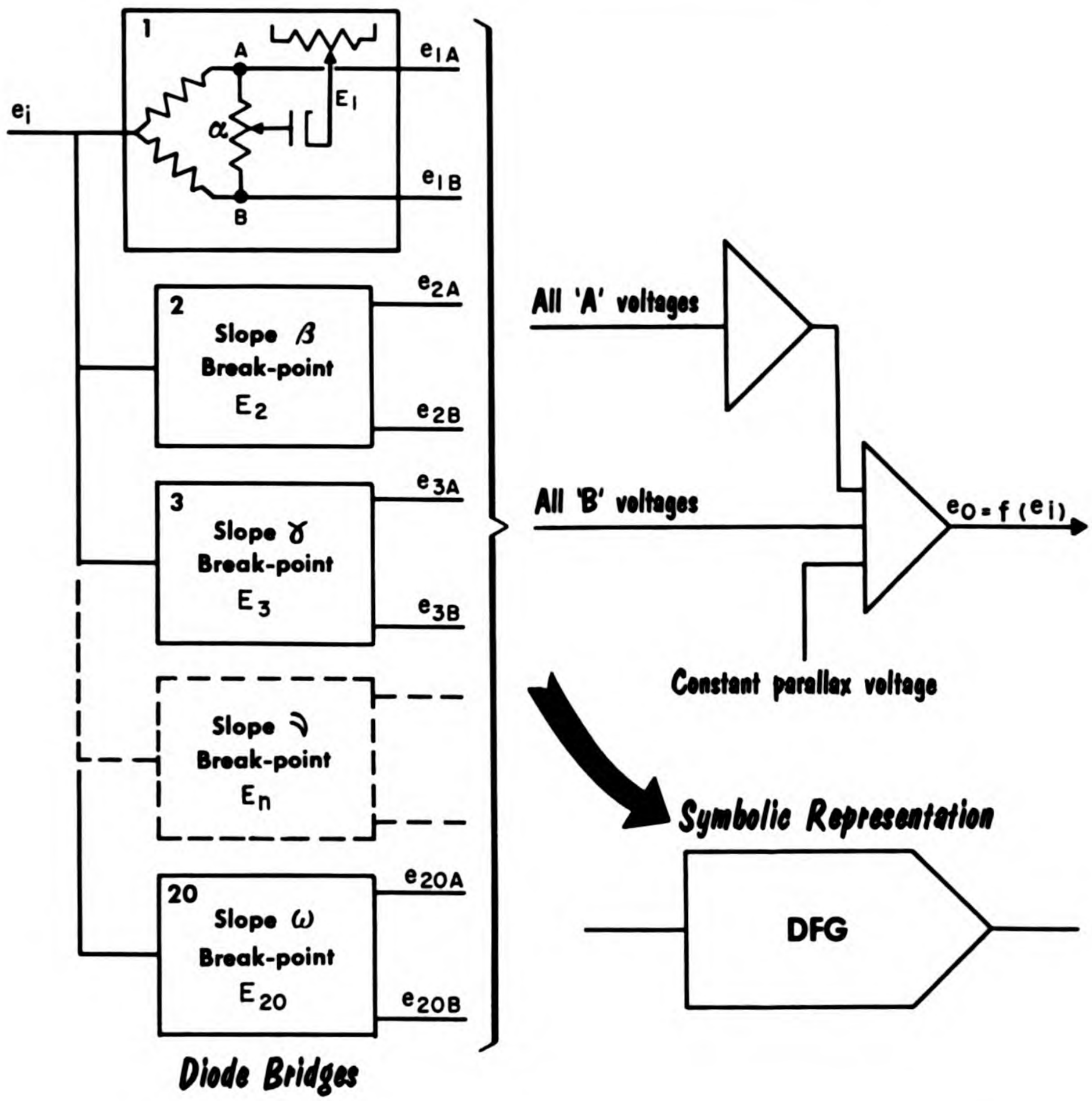


Fig. 4-12 (B)

different from e_i. If R_A is greater than R_B more current flows past point *B* than point *A*, and therefore $(e_A - e_B)$ will begin increasing positively in proportion to $(e_i - E_o)$. The potential difference $(e_A - e_B)$ will continue to change linearly with the increase in e_i [Fig. 4-12 (B)].

The potentiometer determining E_o is known as the *break-point* potentiometer. It can be set to make E_o any value in the range $-100 < E_o < +100$.

ALL-ELECTRONIC ADJUSTABLE SEGMENTS



By placing many diode bridges in parallel, a close approximation to a function can be obtained

Fig. 4-13A

The potentiometer setting that fixes the values of R_A and R_B determines the change in ($e_A - e_B$) for a given change ($e_i - E_o$). It is therefore known as the *slope* potentiometer. Note that ($e_A - e_B$) can change positively or negatively for a positive change in ($e_i - E_o$), depending on whether R_A is greater than or less than R_B.

The Diode-Function Generator

By placing in parallel a number of diode bridges the characteristics of each can be added. Thus as e_i increases from a value of -100 volts, the diodes will begin conducting in turn, the order and the break values of e_i depending on the values of the voltages set on the break-point potentiometers (Fig. 4-13A). Each bridge will contribute to the output of the device a voltage which changes linearly after the bridge diode begins conducting. The slope of each contributing voltage depends on the setting of the corresponding slope potentiometer.

At any desired value of e_i a bridge can be made to begin contributing to the output by an appropriate setting of its break-point potentiometer. The contribution can be made to change the resulting output voltage slope by any amount within the capabilities of the slope potentiometer. In this way a straight-line approximation can be built to approximate any given non-linear relationship.

The diode-function generator has the advantage over other function gen-

SEGMENTS of the DIODE FUNCTION GENERATOR are COMPLETELY ADJUSTABLE

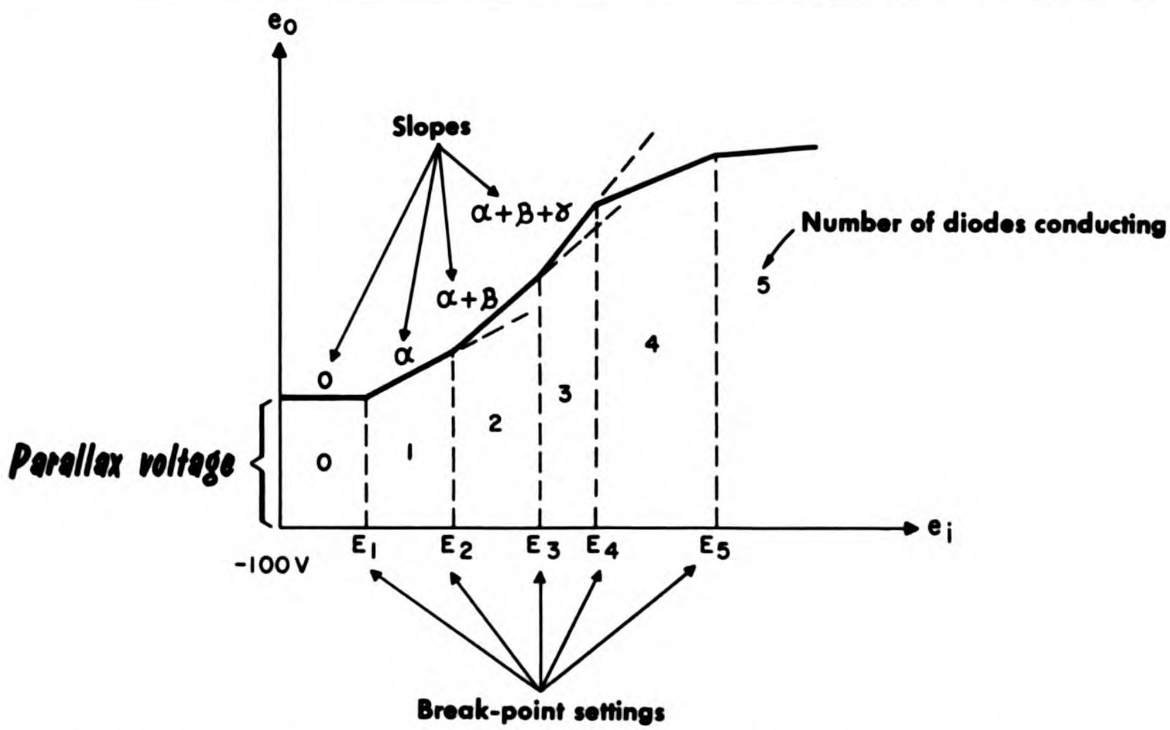


Fig. 4-13B The diodes break at different values of e_i and contribute to the output voltage over different ranges

erators discussed here, of being an all-electronic device. It has a wide bandwidth, and for all practical purposes there are no restrictions placed on the allowed rates of change of the input voltage. The output correctly follows the input without any delay. A further advantage of the diode-function generator is the control one has over the positioning of the straight-line segments. In contrast to the fixed equal segments of the tapped potentiometer, those of the diode function generator are completely adjustable, permitting the more accurate generation of sharply-changing, but unbalanced functions similar to that shown in Fig. 4-13B.

QUESTIONS

1. Explain the operation of a resolver. Why is automatic gain control required?
2. Why is the unloading of a resolver potentiometer not the same as that of a multiplying potentiometer?
3. What is meant by polar resolution, given an analog computer circuit to achieve this operation?
4. Why do we require variable function generators? Give some examples of their applications.
5. Compare the tapped-potentiometer and diode-function generators.
6. Explain the operation of the diode bridge used in a diode-function generator. What are the purposes of the *break-point* and the *slope* potentiometers?

Volume 3

USING THE D-C ANALOG COMPUTER

Chapter 1

MONITORING AND CONTROL

Introduction

The basic computing components discussed in detail in Volume 2 are housed in a console to form a general purpose computer. The number of components of any one type depends on the requirements of the common form of problem to be solved on the computer. A computer solving problems in chemical plant design will contain a relatively large number of variable function generators, while one devoted to problems in electron ballistics will require chiefly linear components. The size of any one computer is limited only by the component terminations available at the problem patching bay, and when more components are needed than can be terminated at one bay, two or more computer consoles are interconnected and operated simultaneously by a single control station.

VOLTMETERS

In operating the computer two forms of voltage supply are required. One is for the plate-supply voltages for the electron-tube circuits and control-relay operating voltages. The supplies of this form are of conventional design and are series regulated. The other is the reference supply of + and −100 volts which forms the source of all signal voltages throughout the computer, and must therefore be carefully regulated (Fig. 1-1A). A commonly-used circuit for the reference voltage generation employs standard high-gain amplifiers as shown in the figure. The standard cell voltage is accurately amplified to −100 volts, and then by inversion +100 volts is produced. An amplitude and a balance adjustment are included in the circuit. A connection from the second amplifier back to the standard cell provides an unloading action, similar to that described previously when discussing the resolver. Little if any current is drawn from the cell, because the low-impedance amplifier output provides an ample source. It is not unusual for the amplifiers in such a circuit to be capable of supplying 1 ampere without any noticeable change in output voltage. This large cur-

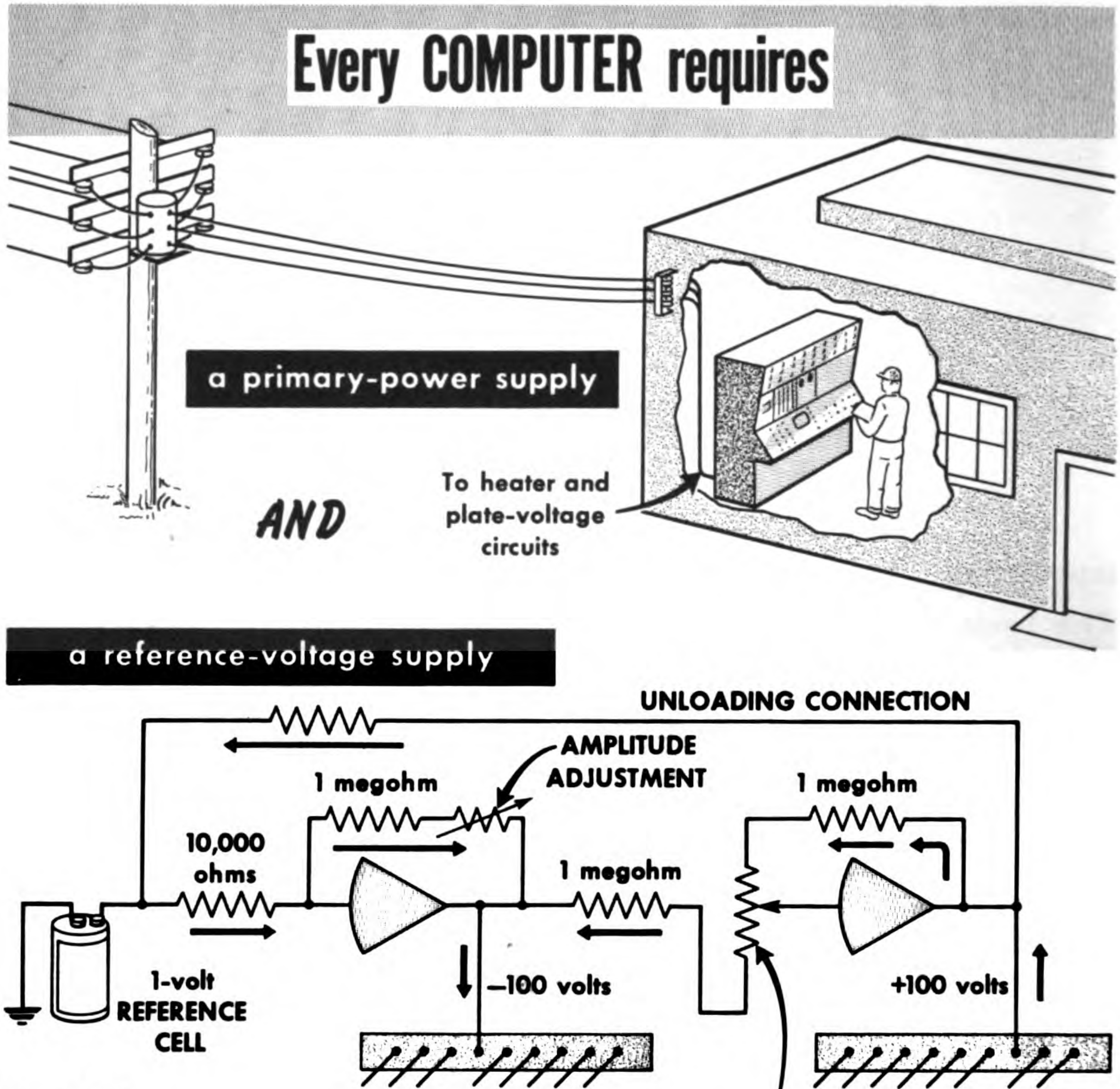


Fig. 1-1A

rent is frequently required by the many computing circuits connected through the patch bay to the reference supply.

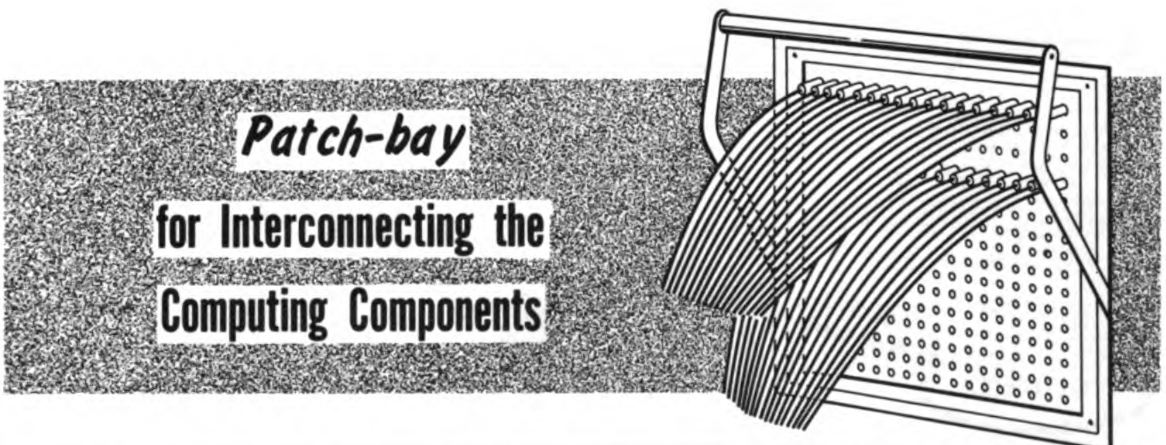
All signal terminals to and from computing components are brought to the patch bay, where with cords and plugs in the prepatch panel, they can be interconnected as dictated by the program for the problem at hand. The output of any computing component could be connected to the input of any other. However because of changing impedance levels, some connections are not permitted. The table given here indicates which connections are allowed (Fig. 1-1B).

Monitoring Systems

The output of any computing component can be monitored on a digital voltmeter. This enables any stationary value of voltage to be checked im-

mediately, and, if desired, printed out by an automatic typewriter or paper-tape punch. The output of any amplifier can be connected through the patch panel to the input of a strip-chart recorder, plotting table, or oscilloscope, allowing the change in the corresponding voltage to be recorded (Fig. 1-2).

The computing operation of all components is controlled manually by one switch or push-button system. With power supplied to the computer, and the plates, control relay and reference power supplies working, all components except the integrating amplifiers accept their input voltages, and produce corresponding output voltages related algebraically to the input voltages. Dynamic solutions are initiated by simultaneously causing the integrators to begin operation with the resulting changes in voltage levels



Question: *Can the output of A be connected to the input of B?*

A ↓ / B →	1	2	3	4	5	6	7
1. Amplifiers	Yes	Yes	Yes	Yes	Yes	Yes	Yes
2. Attenuators	Yes	Yes	No	Yes	No	No	No
3. Servomultipliers and Resolvers	Yes	No	No	Yes	No	No	No
4. Time-Division Multiplier	Yes	Yes	Yes	Yes	Yes	Yes	Yes
5. Quarter-Square Multiplier	Yes	Yes	Yes	Yes	Yes	Yes	Yes
6. Diode-Function Generator	Yes	Yes	Yes	Yes	Yes	Yes	Yes
7. Tapped-Potentiometer Function Generator	Yes	No	No	Yes	No	No	No

Fig. 1-1B

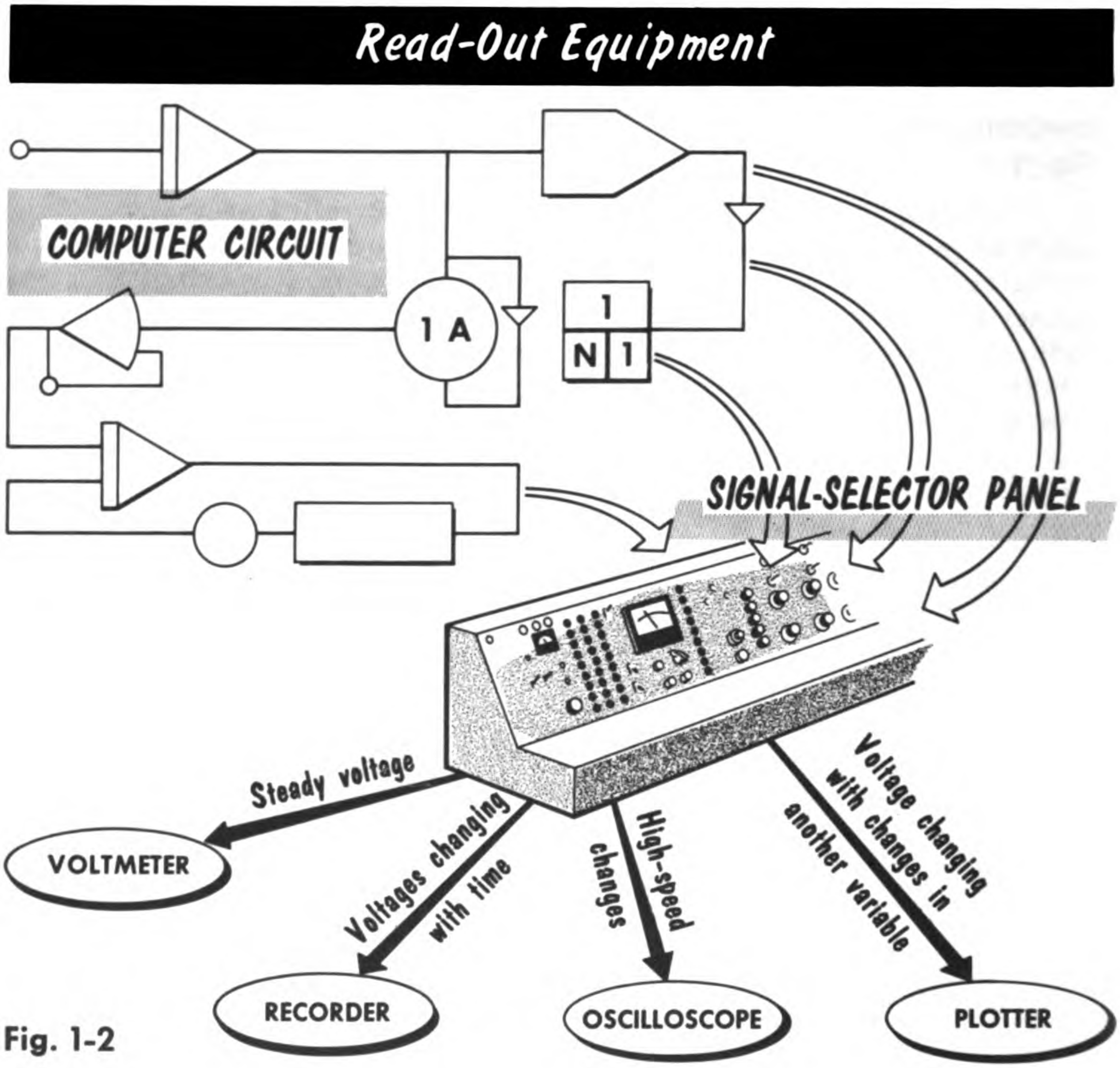


Fig. 1-2

throughout the circuit. Stop the integrators and the solution stops. Thus solution control depends solely on control of the integrating components.

Servo Voltmeters

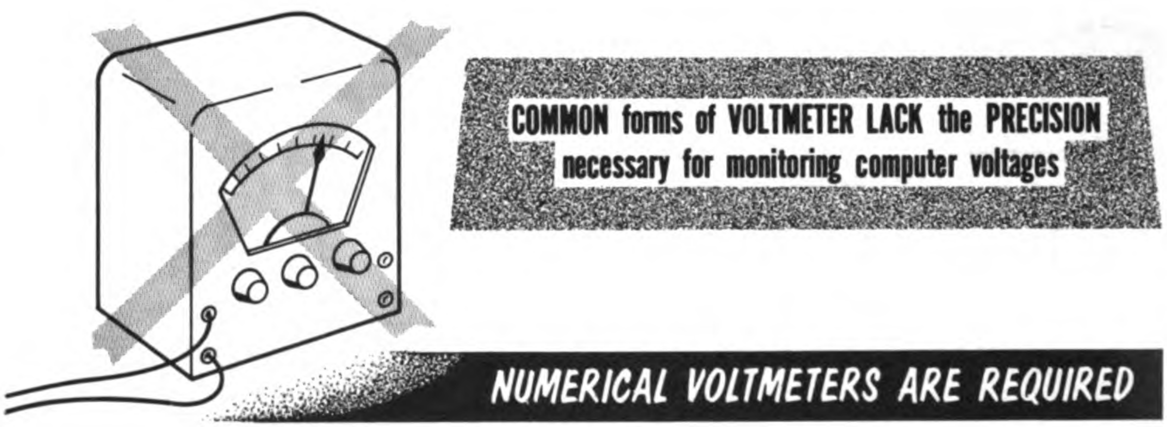
The stationary values of the signal voltages in an analog-computer circuit must be monitored with an accuracy that exceeds the capability of the best form of voltmeter having a moving pointer indication. Futhermore, the speed with which such a meter can be viewed and the reading interpreted, is too slow for this type of work. Consequently, in place of the common forms of voltmeter the analog-computer uses the more modern kind of numerical or digital voltmeter (Fig. 1-3).

A simple form of numerical voltmeter (one in which the reading is given in terms of a three- or four-figure number display) uses a position-servo system similar to that of the servomultiplier. The monitored voltage is compared with one taken from a precision follow-up potentiometer, and any

difference between them is used as an error voltage in a high-gain position servomechanism driving a shaft to which the wiper of the potentiometer is connected. In this manner the shaft position is made to correspond quite accurately to the input voltage. To the shaft is also connected a counter having three decades, and thus for any input voltage a counter-reading correct to three figures is obtained.

It is usual in such a unit to minimize settling time, and also to save counters, by indicating only the absolute value of the voltage on the counter, and displaying the sign in some simple way, such as by an indicator light. The follow-up potentiometer is energized with a positive voltage if the unknown voltage is positive, with a negative voltage if it is negative. A reversal of the control-winding connections to the servo motor takes care of the negative feedback requirements of the position controller as the follow-up voltage changes sign.

This unit is capable of measuring a voltage correctly to 0.1% of the



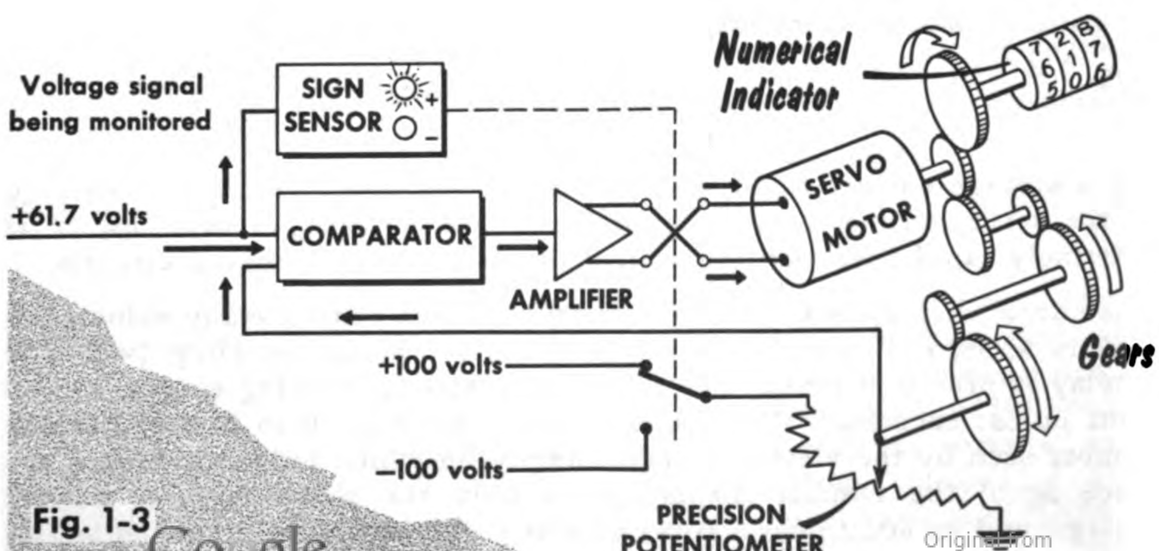


Fig. 1-3

reference voltage applied to the follow-up potentiometer which is a 10-turn, wirewound precision potentiometer having a linearity of 0.02%.

Digital Voltmeters

More modern and desirable in an analog computer than the simple servo voltmeter is the digital voltmeter. Still employing the null or balancing principle of the former device, the digital voltmeter replaces the position servomechanism containing a motor and follow-up potentiometer by an electromechanical stepping-relay system or, in later models, by a very fast-operating solid-state diode switching circuit. The basic operation of the device remains the same as that of the servo voltmeter — a voltage is generated within the voltmeter to balance the unknown one being monitored, and when a null is achieved the condition of the internal voltage source determines the number shown on the output display. In the digital voltmeter the 10-turn, wirewound follow-up potentiometer of the servo voltmeter is replaced by a more accurate voltage source capable of supply-

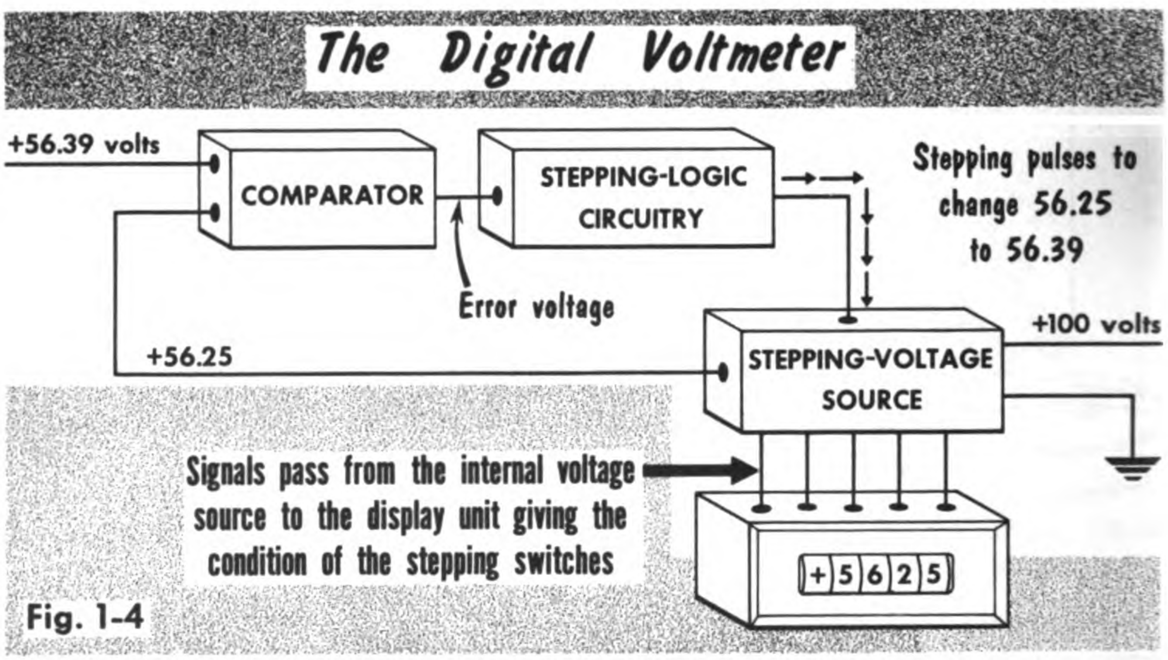


Fig. 1-4

ing a voltage correct to 10 mv. The two-phase induction motor required in the servo voltmeter is replaced by a bank of stepper-switches, or if extremely rapid operation is required, by solid-state switching circuits.

The output display is a four-figure number which can have any value from 0000 to 99.99, indicating any voltage value in this range (Fig. 1-4). The display is produced by number tubes containing a focusing system and 10 light bulbs; each bulb illuminates one number from 0 to 9 and thus the number seen by the viewer depends simply on which bulb is switched on. Once again the number display gives only the absolute value of the voltage, and an additional display tube is required for the sign.

As the INDICATOR in a DIGITAL VOLTMETER can ONLY CHANGE in STEPS equivalent to 10 mv, the INTERNAL VOLTAGE SOURCE need ONLY CHANGE in STEPS

The CONTINUOUS POTENTIOMETER can be replaced by a STEPPING POTENTIOMETER

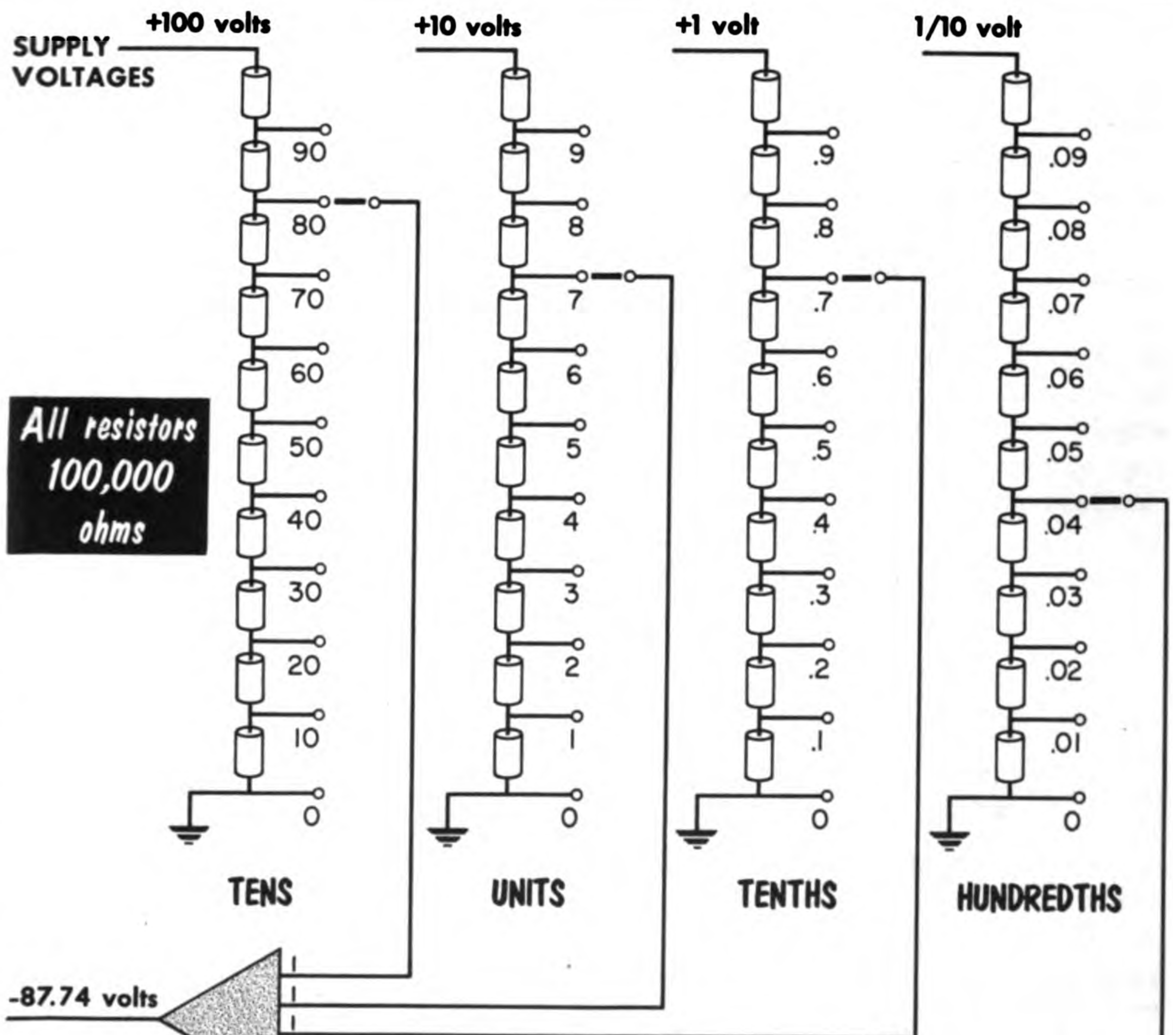


Fig. 1-5A

In the first kind of digital voltmeter we describe here, the potentiometer follow-up element present in the servo-balancing system previously discussed is replaced by a stepping, potential divider. A continuous potentiometer supplied with +100 volts is capable of producing a wiper voltage with a resolution determined largely by the number of turns of wire (Fig. 1-5A). If there are 10,000 turns, the output voltage changes by 10 mv as the wiper moves from turn to turn. The mechanical movement might be

extremely small and various difficulties of positioning "dead-zone" might interfere with obtaining very small changes in voltage, but the ultimate limit would be fixed by the number of turns. When the potentiometer is used in a servo voltmeter the display is connected mechanically to the

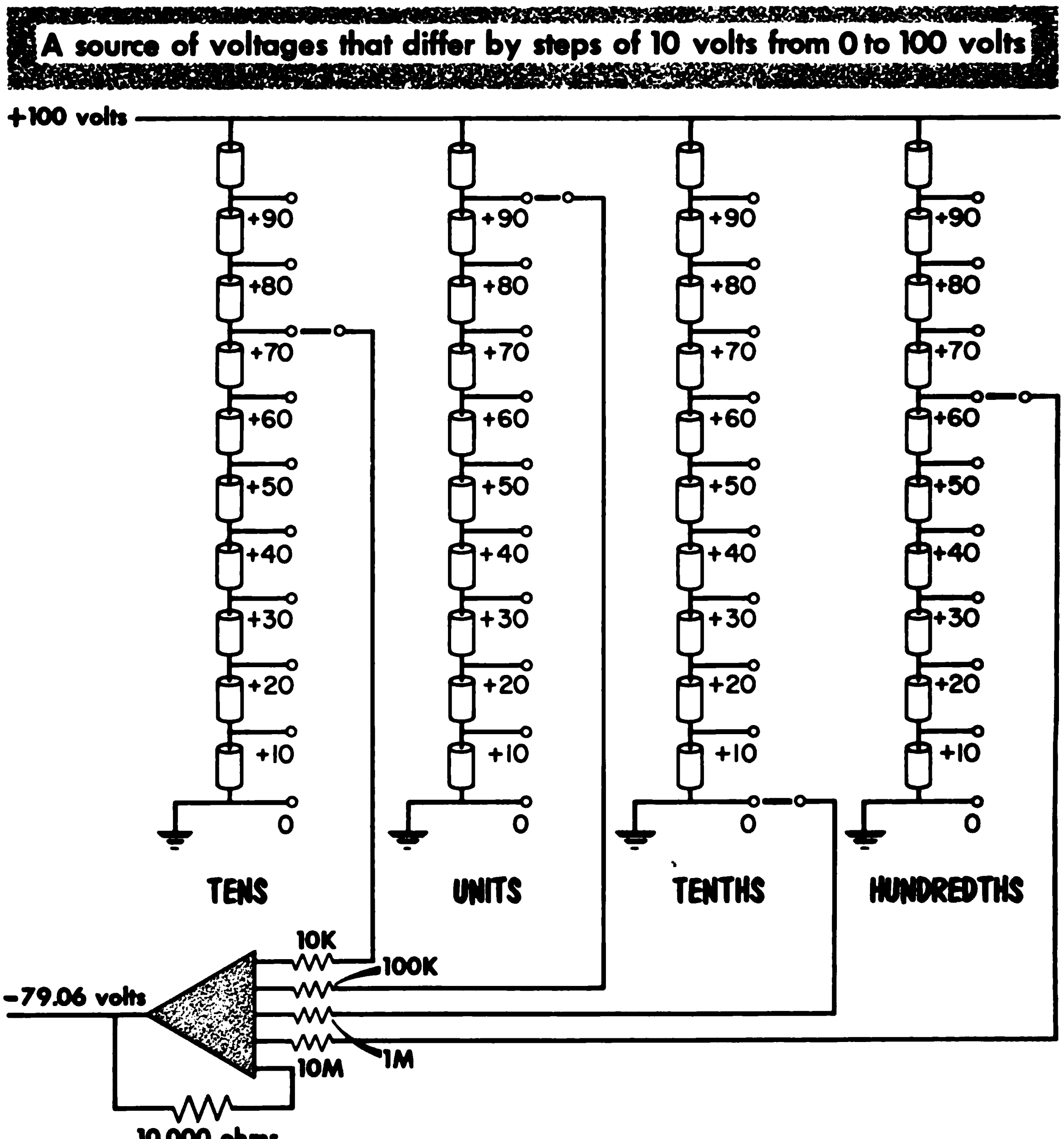


Fig. 1-5B

wiper, and any nonlinearity in the winding of the potentiometer will cause errors in the value of voltage displayed. If the resistance per inch of wiper travel is not constant, the number displayed by the counter will change through different increments for similar changes in voltage. Both of these possible causes of error — insufficient resolution and nonlinearity —

cause us to seek a more accurate adjustable voltage source when readings of voltage correct to 1/100 volt are required.

As the voltage display is to change in steps of 1/100 volt the voltage source is also required only to be adjustable in steps of 1/100 volt. Thus a chain of precision resistors can be used in the form of a potential divider, and the follow-up voltage can be taken from the appropriate junction. Consider for example, a source of voltages which differ by steps of 10 volts from 0 to 100 volts (Fig. 1-5B). It could be a chain of 10 precision 100,000-ohm resistors. To whatever voltage is obtained from this potential divider can be added another from 10 series resistors energized with 10 volts, and thus capable of being changed in steps of 1 volt. Similarly, by using two more decades energized by voltages of 1 volt and 1/10 volt, one can by summation easily produce any voltage between 0 and +100 volts, with a resolution of 1/100 volt. Changing the sign of all energizing voltages allows any voltage between —100 volts and 0 to be similarly produced.

A slight variation in this voltage source would be to use for all decades an energizing voltage of 100 volts, and then to sum the four voltages in an operational amplifier with different gains — 1, 1/10, 1/100, 1/1000. The feedback resistor of the amplifier might have a value of 10,000 ohms, and then the input resistors would be 10,000 ohms, 100,000 ohms, 1 megohm and 10 megohms. The resistors of the decades need not be the same value, and in any case slight adjustments have to be made for the correction of loading errors.

The Thomson-Varley Bridge

An alternative scheme uses a Thomson-Varley bridge (Fig. 1-5C). The 10 resistors in the first decade are similar to each other in value, 100 ohms say, and a stepping relay makes contact with the junctions between the resistors. The ends of the decade are connected by two ganged stepping relays across two adjacent resistors in a second decade which has 11 500-ohm resistors. The combined parallel resistance of the first decade and any adjacent two resistors in the second decade is 500 ohms, and thus the second decade has a total resistance of 5000 ohms. This is placed, through two ganged stepping relays, across two adjacent resistors of a third decade having 11 2500-ohm resistors. In combination with the first two decades the total resistance of this third decade is 25,000 ohms, and through two ganged stepping relays it is placed across any two adjacent resistors of a fourth decade having 11 12,500-ohm resistors. To the fourth decade is applied the reference-supply voltage of 100 volts, and thus to the third decade there is applied a 10-volt voltage difference 10-0 volts, 20-10 volts, 30-20 volts . . . or 100-90 volts. Let it be, for example, 40-30 volts. To the second decade will be applied a voltage difference of 1 volt, 36-35 volts, say. To the first decade will be applied a voltage difference of 0.1 volt, 35.8-35.7 volts, say. Finally, the voltage at the relay contact on the first decade can be stepped in increments of 1/100 volt and will therefore be, let us say, an accurate 35.74 volts.

Whichever scheme is preferred, it is obvious that by using approximately 40 precision resistors and 4 stepping switches, a source of precise voltages

The Thomson-Varley Bridge

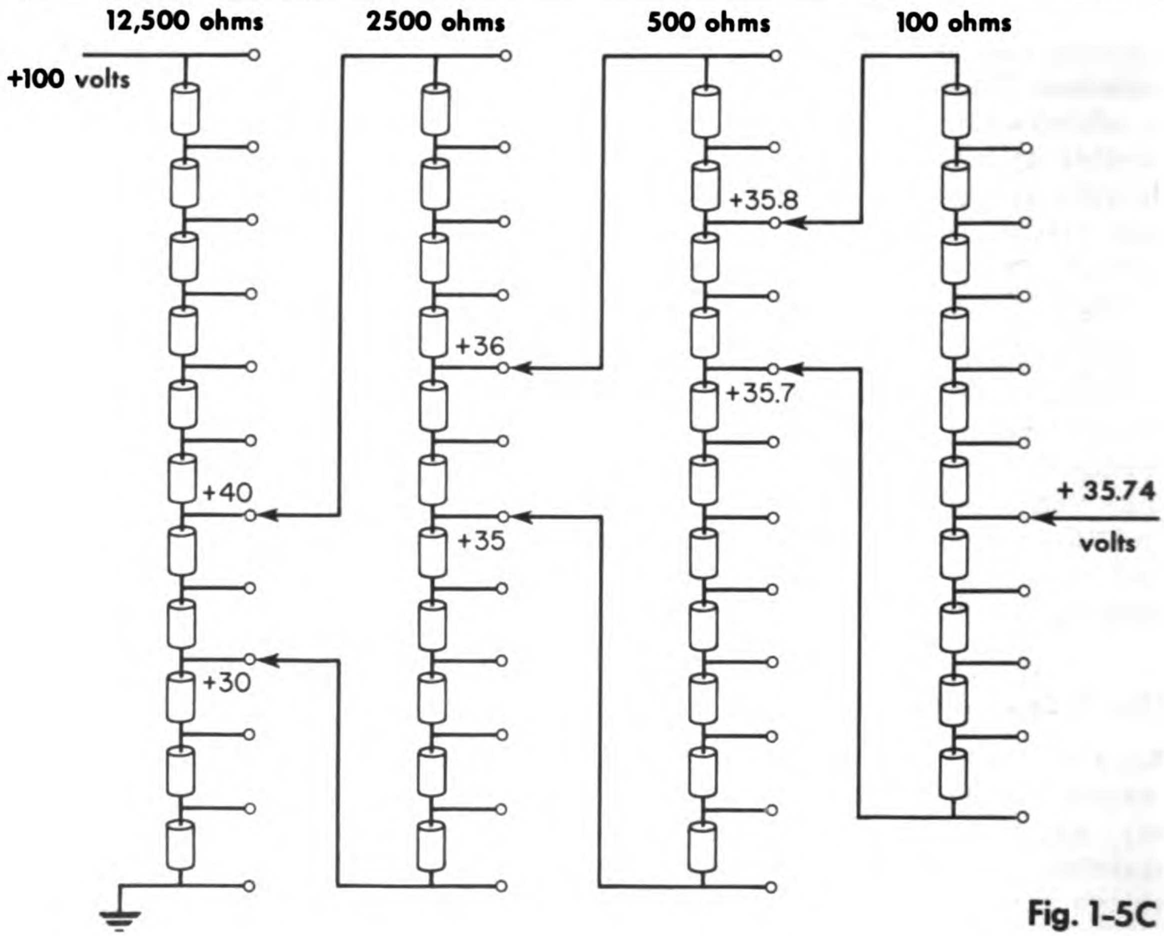


Fig. 1-5C

in the range 0 to +100 volts can be obtained with a resolution of 1/100 volt.

Stepping Logic

Together with the source of adjustable precise voltage, a logical scheme for positioning the contacts of the stepping relays is required. The voltage being monitored is compared with the voltage from the potential divider and any difference between them is used to initiate stepping of the relays.

Whenever a difference is present, the first step is to check the sign of the unknown voltage. The potential-divider voltage is momentarily set to zero by disconnecting the 100-volt supply. If the resulting signal from the error amplifier is positive, the unknown voltage must be negative, and no change is made in the polarity switch position. If the resulting signal from the

STEPPING LOGIC

In settling to a balanced condition with an input voltage of +16.31 volts, the internal voltage source goes through the following conditions:

	Voltage Output	Sign of Error	Resulting Command
1.	0000	-	Invert unknown voltage
2.	9999	-	Shift
3.	0999	+	Count
4.	1999	-	Shift
5.	1099	+	Count
6.	1199	+	Count
7.	1299	+	Count
8.	1399	+	Count
9.	1499	+	Count
10.	1599	+	Count
11.	1699	-	Shift
12.	1609	+	Count
13.	1619	+	Count
14.	1629	+	Count
15.	1639	-	Shift
16.	1630	+	Count
17.	1631	?	Shift and display

Fig. 1-6

error amplifier is negative, a current pulse is applied to the bistable polarity switch causing the error voltage to become positive. No matter what the initial situation, after this first step the contribution of the unknown voltage to the error voltage will be positive. The position of the polarity switch determines, through a pair of ganged contacts, the sign illuminated on the output display.

With the unknown voltage always appearing to be negative, it can be balanced by a positive voltage from the potential-divider circuit. Let us consider a voltage of 16.31 volts (Fig. 1-6). The second step in achieving a balance and thus a correct number in the display unit, is to set all decade contacts to their highest position. This ensures that provided the amplitude of the unknown voltage is less than 100 volts it will be less than that coming from the potential divider, and thus the error voltage will be nega-

tive. The error voltage controls a pulse amplifier producing approximately 100 stepping-relay pulses per second. These pulses are *counting* pulses, causing one or other decade to step upwards, when the error voltage is positive; they are *shifting* pulses, causing the error-amplifier output to be

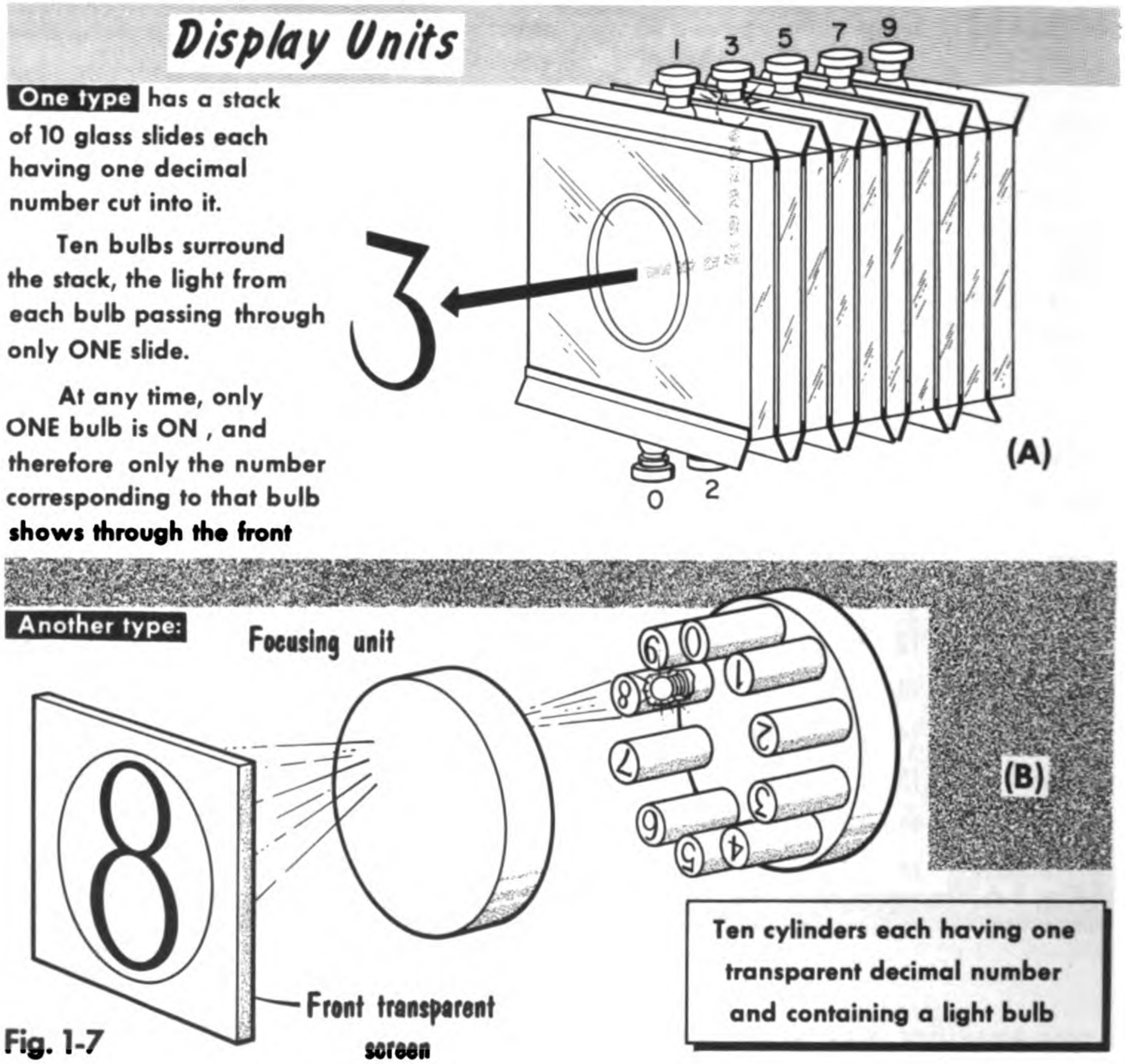


Fig. 1-7

switched to a lower-level decade, and for that decade to be switched to zero, when the error voltage is negative. With the potential-divider voltage set to its highest value the error voltage is negative, and a shift pulse is produced causing the potential-divider voltage to fall to 09.99 volts. This is less than the unknown voltage of 16.31 volts, and thus the positive-error voltage causes a counting pulse to "step" the fourth decade producing a potential-divider voltage of 19.99 volts. The error voltage changes sign, and the next pulse is a shift pulse producing a potential-divider voltage of 10.99 volts. The error voltage is once again positive and thus counting pulses are applied to the third decade, giving 11.99, 12.99, 13.99, and so on, until the potential-divider voltage reaches 16.99 volts. Now a negative error

voltage appears, causing a shift voltage and 16.09 volts from the potential-divider. Counting pulses occur until 16.39 volts is achieved, and then a shift pulse to 16.30 volts. Finally, one more counting pulse produces 16.31 volts, the required balancing voltage. With an error of less than 5 mv, no pulses are generated, and thus the digital voltmeter is balanced and stationary, displaying a number corresponding to the unknown monitored voltage. The total setting time for the voltage used in the example would be approximately 17/100 seconds, as 17 steps are required for balance.

The Output Display

In parallel with the stepping relays of the decade resistors, are others supplying current to one light bulb in each of four-number display units. The position of each decade-stepping relay determines which numeral from 0 to 9 is illuminated in the corresponding display window. A number of different display units are available and two of them are illustrated in Fig. 1-7, parts (A) and (B).

The pulsing operations that step the decade relays occur so rapidly that only when the voltmeter has achieved a balanced condition is any number intelligibly displayed. This number is retained fairly constantly until a change in the input unknown voltage causes a recycle of the balancing operation.

Diode Switches and Their Control

The digital voltmeter using mechanical stepping relays is much more desirable than the servo voltmeter, but its mechanical components limit its speed of operation, its reliability, and its freedom from trouble. Mechanical components become worn and sometimes even break; the contacts present

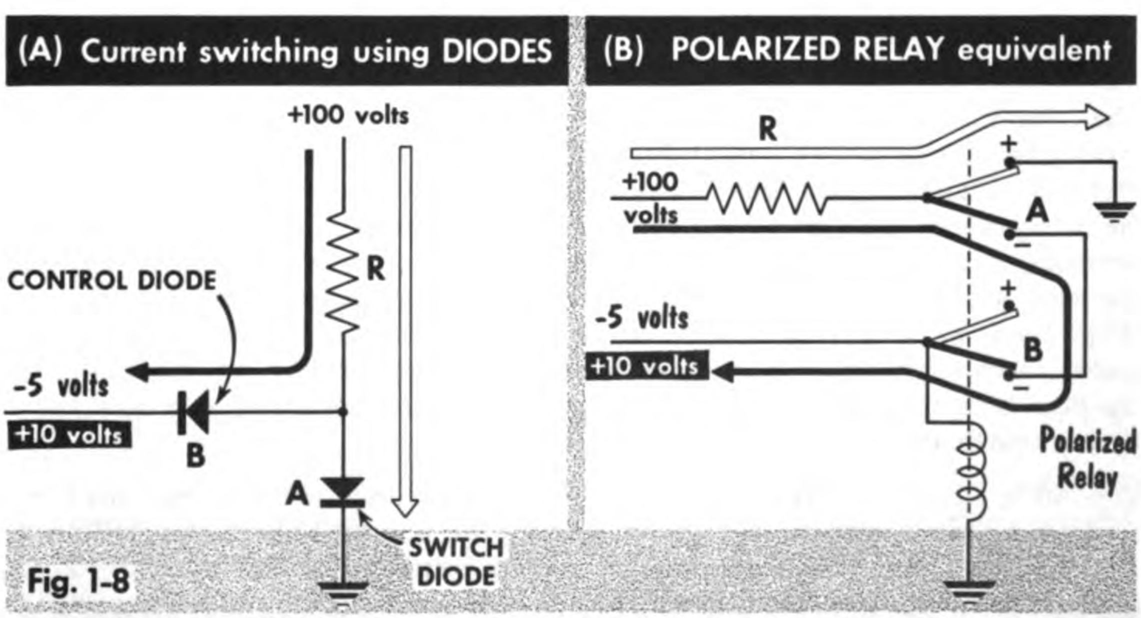


Fig. 1-8

difficulties of high-contact resistance. An obvious improvement might therefore be sought by trying to remove as many of the mechanical relays as possible by using electronic switches in their place. A diode is an electronic switch, and with the introduction of solid-state rather than thermionic diodes, a usable circuit without mechanical relays is practical.

Whereas the control of a stepping relay is electromechanical in nature, that of a diode is purely electrical, permitting faster and more reliable operation. As has been explained previously, a diode will conduct if the plate is more positive than the cathode, and when conducting the plate potential is very close to that of the cathode. Consider the diode circuit shown in Fig. 1-8, part (A). With +100 volts applied to the circuit, diode *A* will conduct, and a current inversely proportional to resistance R will flow through the conducting diode. Diode *B* is cut off because its cathode is more positive than the plate, which is connected directly to the plate of *A* and is therefore at approximately ground potential. If however, in place of the +10 volts shown applied to the cathode of *B*, —5 volts is applied, then diode *B* will conduct, bringing its plate potential and that of the plate of *A* to approximately —5 volts, cutting off diode *A*. Thus the current flowing through diode *A* can, by a voltage applied to diode *B* be controlled to be a constant value or zero. An equivalent mechanical switching circuit is shown in part (B) of the figure.

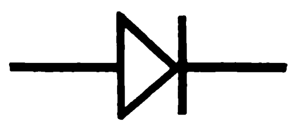
With the introduction of solid-state diodes rather than mechanical switches it is found desirable to change to a different form of circuit employing binary rather than decimal logic. Binary logic takes advantage of the two-state character (conducting or nonconducting) of the diode, and replaces the 10-symbol numbering system that is commonly understood and used by everyone, by a 2-symbol numbering system. The decade (10 positions) switching system used in the digital voltmeter is replaced by a binary-coded decimal system. In place of one 10-position mechanical switch, the latter system requires four two-state diodes as will be explained shortly.

Binary Coding

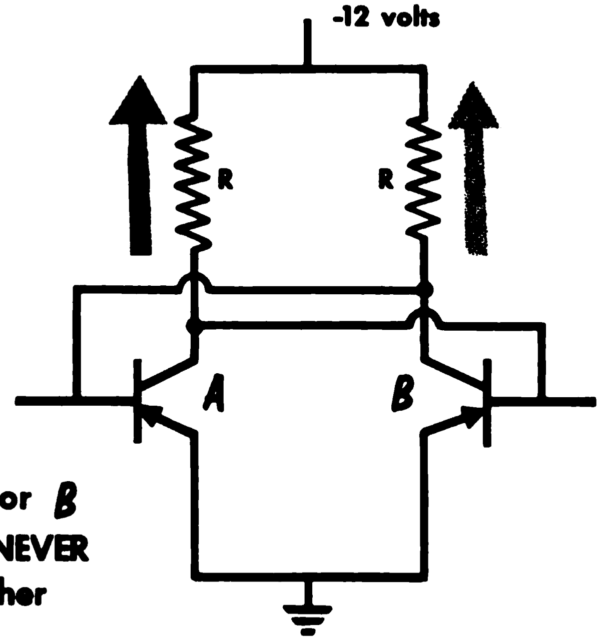
The commonly used decimal system of numbers has 10 symbols. Unfortunately, it is rather unsuitable for use in data transmission and storage systems because it requires devices having 10 different states to represent the 10 symbols. Far more convenient is the binary system having only 2 symbols, for these can be represented easily in an electronic circuit by the two stable conditions of conduction and nonconduction of current [Fig. 1-9A, part (a)]. A diode can be used to control the condition of the circuit and thus the symbol represented. On the other hand, a bistable flip-flop is also a suitable component for use in counting circuits for a binary arithmetical system.

The table given in Fig. 1-9A, part (b), shows how the decimal numbers of 0 to 9 correspond to equivalent binary numbers. The reader will note that 4 binary symbols are required to represent 1 decimal symbol. Thus, in place of a hypothetical decimal component we need 4 binary components,

Two-state Devices



DIODE
conducts or is cut off



FLIP-FLOP
EITHER *A* or *B* conducts, NEVER both together

(a)

BINARY NUMBERS FOR DECIMAL NUMBERS BETWEEN 0 AND 9

DECIMAL NUMBER	BINARY EQUIVALENT			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1

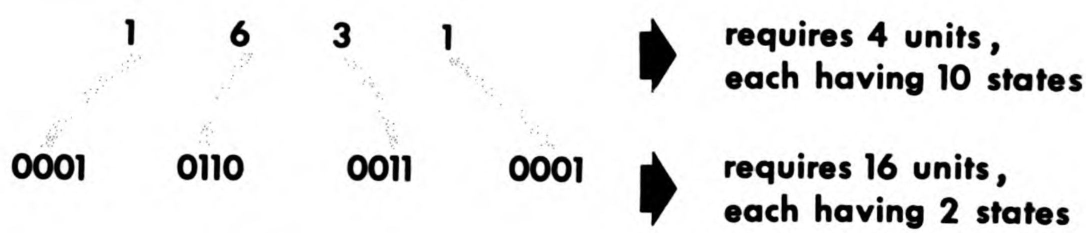
(b)

Fig. 1-9A

diodes or flip-flops, and the condition of these four components determines the decimal number represented. Some combinations of conditions are not required as they would correspond to decimal numbers above 9. Above this number, 2 decimal symbols are used, and we choose then for simplicity to use two sets of 4 binary symbols. This system using 4 binary symbols for each decimal symbol in a number is called *binary-coded decimal.*

In place of the 10-position stepping switch of a decade in the previously

BINARY-CODED DECIMAL



Internal voltage source for electronic DVM

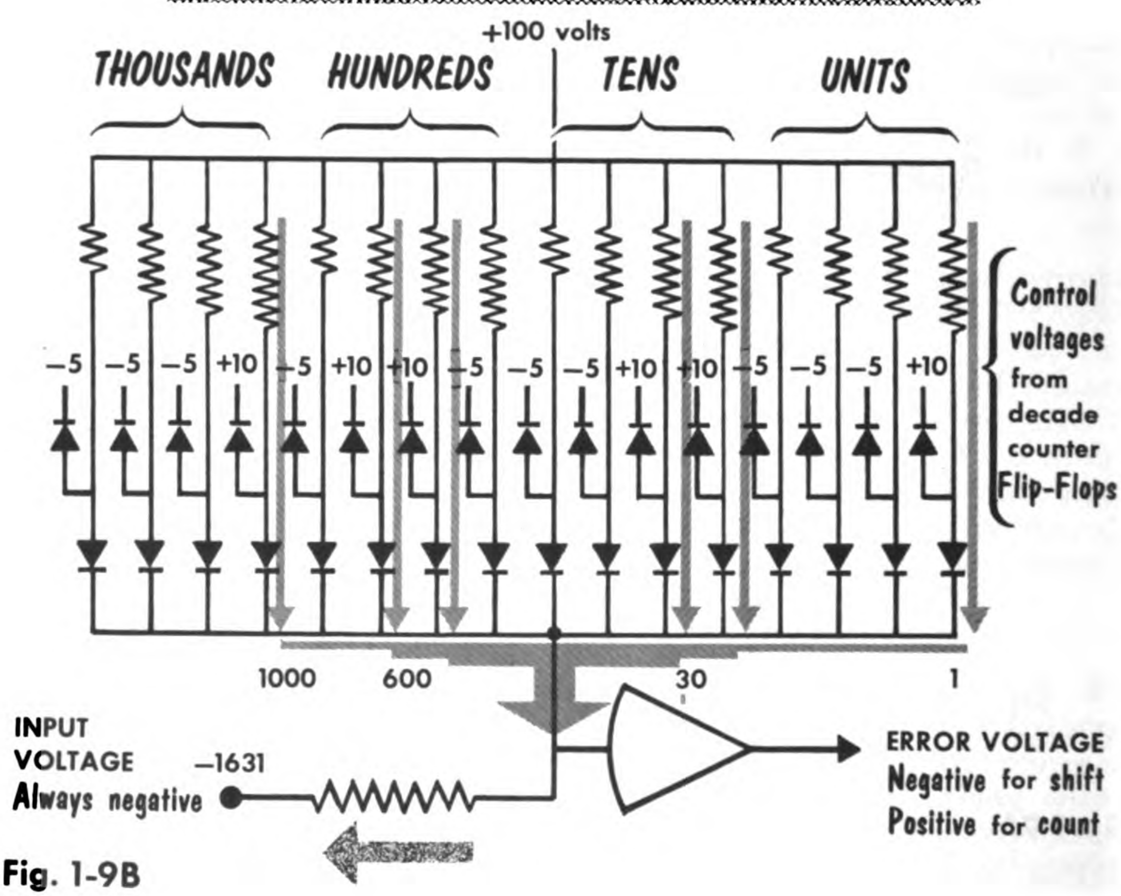


Fig. 1-9B

described digital voltmeter, a unit containing 4 flip-flops controlling the conduction of 4 gating diodes enables the current arriving at the grid of a high-gain amplifier to correspond to any decimal number between 0 and 9. By using four such units with suitably scaled resistors, the condition of 16 flip-flops and the conduction or nonconduction of 16 corresponding diodes determines the current supplied to the grid of a high-gain amplifier so

that it is proportional to any voltage between 0 and 99.99 volts. The condition of the circuit shown in the figure corresponds to a voltage of 16.31 volts (Fig. 1-9B).

In such a circuit the switching logic that determines the appropriate *setting* and *resetting* of the flip-flops is quite similar to that previously

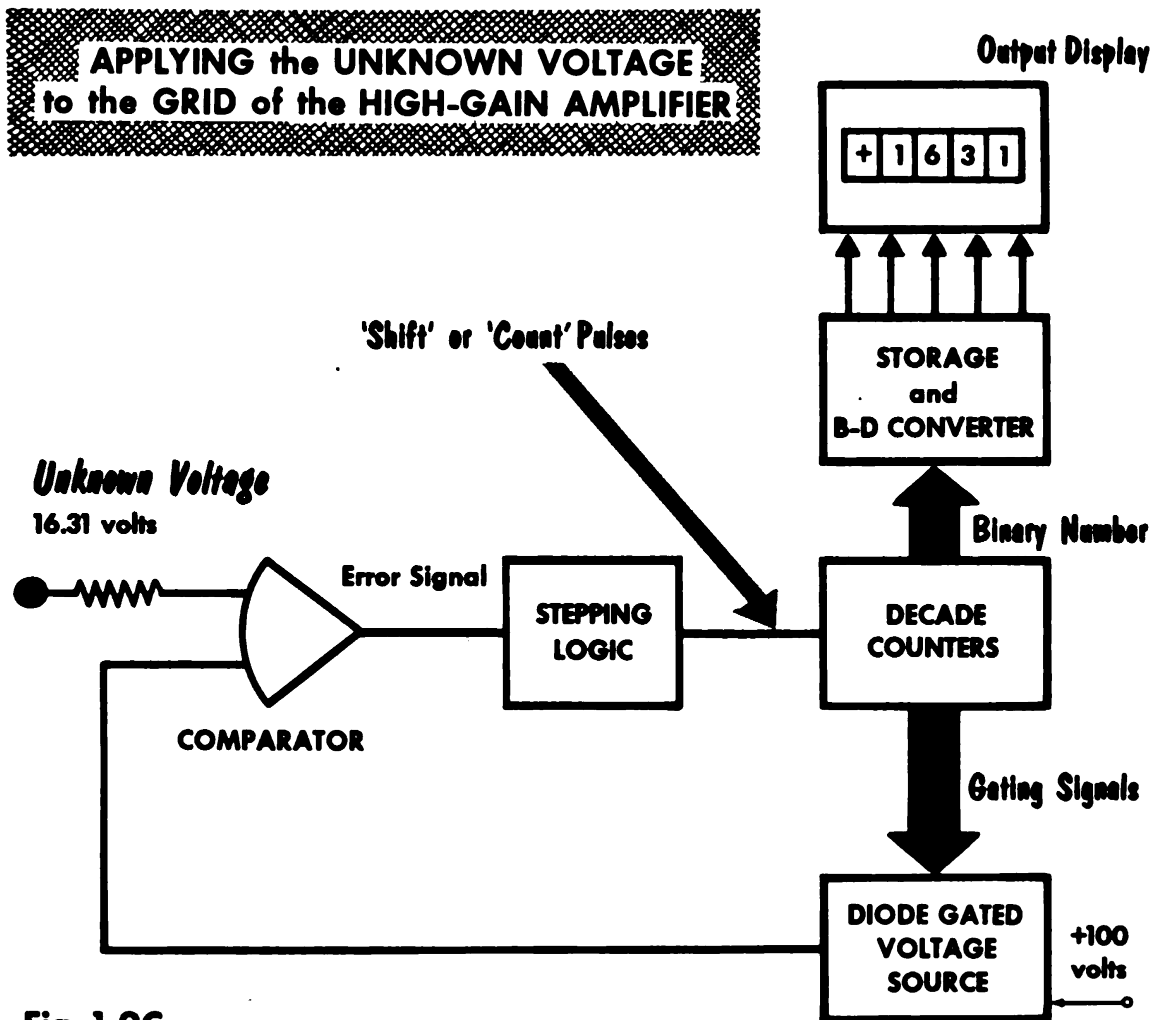
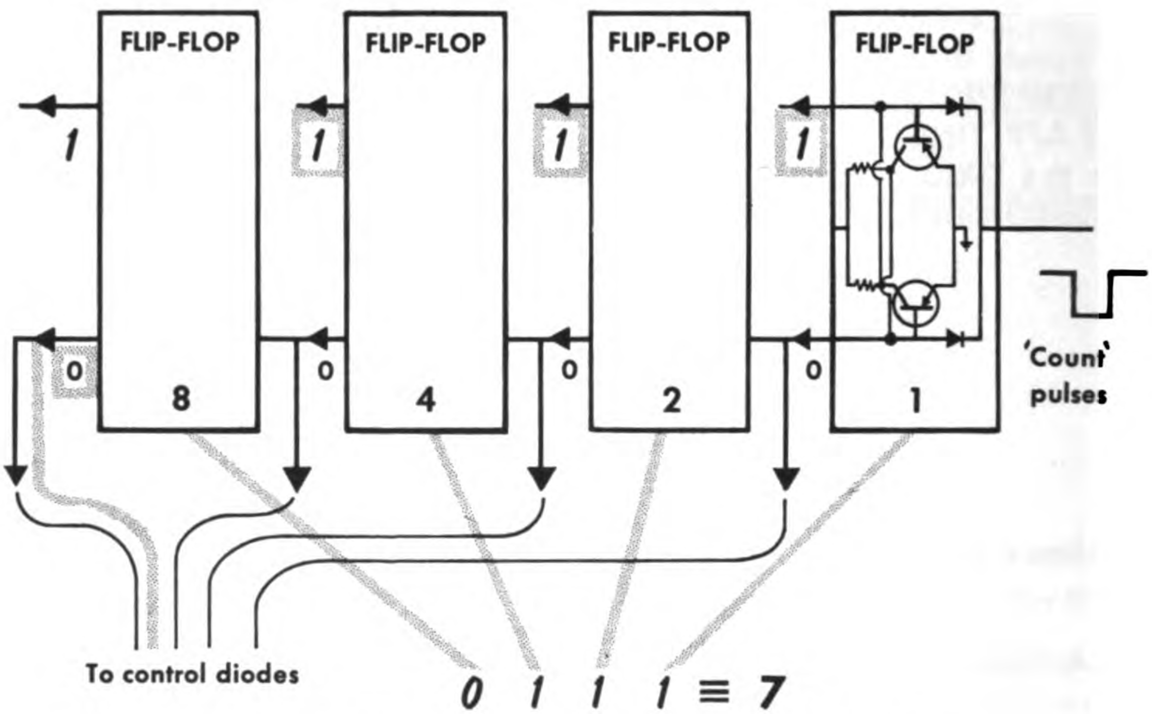


Fig. 1-9C

discussed. To the grid of the high-gain amplifier the unknown voltage is applied with a negative sign through a resistor, causing a current to flow which is to be balanced by the current from the diode circuits (Fig. 1-9C). When the current from the diodes exceeds that from the unknown voltage, shifting results; when it is less than that from the unknown voltage, counting pulses actuate the appropriate flip-flops. When the final decade counter has been set, the condition of the 4 counters corresponds to the unknown voltage, and the last shift signal transmits this condition to the output display. Only when the counters are finally set is the value transmitted to the output display where it is retained.

One-Binary Decade Counter



State of the counter after seven pulses

Next pulse causes	1	to reset	(0)
which causes	2	to reset	(0)
which causes	4	to reset	(0)
which causes	8	to set	(1)

giving **1 0 0 0 ≡ 8**

Fig. 1-10

Binary Counters

In place of each 10-position mechanical switch the electronic digital voltmeter has a binary counter of 4 flip-flops (Fig. 1-10). Each flip-flop can be SET corresponding to a binary number 1, or RESET, corresponding to a binary number 0.

The 4 flip-flops can be considered for counting purposes to be in series. All incoming counting pulses act on the first flip-flop causing it to set and

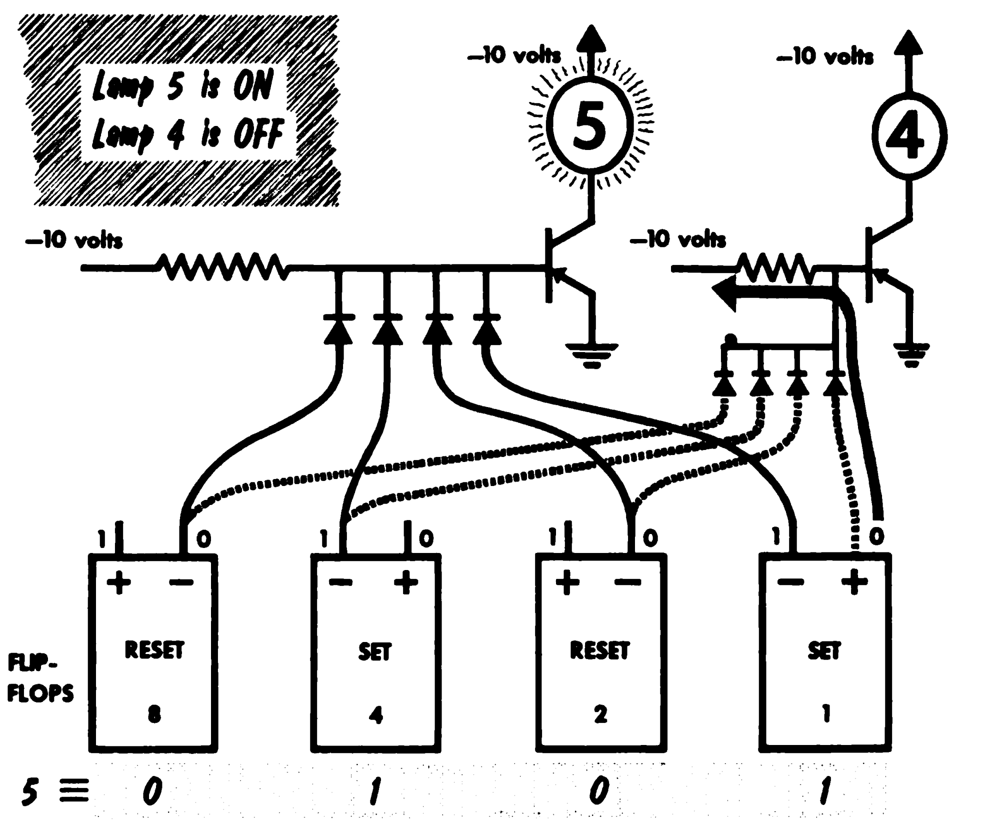
reset in turn. When the first flip-flop resets, a pulse is passed to the second flip-flop causing its state to change. In turn, only when the second flip-flop resets does it send a pulse to cause the third flip-flop to change its state. Similarly, the third flip-flop acts on the fourth. It is in this way that the states of the 4 flip-flops are made to record in binary terms the number of counting pulses that have been applied to the first flip-flop.

Shift pulses from the logic-control circuit are required to open a counter for counting, and to reset all its flip-flops. The resetting is achieved by applying an appropriate pulse to all flip-flops.

The final condition of the flip-flops determines which of the diode gates at the summing junction of the comparator amplifier are conducting. Those flip-flops that are RESET apply negative-control voltages to the control diodes (B), cutting off the corresponding gating diodes (A). Thus the current flowing to the summing junction from the diode gates is determined by the SET flip-flops.

Binary-to-Decimal Conversion

The information transmitted from the counters to the display unit, at the end of the counting sequence, is in binary form. This must be converted



STORAGE COUNTER and BINARY-to-DECIMAL CONVERTER

Fig. 1-11

to decimal form before it can be used to energize the decimal output display. The display unit contains storage counters that are set to the same condition as those activated by the logic circuits when the counting sequence ends. The storage counters act on a binary-to-decimal converter that energizes the lamps in the display output.

Figure 1-11 illustrates the nature of the binary-to-decimal converter. The decimal number 5 corresponds to the binary number 0101, and thus when a number 5 is to be displayed, the first and third flip-flops are set, the second and fourth are reset. When a flip-flop is set, the *1* output is at a negative potential, and the *0* output is positive, and vice versa. Thus for the binary number 0101 the anodes of the 4 diodes are all held at a negative potential, and none can conduct. This ensures that the base of the transistor is negative with respect to the emitter and therefore a current passes through the transistor to energize the lamp. Should any of the flip-flops be in the reverse condition, SET instead of RESET, or RESET instead of SET, a positive potential would be applied to the anode of a diode causing it to conduct, and placing a positive potential on the transistor base. Thus if the state of the flip-flops corresponds to a number other than 5, the transistor will be cut off and the lamp de-energized.

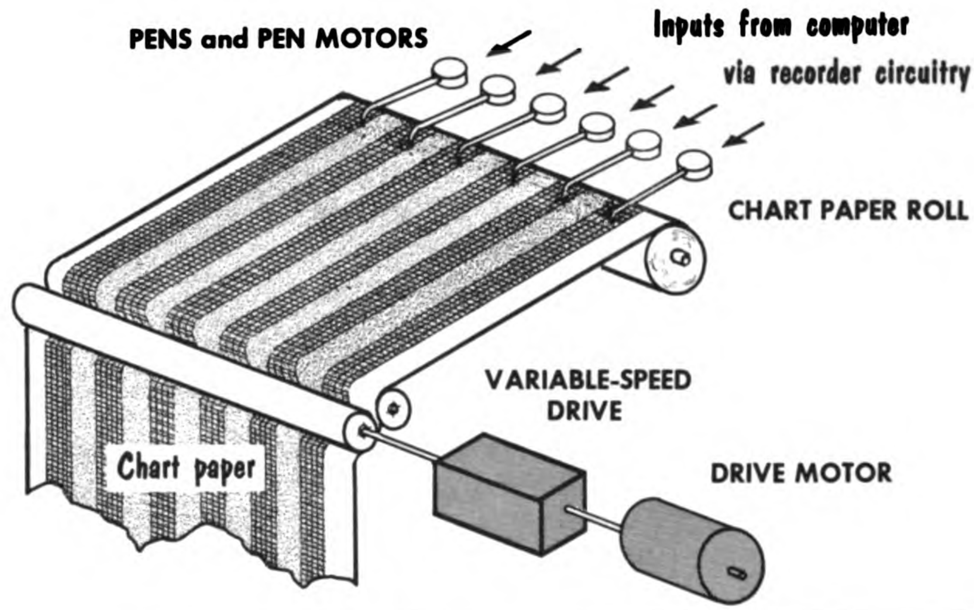
Connections similar to those shown are made for all the other 9 indicator lamps, causing only 1 lamp to be energized at any one time.

RECORDERS AND PLOTTERS

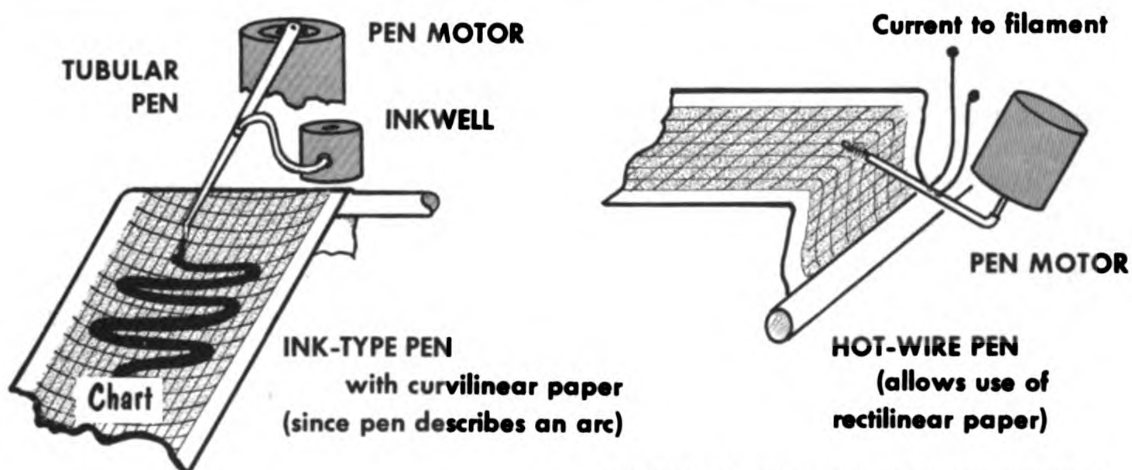
Strip-Chart Recorders

A frequent requirement in using an analog computer is to be able to view easily the variation of a number of voltages as they change with time. These variations are similar to those that occur in the behavior of quantities important in understanding the primary physical system — displacements, velocities, temperatures, concentrations, forces, etc. While viewing the voltage variations the engineer will immediately interpret them in terms of the quantities of the primary system. Thus it would be desirable to have them portrayed in a fashion similar to that common in taking records in the primary system. A common form of experimental or operational record uses a strip-chart recorder (Fig. 1-12). The reader will no doubt be familiar with the use made of this instrument in recording the variations of atmospheric temperature and barometric pressure in meteorological studies, steam temperature and pressure in power plants, wind velocity in a wind tunnel, etc. In each, a piece of graph paper is drawn past a pen that can be deflected perpendicular to the direction of paper motion to the extent that the distance of the trace it draws on the paper from a zero line, is proportional to the actuating physical quantity.

In a strip-chart recorder for use with an analog computer a fair degree of precision is required, and considerable versatility in paper speed, pen zero position, deflection sensitivity, and frequency response, is desirable to



STRIP-CHART RECORDER with 6 CHANNELS for PORTRAYING INFORMATION



Methods of Writing

Fig. 1-12

record conveniently the different behaviors of variables from widely different problems. Although the basic pen-deflection mechanism of an electromagnetically positioned motor is common to most strip-chart recorders, the writing schemes available are many, and the circuitry preceding the electromechanical transducer are varied and peculiar to the design of a particular manufacturer. One circuit will be discussed later. Some writing schemes are shown in the figure.

Rectilinear Operation

The most common form of strip-chart recorder employs an ink pen that is angularly deflected by an electromechanical transducer. The pen does not travel perpendicular to the paper motion but moves along the arc of a circle. Thus the graph imprinted on the paper and against which one views the trace, has to be *curvilinear*, rather than *rectilinear*. This leads to a small difficulty in interpreting the traces — a sinusoid does not look like a sinusoid — and a change to a writing scheme that results in rectilinear traces is desirable.

By using heat-sensitive paper and a heated stylus pressing against the paper as it is drawn over the edge of a table, a rectilinear trace is possible [Fig. 1-13, part (A)]. The stylus is deflected angularly, and thus for a reasonably accurate record to be obtained the stylus must be long, compared with the size of the deflection. The angle of deflection is proportional to the actuating signal and the height of the trace is proportional to the tangent of this angle. For small angles, $\tan \theta \cong \theta$, and thus the height of the trace is proportional to the actuating signal, as shown in part (B) of the figure. The graph paper has a thin wax-like, white coating which is removed by the hot stylus to form a very narrow line. With this form of writing the temperature of the stylus has to be increased when the paper speed is increased, if the trace is to remain legible.

A second technique continues to use ink as the writing medium, but now the ink is sprayed in a fine jet from a nozzle (part C of the figure) which can be deflected. Once again the angle of deflection must be comparatively small if a precise trace is to be produced. The particular advantage of this technique is that it has a very good high-frequency response, extending up to relatively high frequencies (200-500 cps); for the moment of inertia of the moving parts can be quite small.

A third technique for rectilinear recording continues to use a conventional pen-and-ink writing scheme, but a mechanical-linkage arrangement translates the curvilinear motion to rectilinear motion.

Channel Circuitry

A strip-chart recorder commonly has as many as 8 or 12 parallel channels each with its own pen mechanism and control circuitry, and each deflected simultaneously on parallel strips of the recording paper. The channels are similar, and in Fig. 1-14 we see an indication of the circuitry required to control one pen.

The computer signal voltage is applied through an adjustable input resistor to a conventional high-gain d-c amplifier (Fig. 1-14). With a fixed feedback resistor around the amplifier the *sensitivity* of the channel depends on the value of the input resistor, and this is adjustable in steps to give a range of sensitivities. The output of the amplifier goes to the electromechanical transducer, which positions the pen. With no other inputs to the amplifier, a mechanical adjustment of the pen allows it to be

Rectilinear Recorder Mechanisms

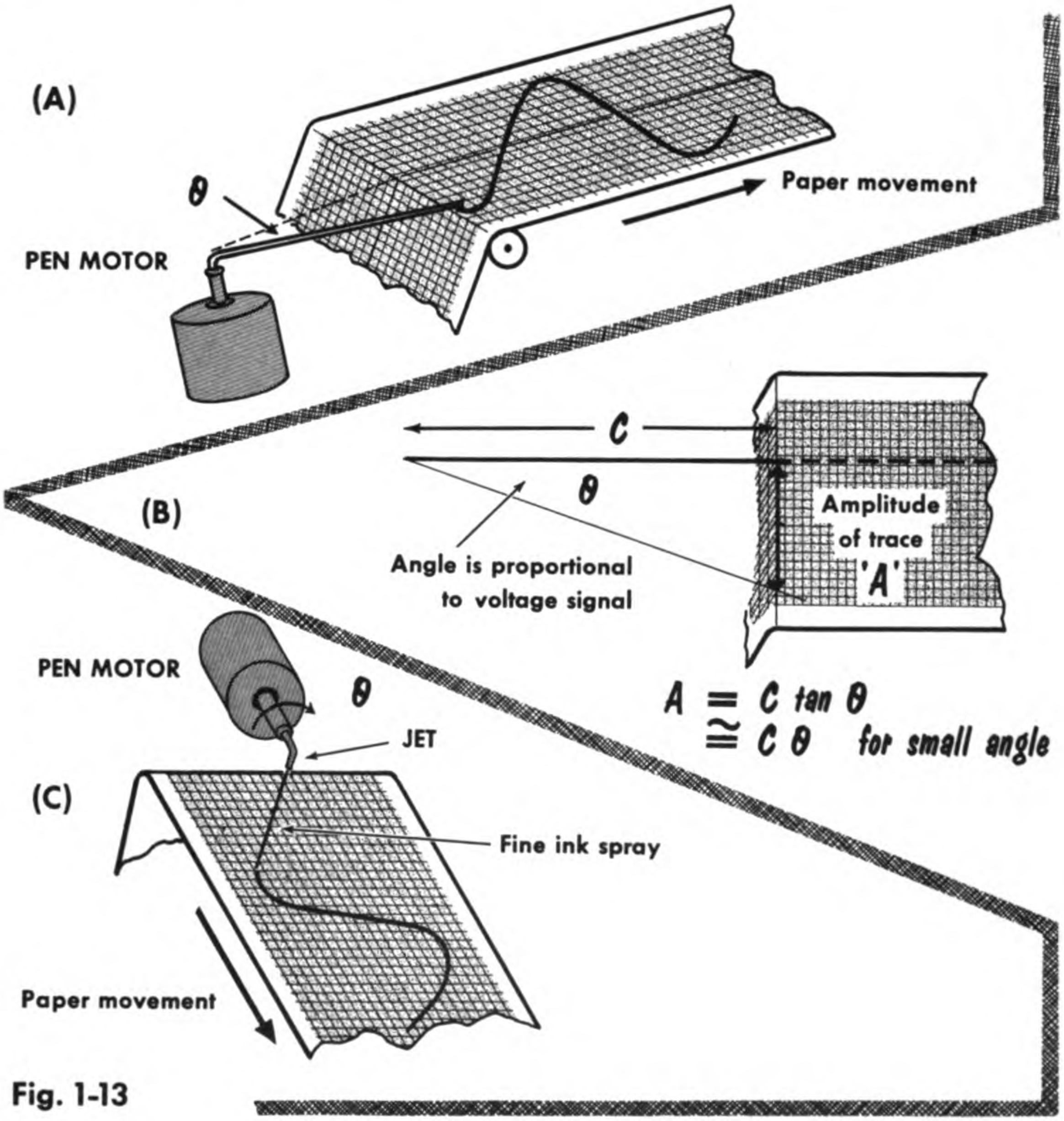


Fig. 1-13

positioned on the zero line of the graph for a zero-voltage signal. If a deflection is required for a zero input (for example, when the computer voltage is known to be always positive, the pen could be deflected to its negative limit for a zero input), then applying a second voltage to the amplifier from the ZERO-ADJUST control causes the pen to be offset as desired.

The diodes placed around the high-gain amplifier limit the voltage at the output of the amplifier, ensuring that the pen deflection never exceeds that allowed by the width of the graph on the paper. When the voltage at

the amplifier output is small neither diode conducts because each is biased so that the plate is at a negative potential with respect to that of the cathode. In this condition the diodes do not interfere with the normal operation of the amplifier. When the voltage amplitude rises to a certain value one or the other diode conducts, and effectively shorts the normal

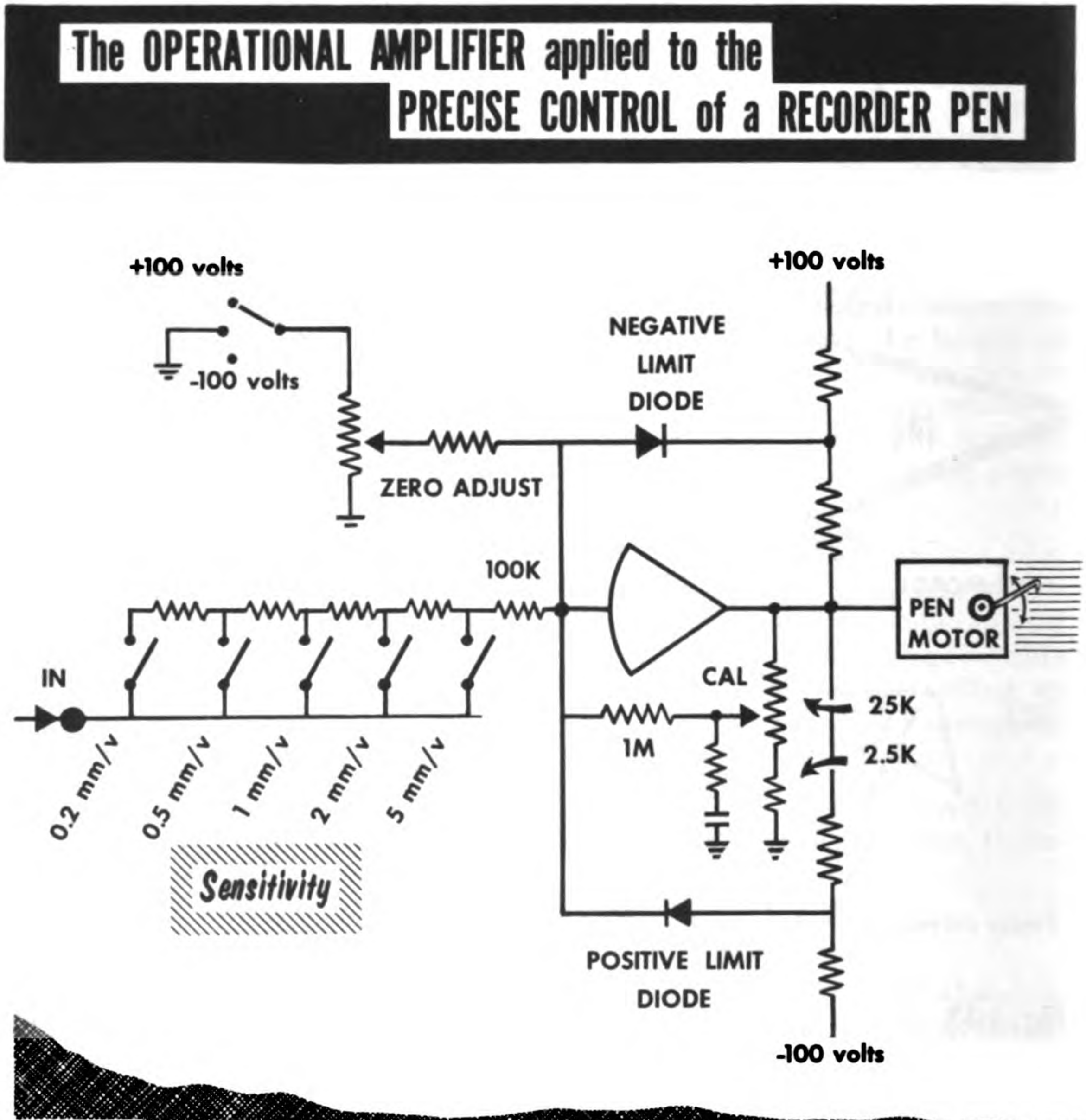


Fig. 1-14

feedback resistor of the amplifier by a low-impedance path. In this condition the amplifier gain is extremely low and the output voltage is essentially held constant at the critical value — that corresponding to maximum allowable pen deflection.

The normal feedback around the amplifier goes through a CALIBRATE

potentiometer, which permits the gain of the amplifier in its nonlimited operation to be adjusted to give the correct pen deflection for a selected sensitivity and actuating signal. Note that in the circuit shown the maximum amplifier gain would be approximately 100.

Some frequency compensation is added to the normal feedback circuit to improve the frequency response of the recorder. The mechanical inertia of the pen, and delay of the magnetic circuit, causes the amplitude of the pen deflection to be less for a 1-volt sinusoid of 50 cps, than for one of 10 cps. Some compensation of this effect is achieved by simple circuits within the amplifier, permitting a fairly flat response up to 60-100 cps.

Plotting Tables

Instead of viewing the variation of a quantity as it changes with time, a plot of the values of one quantity against those of another as both change with time, is frequently desirable. It is then necessary to control two perpendicular displacements instead of just the one controlled in strip-chart recordings. A plotting table is used. A pen travels along a straight arm and is positioned to correspond to the value of one of the interesting quantities. The arm travels along a table perpendicularly to the movement of the pen, and is positioned to correspond to the value of the second quantity. On the table, a sheet of graph paper (Fig. 1-15) is suitably fixed, and as the pen and/or arm moves, the pen produces an ink trace on the paper.

The positioning systems for pen and arm are quite similar, differing only in the values of certain compensating components so that they shall have similar performances in spite of the difference in masses, inertias, and hence the power requirements. A linear precision potentiometer provides a voltage corresponding to the position achieved by the pen, say, and this is compared with the actuating voltage from the computer. Any difference between them is applied to a position-servo system to move the pen until the wiper of the potentiometer attached to the pen achieves a voltage equal to that of the actuating signal. It is not uncommon for the pen of a modern plotting table to be able to produce a precise trace while moving at a speed of 10 or more inches per second.

Scale and Zero Adjustments

For greater convenience, the plotting table must be able to accept signal voltages that change over large ranges of values, or ones that change only slightly. To maintain an acceptable plot of such different variables the plotting table must have an adjustable scale for both pen and arm servos. One simple scheme for making discrete changes in the scale of the servo, allowing the motion of the pen for a 1-volt change in the driving variable, to change from 1 mm to 20 cm, is shown in Fig. 1-16.

A zero adjustment is also included in the circuitry. This adjustment permits the operator to locate at any desired spot on the graph paper, the point of the x-y plot for which x and y are both equal to zero.

To PLOT one VARIABLE AGAINST ANOTHER

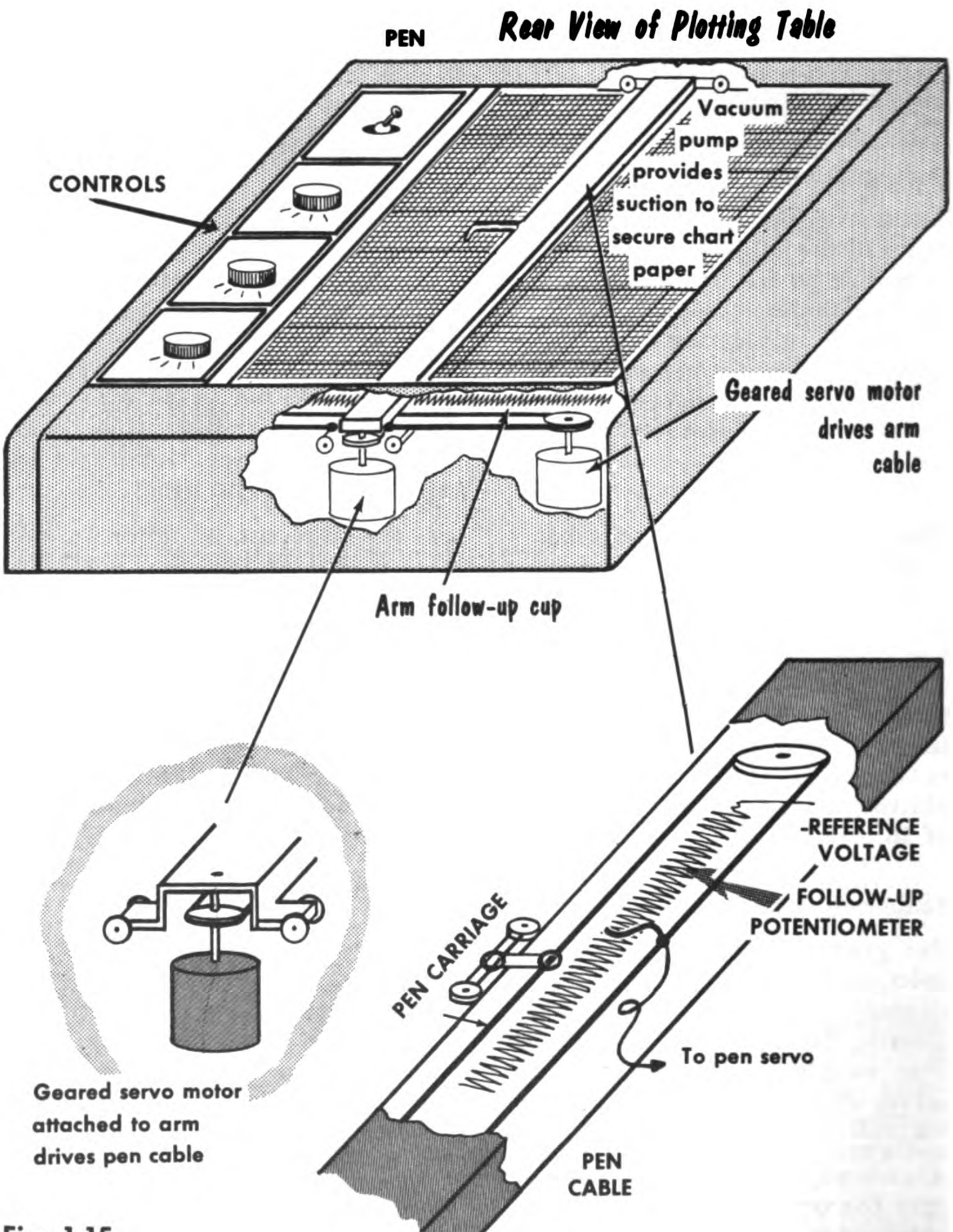
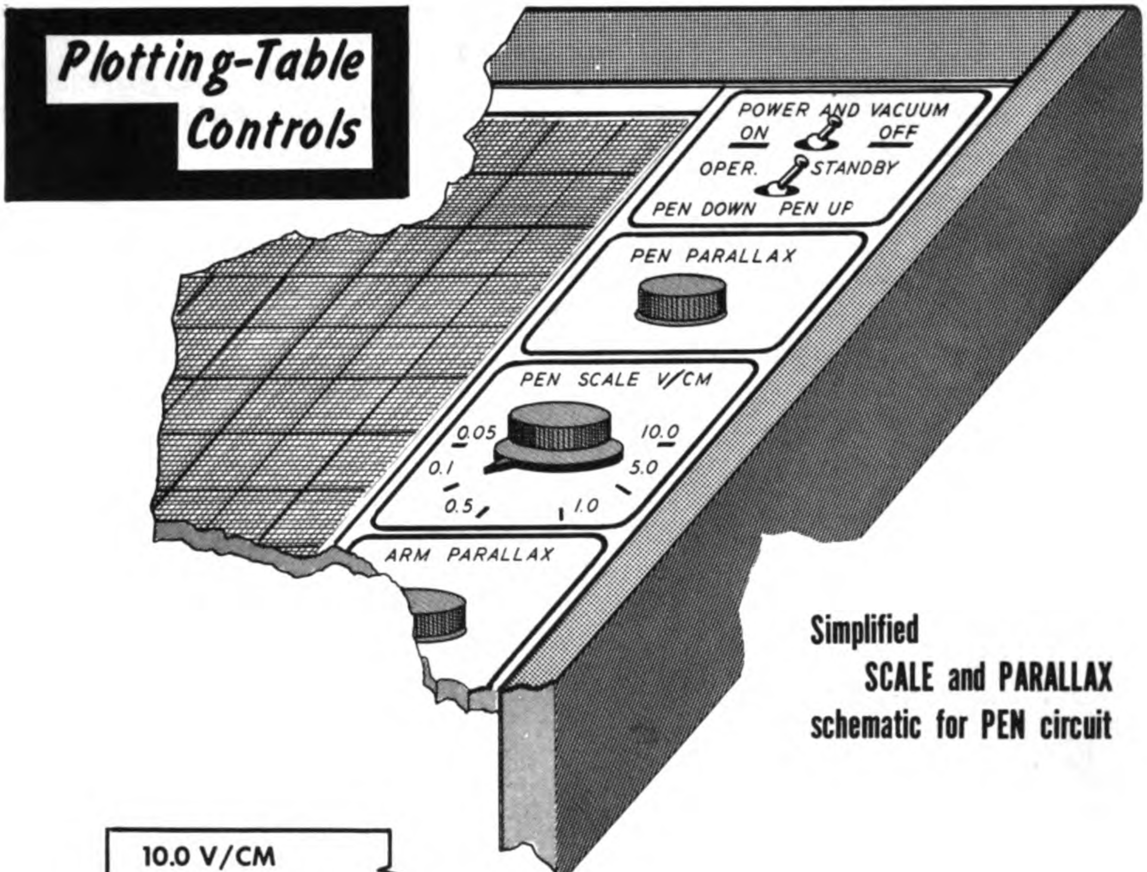


Fig. 1-15



Simplified SCALE and PARALLAX schematic for PEN circuit

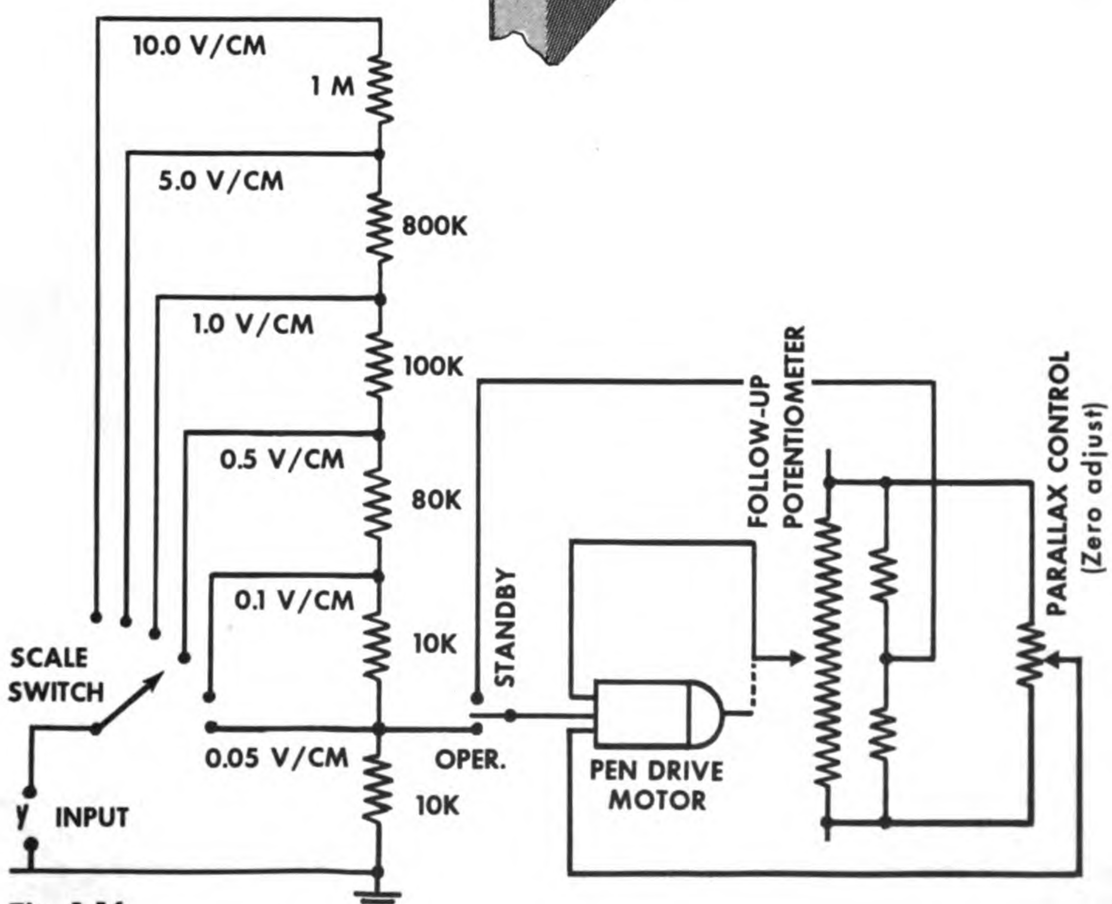


Fig. 1-16

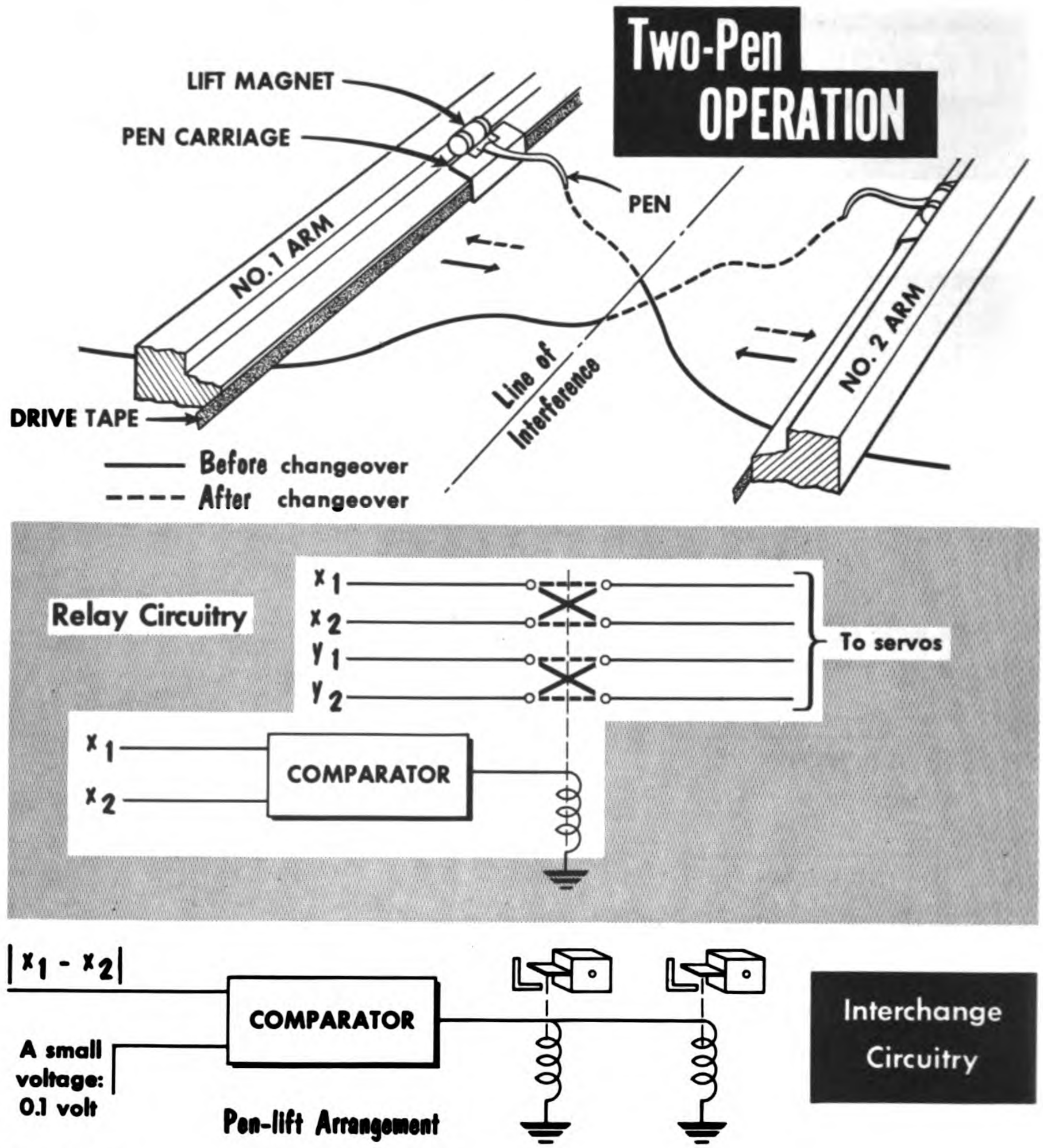


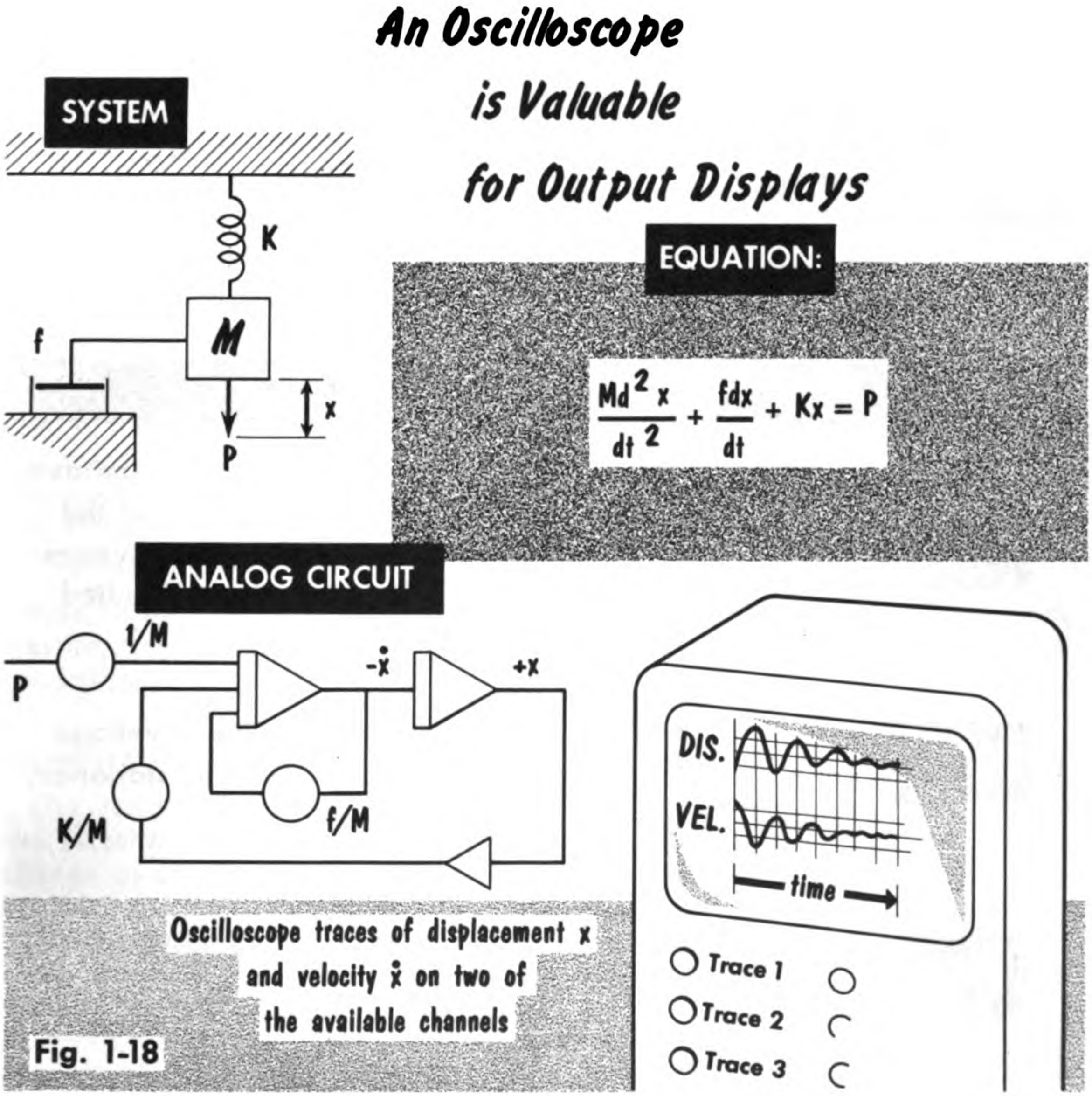
Fig. 1-17

Two-pen Operation

In some plotting tables two pens are included permitting two graphs, y_1 against x_1 and y_2 against x_2, to be recorded. To prevent the pens from interfering, a pen interchange circuit is included which switches the variables being recorded should the arms approach to a point of crossover. The interchange circuitry compares the signals x_1 and x_2, driving the two arm servos, and when they reach the same value, relays are energized to interchange x_1 and x_2, y_1 and y_2. During the time the change is taking place both pens are raised from the paper (Fig. 1-17).

Oscilloscope Displays

Just as there is a need in some applications for all-electronic computing components to give precision under rapidly changing voltage values, so with output-display equipment an all-electronic unit is necessary. An



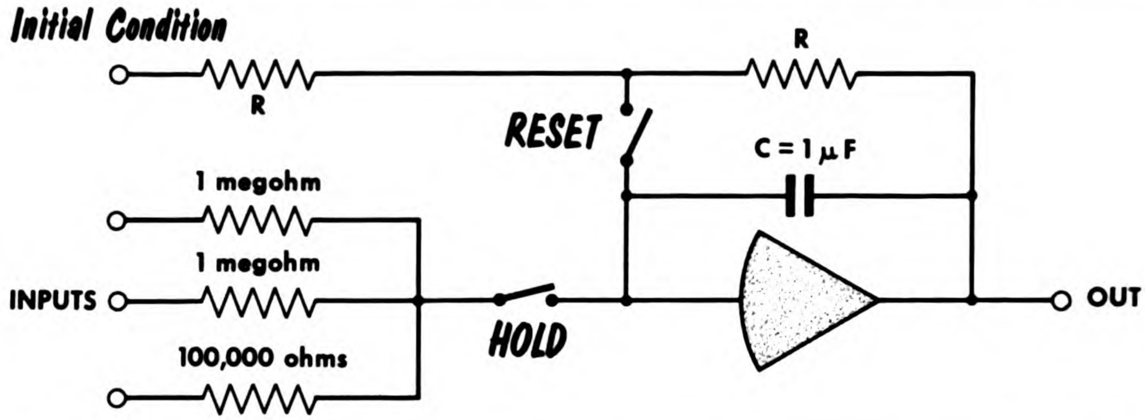
Oscilloscope traces of displacement x and velocity $\dot{x}$ on two of the available channels

Fig. 1-18

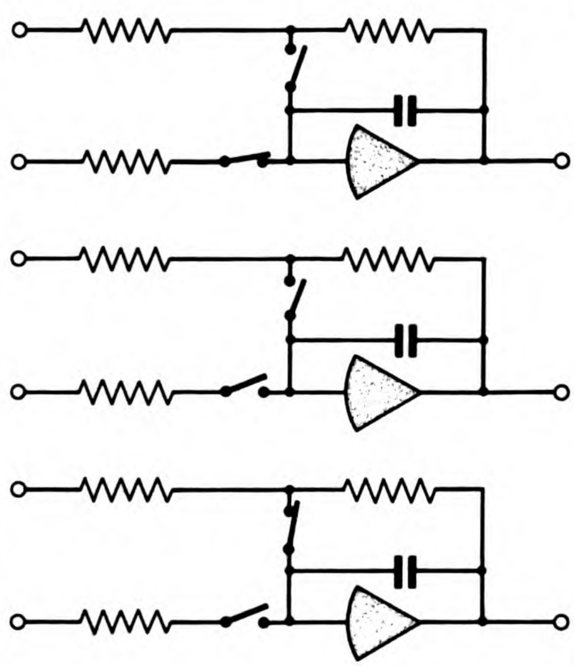
oscilloscope is found valuable for this purpose. Usually having at least a 17-inch display tube, the oscilloscope commonly contains a sampling circuit which permits four or six signals to be shown at one time. The sweep time is adjustable, and with a long-persistence phosphor, traces as long as a few seconds are possible (Fig. 1-18).

The record of the variation of the signal against time is not permanent. However, by using a camera a photograph can be taken, if a permanent record is desired.

The ANALOG MODEL is CONTROLLED by CONTROLLING the INTEGRATORS



Three conditions are possible



1. **OPERATE** — The model behaves as would the physical system being studied

2. **HOLD** — Every voltage remains stationary

3. **RESET** — Initial values are applied

Fig. 1-19

CONTROL CIRCUITRY

Mode-Control Relays

The operational control of an analog computer — the control which permits a model built on the computer to start behaving as the primary physical system does — is achieved with the integrating amplifiers. Once the model has been built by interconnecting the computing components and setting the attenuators to the appropriate values, everything is ready to obtain results. Every computation is being performed as required except for the

time integrations, and it is by control of these operations that a solution is started and stopped.

Two relays with each integrator are involved in the mode control and they are both energized from the master control switch (Fig. 1-19). The HOLD relay, when energized, disconnects from the grid of the operational amplifier all input resistor connections, rendering the amplifier unable to integrate the input voltages. The RESET relay, when energized, places across the 1-μf feedback capacitor a shorting resistor, and connects to the amplifier grid an additional input resistor for initial conditions, equal in value to the resistor shorting the capacitor.

The master-control switch has three important mode-control positions. 1. OPERATE: Both relays associated with each integrating amplifier are de-energized, permitting proper operation of these amplifiers and thus causing the analog model to "work". 2. HOLD: All hold relays are energized, open-circuiting the integrators' input connections and halting the behavior of the model. 3. RESET: All reset relays and hold relays are energized, causing the voltages at the outputs of integrators to be stationary, and equal to the initial values determined by the voltages applied to the initial-condition inputs. The integrating amplifiers are here acting as low-pass filters with relatively short time constants, and thus their output voltages quickly assume the value appropriate to the initial-condition voltages driving them.

These three positions of the mode-control switch allow experiments to be started, stopped at any time, continued, or reset to different initial conditions.

Repetitive Operation

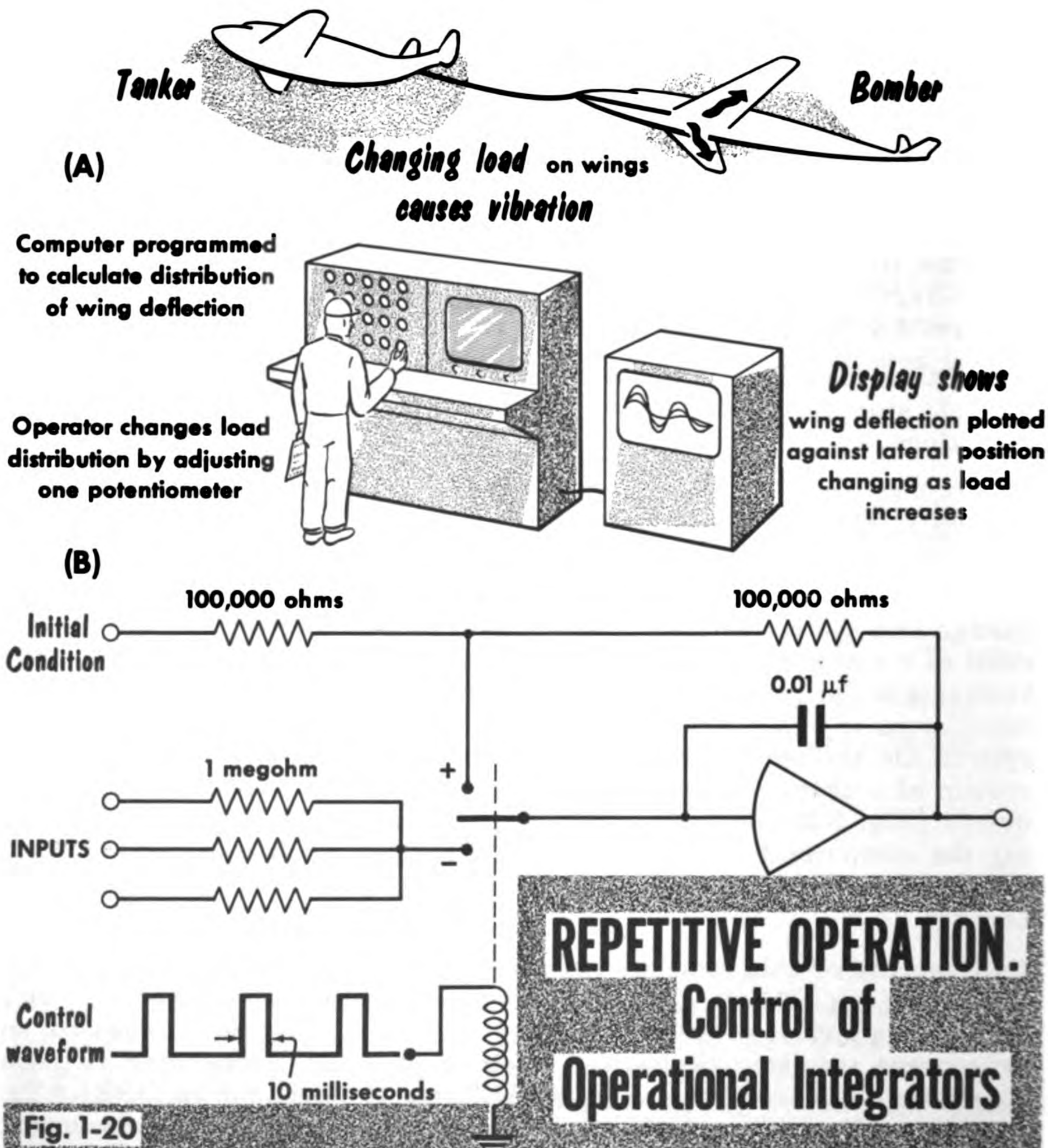
Quite frequently, the initial conditions of an analog experiment remain the same while one investigates the different behaviors resulting from a change in a system parameter — the gain of an amplifier, the spring constant of a suspension system, the mass of a pilot-valve piston, the specific heat of a body, etc. Any one of these parameters might be changed from experiment to experiment, to determine their importance in the physical system. On the other hand, the effect on the behavior of a fixed physical system of a change in one of the initial conditions is also frequently required [Fig. 1-20 (A)]. Investigations of this kind would require switching the computer from RESET to OPERATE and back again, as many times as desired, with only a change in the setting of one attenuator in between.

Experiments of this kind are greatly facilitated by the use of *repetitive operation* [Fig. 1-20 (B)]. By replacing the 1-μf integrating capacitors with ones having a value of 0.01-μf, the speed of solution is greatly increased, an experiment requiring typically 0.1 second. The mode switching is made automatic — the computer is in RESET for 10 msec, and in OPERATE for a time adjustable to 1 second. With such a fast operation an attenuator

setting can be continually and slowly adjusted while the experiments are being performed. Its effect on the behavior will be seen immediately and as though it were being changed between experiments.

To go with the fast operation one requires all-electronic components, including an oscilloscope output-display unit on which to view the computer variables.

In repetitive operation the reset and hold relays of an integrator are replaced by one relay which is controlled by a rectangular switching waveform that switches the amplifier grid between the initial-condition resistors and the input resistors, as shown in the figure.



REPETITIVE OPERATION. Control of Operational Integrators

Fig. 1-20

QUESTIONS

1. Explain the operation of the servo voltmeter. Indicate its superiority over ordinary voltmeters, and discuss its *dis*advantages.
2. Describe in detail the Thompson-Varley bridge. Show how a voltage of 73.46 volts is produced by this unit.
3. Give the binary-coded decimal equivalents of:-

   10.73 39.62 56.14 82.28
4. Explain the operation of a flip-flop and indicate how this device can be used in storing binary numbers.
5. Explain the operation of a binary-to-decimal converter and use for illustration the circuit that converts 0111 into 7.
6. What would be the next control pulse in a digital voltmeter under the following conditions?

Unknown Voltage	Voltmeter Voltage
16.62 v	15.99 v
20.45 v	20.99 v
61.09 v	60.99 v
33.98 v	33.99 v

7. What do you understand by the terms, rectilinear and curvilinear recording? Give two examples of rectilinear recorders.
8. Explain the mode control of an analog computer.
9. What is repetitive operation? Explain how it is achieved on an analog computer.

Chapter 2

PROGRAMMING AND PROBLEMS

USING THE D-C ANALOG COMPUTER

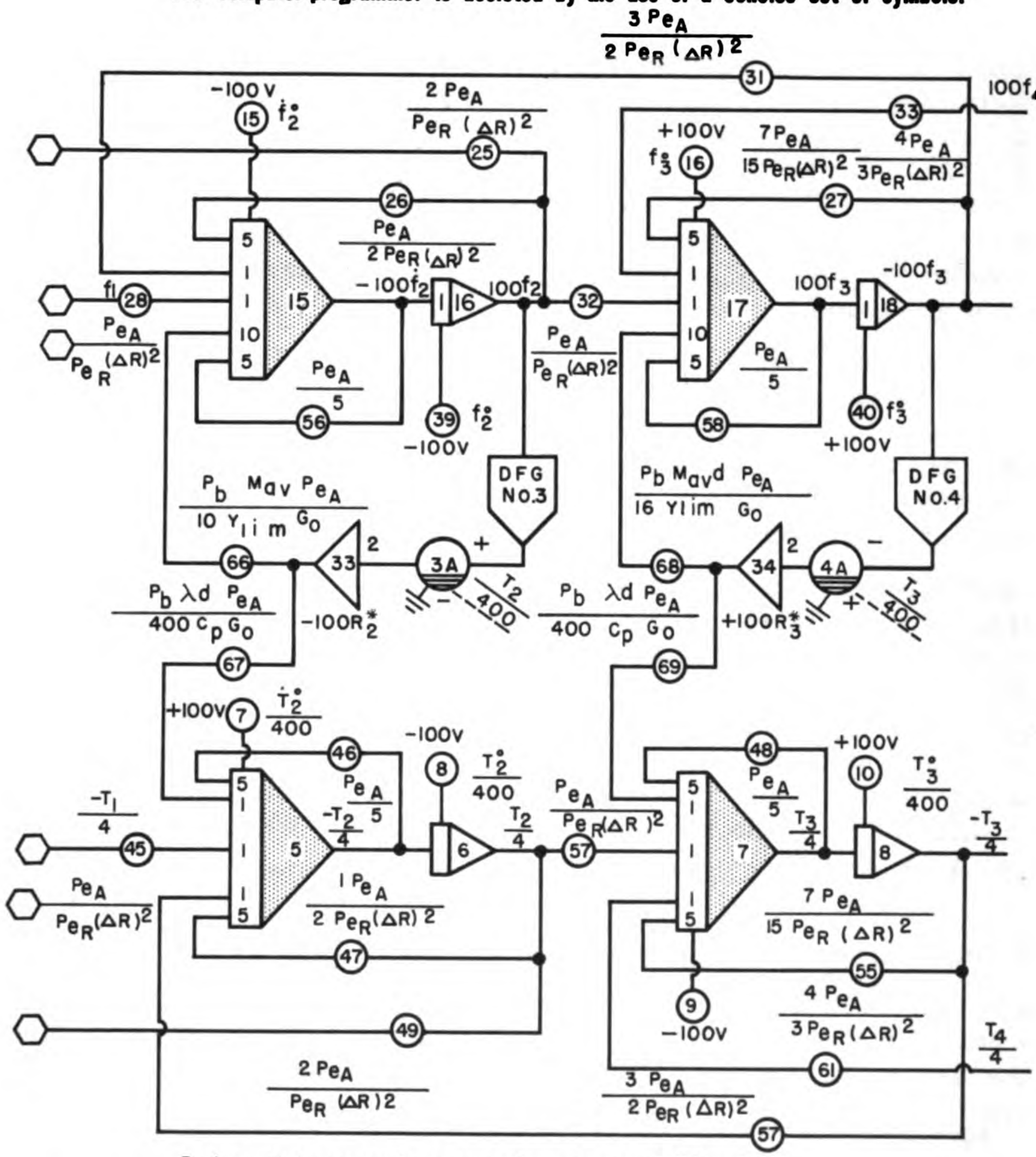
Schematic Diagrams

In all branches of engineering and scientific endeavor, graphs, diagrams, and drawings are heavily relied upon for clear presentation of complex functional relationships and detailed features of objects, and even of ideas. Somewhat inadequate in effective verbal expression, engineers are wont to devise most elaborate graphic means of description. The field of analog computing is not lacking in this respect, for it has its own brand of symbols and schematic diagrams. Fortunately, the use of analog computer schematic diagrams is not at all hard to master. After one is familiar with the basic building block symbols, which have already been presented, understanding of complex drawings is rapidly acquired with practice. With a little experience one can quickly deduce from a well-drawn computer schematic, an astounding amount of information about the physical system being simulated, about the form of the solution, about the effect of component inaccuracies upon the solution, and even about the experience of the programmer.

In the d-c analog computer it is always voltage which is the computer variable, never current or charge or magnetic flux, although we know that these quantities vary according to precise physical laws and therefore *might* be used were it convenient. To transmit a voltage from one circuit to another always requires *two* conductors, for what we are really doing is transmitting a potential *difference* — that is, a voltage measured with respect to some reference potential. Most commonly this reference potential is assigned the value of 0 volts and is called *ground*. When all voltages are measured with respect to a common ground then only one conductor is required for each signal, provided the common ground plays the role of the second conductor for every signal. Such a common-ground system is used in analog computers, and hence the general practice is to draw single line diagrams to show the transmission of a signal without ever indicating

TYPICAL SMALL COMPUTER PROGRAM

The computer programmer is assisted by the use of a concise set of symbols.



Each symbol represents a particular computer building block.

Fig. 2-1 **The completed schematic** ***is the COMPLETE COMPUTER PROGRAM***

the ground line. Lines drawn from the output of a computer symbol to the input of another symbol correspond directly with the actual wires (called *patch cords*) which interconnect the building blocks (usually at a patch panel). Notice that there is no need to place arrows on these lines to indicate the direction of information flow, because the output and input connections are clearly differentiated on each symbol (except the potentiometer). Obviously, information always flows from an output connection to an input connection, through the patch cords.

Every computer has its own reference-voltage system. The most common reference voltages used are + and − 100 volts. Whenever a constant voltage of a precise magnitude (100 or some precise fraction thereof) is required in a computer program, such as the initial condition for an integrator or a constant term in an equation, the programmer selects 100 volts from one of a group of reference-voltage terminals and uses it in the problem (Fig. 2-1). Such connections are indicated by simply +100 volts or −100 volts, written next to the line being supplied with the reference voltage. The two reference voltages are also often available at a switch directly connected to the top of a potentiometer. If the programmer intends to use the switch he shows it by writing +100 or −100 above the "pot" symbol, with no other input shown.

Often, two computers are connected together and it is necessary to bring computer variables from one to the other. Shielded cables (called *trunks*) are provided for this purpose. The symbol for a trunk is a small hexagon. Trunks are also used to connect the program patch panel to a section of one of the racks with many blank connectors to which may be attached special components, and special groupings of extra-function switches.

All connections used in the drawing of schematic diagrams are intended to minimize the likelihood of human error in the use of the program. This is very necessary, since the patch panel of a single computer problem may contain several hundred patch cords — there are several thousand holes in a typical patch panel. Once a complete schematic has been drawn it represents a total statement of the problem, for it is a graphic analog of the equations of the problem. All further programming and analysis is often done directly from the schematic diagram as if it were a real picture of the primary system.

First-Order Systems: Single-Integrator Circuits

The simplest differential equation we could write down and the one which appears most frequently in the mathematical formulation of physical laws is the *first-order, ordinary, linear* differential equation with constant coefficients. That is:

$$a_1 \frac{dx}{dt} + a_0 x = o$$

where $x(t)$ represents some physical variable, and the coefficients a_1 and a_0, are constants characterizing the physical system. Several typical physical systems with first-order dynamic behavior are illustrated below. Generally, the systems have an *energy-storage* device (determines a_1) and an energy-dissipating device (determines a_0). The time required for any initial stored energy to decay to 37% of its value is called the *system time constant*, τ.

$$\tau = \frac{a_1}{a_0}$$

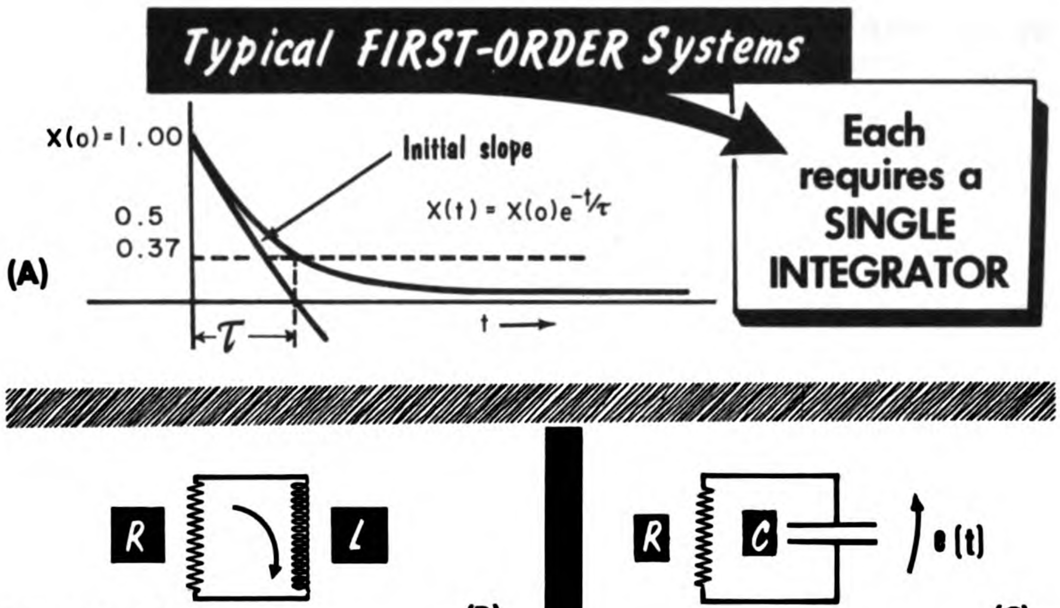


Fig. 2-2 (A) (B) (C)

The reciprocal, $1/\tau$ is a measure of the uppermost useful frequency at which one can cause the variable to alternate.

The solution to the above equation, and hence for all the systems shown, is illustrated graphically in Fig. 2-2, part (A). Analytically the solution has the form

$$x(t) = x(o)\epsilon^{-t/\tau}$$

In each of these systems it is assumed that some amount of energy is initially stored in the energy-storage element and is then dissipated.

1. *R-L circuit* [Fig. 2-2 (B)]. Magnetic field energy is stored by the inductor (L); energy is dissipated as heat by the resistor.

   The equation: $L\dfrac{di}{dt} + Ri = 0$

   The solution: $i(t) = i(o)\,\epsilon^{-\frac{R}{L}t}$, $i(o)$ = initial current

   The stored energy: $W = \dfrac{1}{2}Li^2$

2. *R-C circuit* [Fig. 2-2 (C)]. Capacitor stores electric charge; resistor dissipates energy.

   The equation: $C\dfrac{de}{dt} + \dfrac{1}{R}e = 0$

   The solution: $e(t) = e(o)\,\epsilon^{-\frac{1}{RC}t}$, $e(o)$ = initial voltage

   The stored energy: $W = \dfrac{1}{2}Ce^2$

3. *Moving mass with friction* [Fig. 2-2 (D)]. Kinetic energy stored by mass; friction dissipates energy.

Fig. 2-2 (D)

The equation: $M\dfrac{dv}{dt} + fv = 0$

The solution: $v(t) = v(o)\,\epsilon^{-\frac{f}{M}t}$, $v(o)$ = *initial velocity*

The stored energy: $W = \dfrac{1}{2}Mv^2$

4. *Rotating mass* [Fig. 2-2 (E)]. Kinetic energy stored by mass; friction dissipates energy.

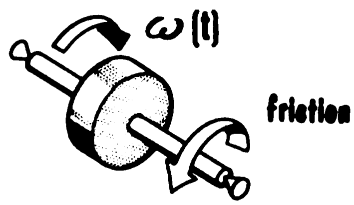


Fig. 2-2 (E)

The equation: $I\dfrac{d\omega}{dt} + f\omega = 0$

The solution: $\omega(t) = \omega(o)\,\epsilon^{-\frac{f}{M}t}$, $\omega(o)$ = initial angular velocity

The stored energy: $W = \dfrac{1}{2}I\omega^2$

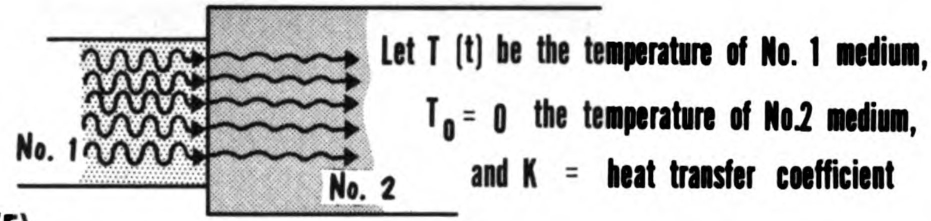
5. *One-dimensional heat transfer across a plane* [Fig. 2-2 (F)]. Heat energy stored by the medium No. 1. Heat energy lost by No. 1 by transfer into medium No. 2.

The solution: $\dfrac{dT_1}{dt} = -K(T_1 - T_0)$ or $\dfrac{dT_1}{dt} + KT_1 = 0$

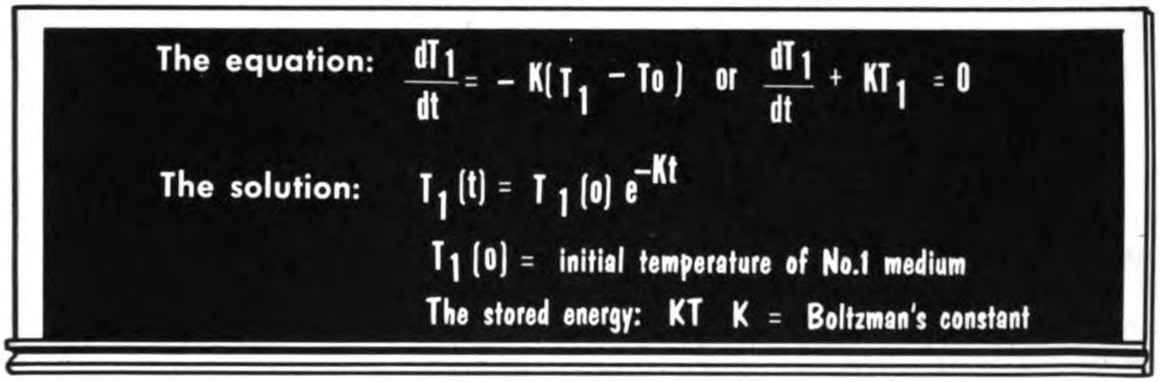
The equation: $T_1(t) = T_1(o)\,e^{-Kt}$, $T_1(o)$ = initial temp. of No. 1 medium

The stored energy: KT, K = Boltzman's constant

The analog computer simulation for all these systems is the same. The



(F)



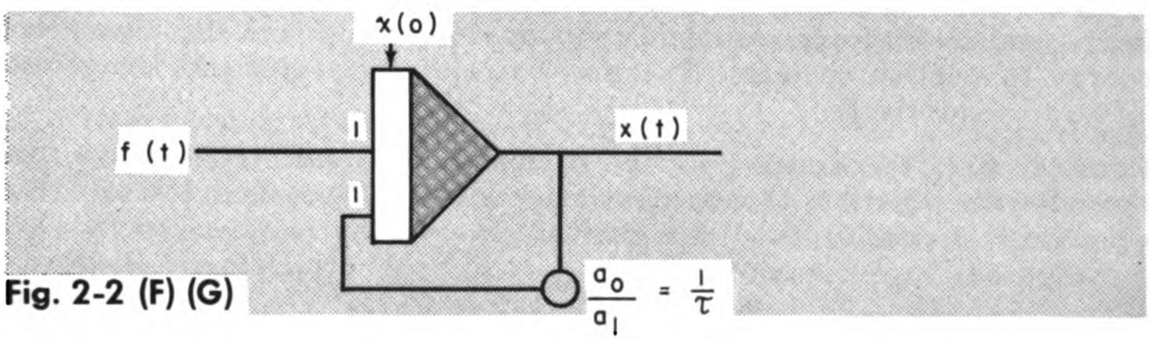


Fig. 2-2 (F) (G)

schematic for the general case, where an external force may be supplying energy to the system, is shown in Fig. 2-2 (G). The equation is:

$$a_1 \frac{dx}{dt} + a_0 x = f(t)$$

Second-Order Systems: Two-Integrator Circuits

The next most common equation occurring in the mathematics of dynamic physical systems is the *second-order*, ordinary, linear differential equation, with constant coefficients:

$$a_2 \frac{d^2x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = f(t)$$

or

$$a_2\ddot{x} + a_1\dot{x} + a_0 x = f(t)$$

$f(t)$ is some arbitrary function representing an external agent supplying or taking energy from the system. The coefficients a_2, a_1, a_0 are determined by the physical properties of the system.

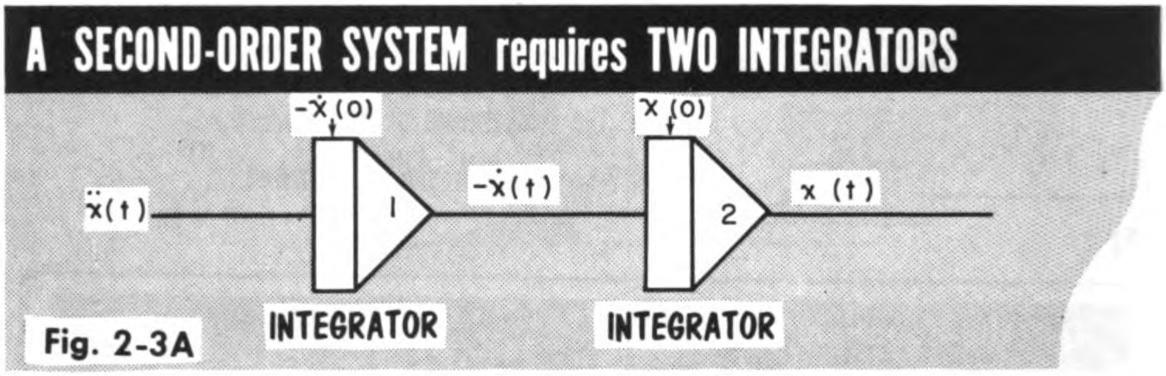


Fig. 2-3A

It has been shown how this equation describes the behavior of an R-L-C circuit and a mass-and-spring system (pp. 1–27; 1–106). In general, this equation or a modified version of it, appears in the mathematical description of every system that can store both kinetic and potential energy (or their electrical analogs: magnetic and electric field energy), and that covers a lot of territory! Characteristic of these systems is the presence of three different kinds of elements: one for storing kinetic energy (a mass), one for potential energy (a spring; elastic diaphram), and one that dissipates energy (a friction surface). The properties of these elements determine a_2, a_0, a_1, respectively.

Consider next the drawing of the computer schematic diagram for the second-order equation. If we proceed as we did in Volume 1, p. 1–114, the equation is first solved for its highest derivative:

$$\ddot{x} = -\frac{a_1}{a_2}\dot{x} - \frac{a_0}{a_2}x + f(t)$$

Then, assuming we have a voltage varying as $\ddot{x}(t)$ at the input to integrator No. 1, we integrate twice to get $x(t)$ with the result shown in Fig. 2-3A.

Next, having $\dot{x}$ and x available, we form the variable $\ddot{x}$ by adding the three

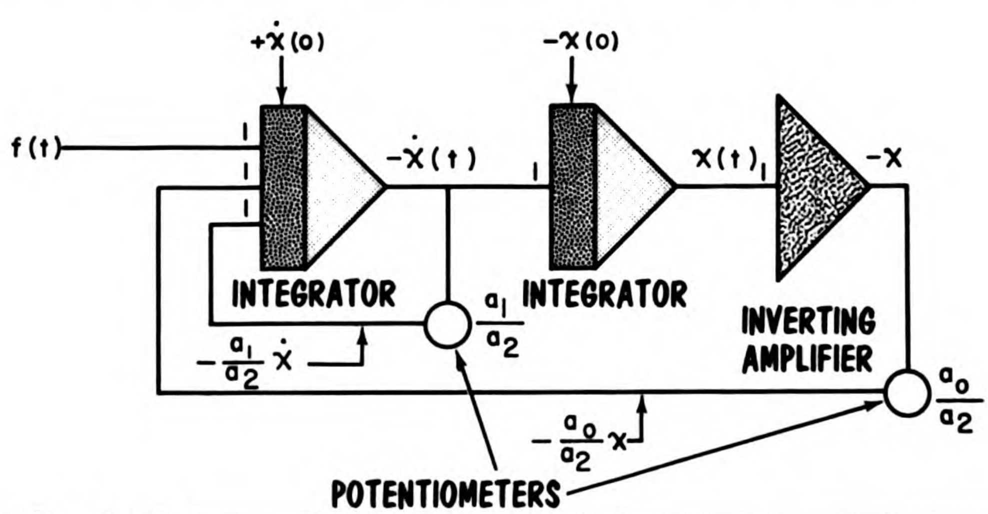


Fig. 2-3B

terms of the above equation at the input of integrator No. 1. Since $[-(a_0/a_2)x]$ is required, an inverting amplifier must be used (Fig. 2-3B).

Second-Order System Responses

In the simulation of real physical systems, the transient solution of the second-order equation can have one of three forms. Which form exists depends upon the relative energy storage and dissipative capacities of the system (thus upon the relative values of a_2, a_1, and a_0).

1. *Low dissipation (underdamped)* [Fig. 2-4 (A)]:

$$a_1^2 < 4a_2a_0$$

2. *Critical damping* [Fig. 2-4 (B)]:

$$a_1^2 = 4a_2a_0$$

3. *High dissipation (overdamped)* [Fig. 2-4 (C)]:

$$a_1^2 > 4a_2a_0$$

For case 3, the analog computer circuit shown in schematic diagram form in Fig. 2-4 (D) is also possible.

If it were possible to construct a frictionless second-order mechanical system, a resistance-free electric circuit, or some other form of ideal device or perpetual-motion machine, the transient solution to the second-order differential equation would take on a fourth form. With no friction or resistance the damping coefficient a_1 would be zero, hence the equation would reduce to

$$\ddot{x} = -\frac{a_0}{a_2}x$$

A solution to this equation is known to be of the form,

$$x(t) = A \sin \omega t$$

where

$$\omega = \sqrt{\frac{a_0}{a_2}}$$

and *A is the largest value of x(t)*. Thus the solution has the form shown in Fig. 2-4 (E).

Furthermore, the first derivative, $\dot{x}(t)$, has the form $B \cos \omega t$ [Fig. 2-4 (F)].

The computer schematic is shown in Fig. 2-4 (G).

Since $a_1 = o$, the potentiometer around the No. 1 integrator can be removed. This computer circuit is very useful, not only because it simulates certain physical systems, but because it produces a continuous *sine-wave* function from integrator No. 2, and a *cosine-wave* function from integrator No. 1. The sine and cosine functions are exceedingly useful functions for testing

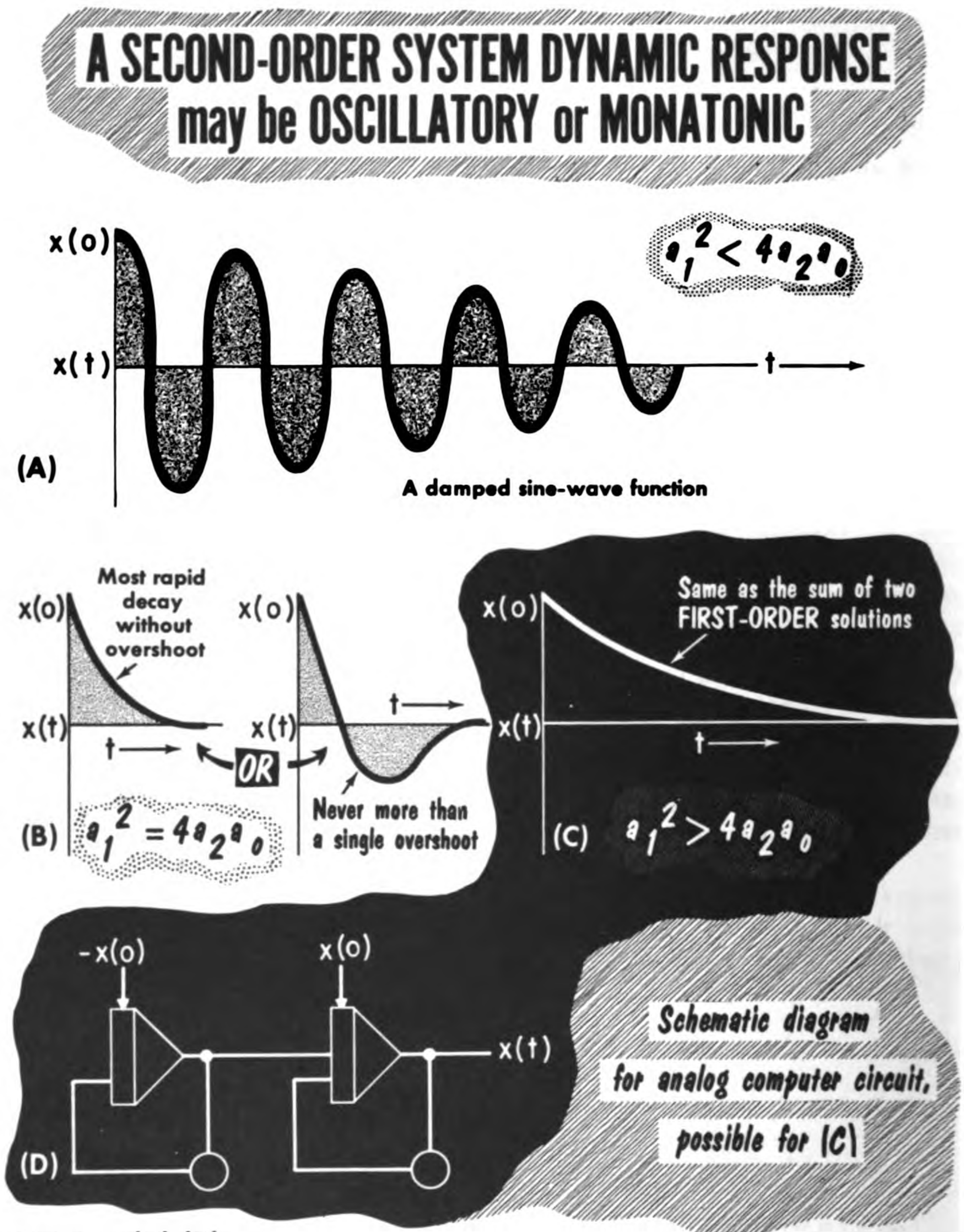


Fig. 2-4 (A)-(D)

the performance of computer models and actual physical systems. For example, the function, $f(t)$, in the equation on p. 39 might well be sin ωt:

$$a_2\ddot{x} + a_1\dot{x} + a_0x = \sin \omega t$$

In such a case, the above circuit (part (G) of the figure), is a sine-function generator [Fig. 2-4 (H)] and is used as a driver for the second-order differential equation setup of integrators No. 2, and No. 3.

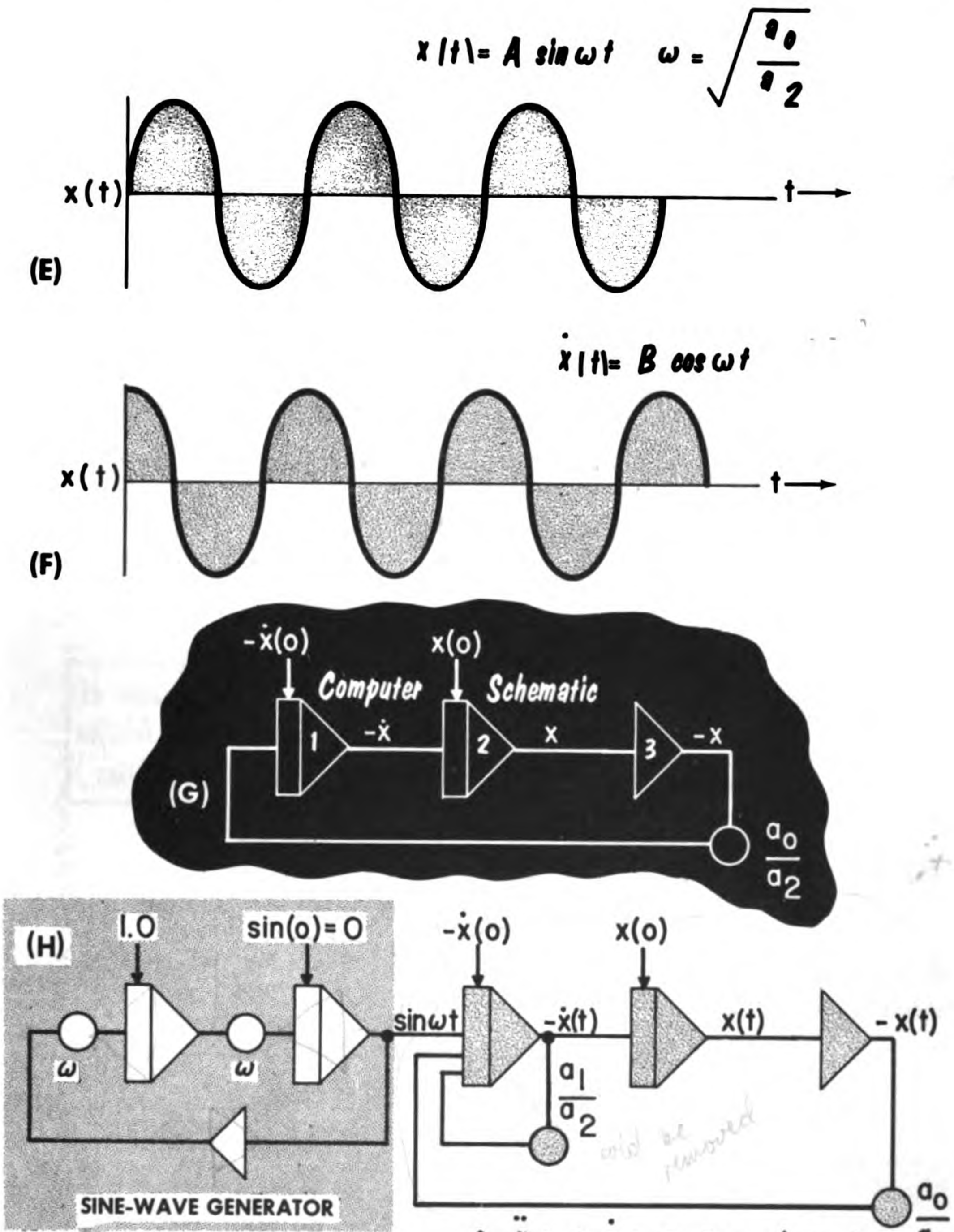


Fig. 2-4 (E)-(H)

Coupled Systems

In Volume 1, p. 1–109, we described a double mass and spring system, and showed the *two linear, second-order, ordinary differential equations* describing the system. These were called *simultaneous equations*, because they cannot be solved independently but must be solved simultaneously. They

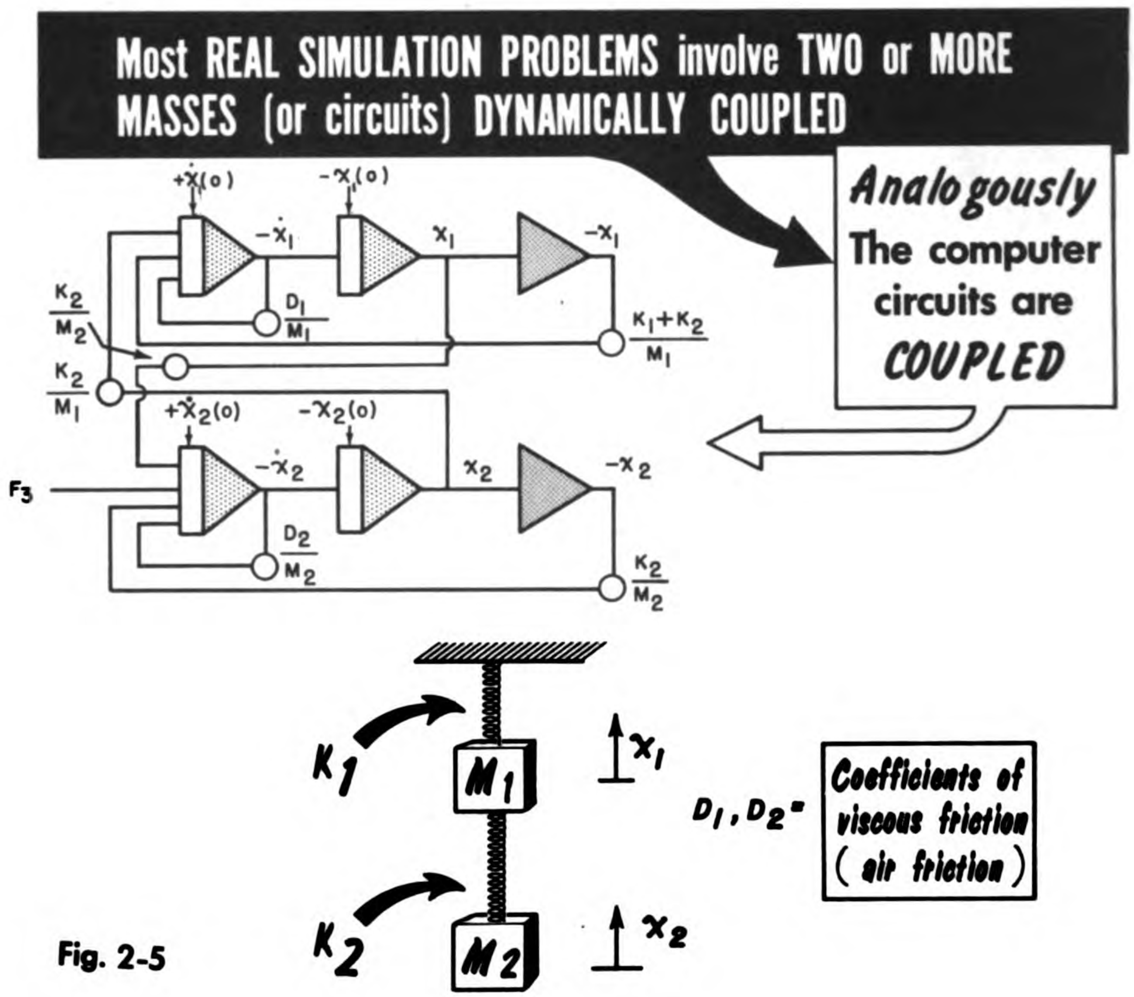


Fig. 2-5

are also called *coupled equations* because they describe two masses *coupled* together by a spring.

$$M_1\ddot{x}_1 + D_1\dot{x}_1 + (K_1 + K_2)\,x_1 - K_2x_2 = 0$$

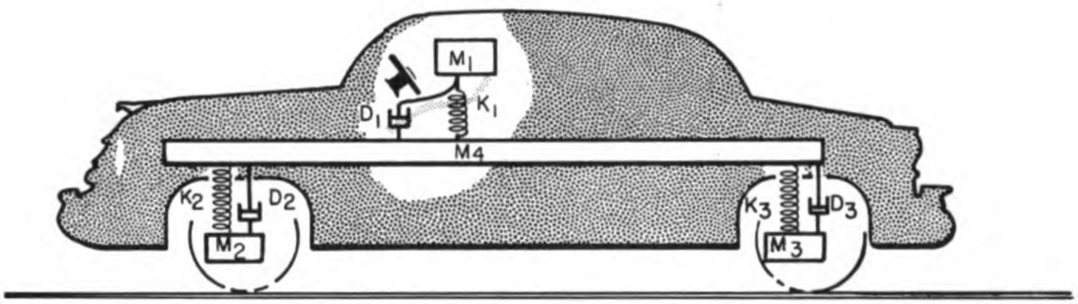
$$M_2\ddot{x}_2 + D_2\dot{x}_2 + K_2x_2 - K_2x_1 = F_3$$

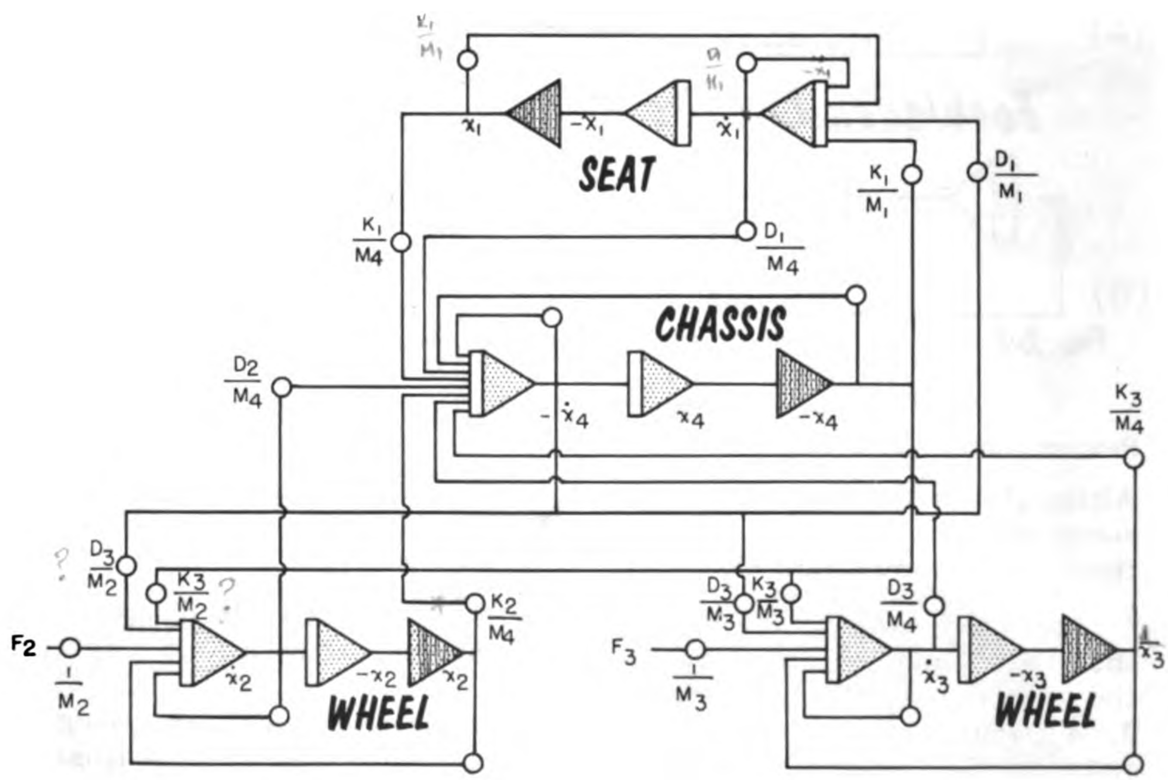
The analog computer schematic diagram for these equations is illustrated in Fig. 2-5.

The equations for a simplified automobile suspension system were presented in Volume 1, p. 1–111. The schematic diagram for the computer program of these equations is shown in Fig. 2-6.

$$\begin{aligned}
&M_1\ddot{x}_1 + D_1\dot{x}_1 && + K_1x_1 && - D_1\dot{x}_4 - K_1x_4 = 0\\
&M_2\ddot{x}_2 + D_2\dot{x}_2 && + K_2x_2 && - D_2\dot{x}_4 - K_2x_4 = F_2\\
&M_3\ddot{x}_3 + D_3\dot{x}_3 && + K_3x_3 && - D_3\dot{x}_4 - K_3x_4 = F_3\\
&M_4\ddot{x}_4 + (D_1 + D_2 + D_3)\dot{x}_4 && + (K_1 + K_2 + K_3)x_4 && - D_1\dot{x}_1 - K_1x_1\\
& && && - D_2\dot{x}_2 - K_2x_2\\
& && && - D_3\dot{x}_3 - K_3x_3 = 0
\end{aligned}$$

Dynamic Simulation of an Automobile Suspension System





Approximation of the real system by four coupled masses permits a realistic study to be made of the effect of various spring and shock-absorber characteristics upon the motion of the passenger

Fig. 2-6

POSITIVE FEEDBACK can cause UNSTABLE COMPUTER CIRCUITS

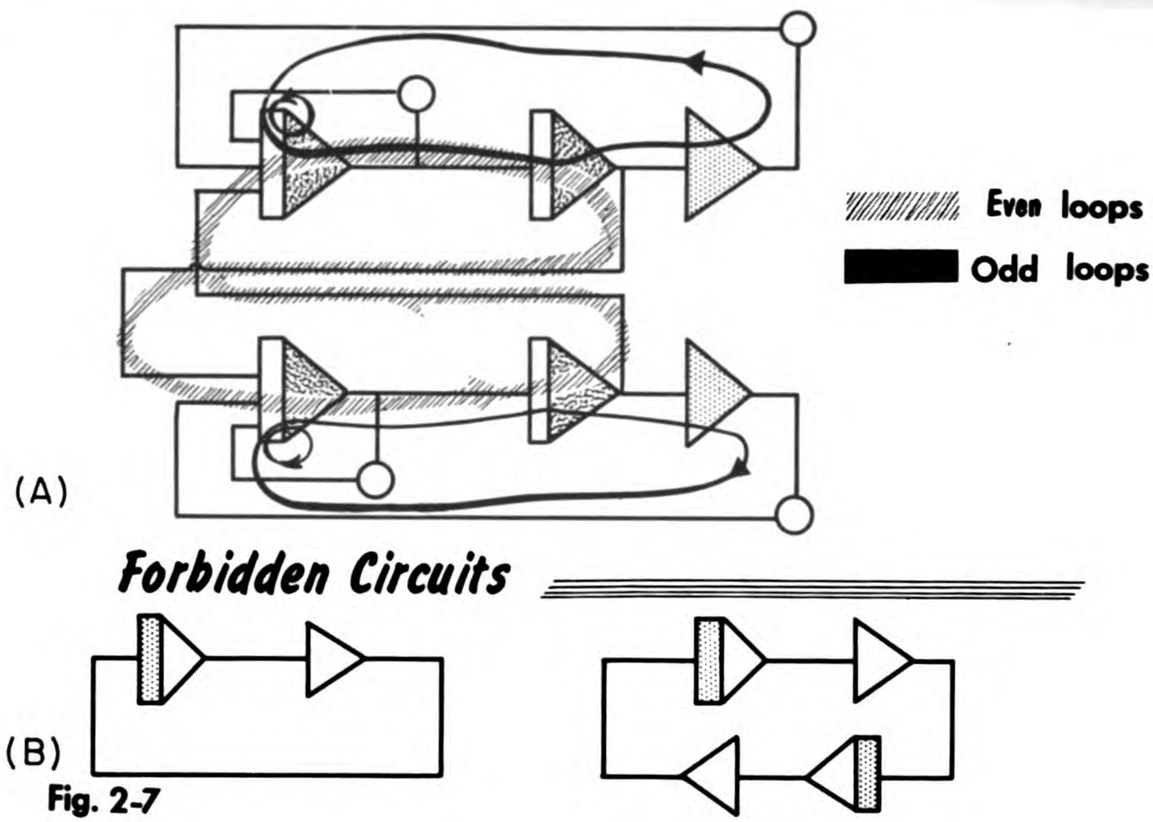


Fig. 2-7

Programming: Two Important Precautions

Although, for the most part, analog computer building blocks can be interconnected in any manner without getting the programmer into hot water, there are two important exceptions to this generality.

1. *Positive feedback:* If a program results in a positive feedback circuit, the programmer will be rapidly informed of the fact by a loud ringing of the computer overload-alarm system.
2. A circuit with *algebraic feedback* can cause errors without giving any warning. Fortunately both situations are readily recognized from the schematic diagram.

Positive feedback can be recognized from the schematic diagram by tracing a path through several computing elements to the origin of the path. If in any such loop there is an *even* number of sign inversions (i.e., amplifiers), an increase in the voltage at any point in the loop will be fed back through the loop as an additional increase, which again is fed back through the loop as a further increase. This is known as *positive feedback*. If the signal feedback is larger than the signal initiating the action, the voltage will increase without bound. Thus a positive feedback loop with a loop gain

(amplification) greater than 1, causes all amplifiers in the loop to saturate at 100–150 volts, actuating the overload alarm. This of course, suggests that there are certain differential equations that cannot be solved with the d-c analog computer. However, it is important to understand that all those equations which cannot be solved because of the resulting unstable computer circuit, correspond to non-real physical systems. That is, variables which increase without bound imply either a perpetual motion device, or a source of infinite energy. Thus the instability of the positive feedback loop is not a serious restriction upon the usefulness of the computer.

Quite often, positive feedback is gainfully employed in simulating regenerative systems, devices with saturation limits, and in coupled systems (as previously referred to). In the double-mass system it can be seen that if mass No. 2 is accelerated briefly, mass No. 1 will also move, and in a direction resulting in an *increase* in the original acceleration of mass No. 2. In such cases, positive feedback is associated with one or more paths of negative feedback. Any loop containing an *odd* number of amplifiers is a negative feedback circuit. Observe the even and odd loops in the double mass-spring schematic illustrated in Fig. 2-7(A).

In all useful computer circuits the negative feedback must dominate the positive, or the amplifiers in the positive loop must have saturation limits (as part of the simulation) [Fig. 2-7 (B)].

Algebraic feedback, or an algebraic loop, exists when the computer schematic indicates a closed d-c path (a loop) through several computing ele-

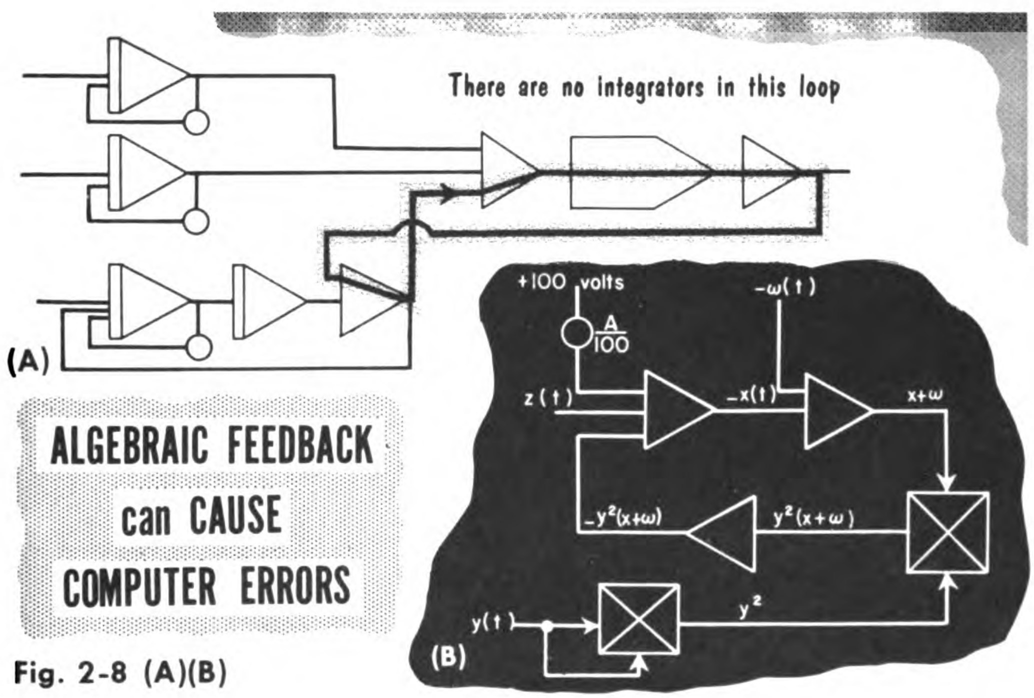


Fig. 2-8 (A)(B)

ments but *not* passing through an integrator. When one is concerned with the solution of differential equations all loops on the schematic ordinarily contain at least 1 integrator. It is possible however, that in a complicated set of simultaneous differential equations there might be some redundant

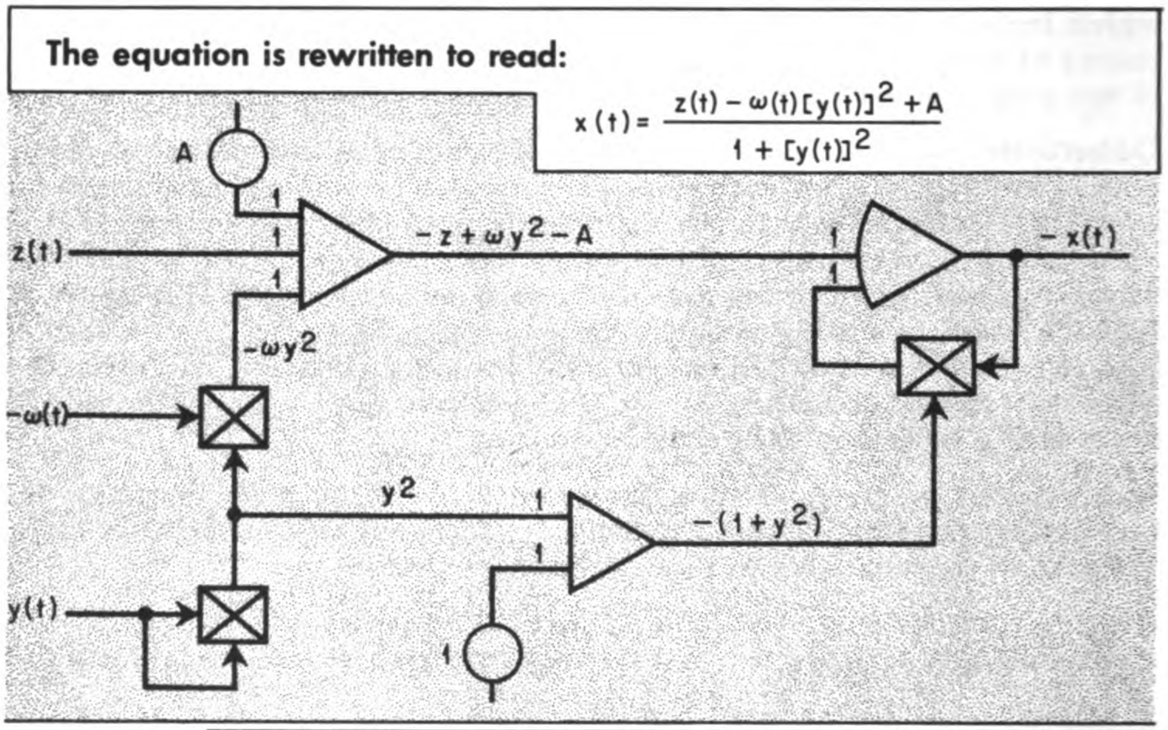


Fig. 2-8C

THIS circuit AVOIDS the ALGEBRAIC LOOP

terms which would result in algebraic loops in the computer program. When such a situation is detected the equations can almost always be reformulated to eliminate the algebraic relationships [Fig. 2-8 (A)].

The name "algebraic loop" comes, of course, from the fact that such circuits attempt to simulate an algebraic rather than a differential equation. For example, consider the algebraic equation

$$x(t) = z(t) - [x(t) + \omega(t)]\,[y(t)]^2 + A$$

The computer circuit would be as shown in Fig. 2-8 (B).

As above, such equations can usually be solved or rewritten to avoid the algebraic loop; one example is shown in Fig. 2-8 (C).

Algebraic loops in analog computer programs are not forbidden. In general, they are to be avoided whenever possible, for they tend to accentuate any noise in the circuit and this may cause undesired oscillations.

Switching Circuits: Diodes and Relays

Diodes conduct in only one direction and consequently pass a voltage signal only when the anode is more positive than the cathode. Relays operate double-pole-double-throw switches and therefore pass or do not pass voltage signals, according to the state of the relay. These elements are often termed *decision elements*, inasmuch as they "decide" to conduct or switch according to the value of some signal. A large number of useful circuits have been devised using diodes and relays. Some are listed below and shown in Fig. 2-9 (A) through (F). Where appropriate, the characteristics of the circuit are indicated by a plot of output voltage *vs.* input voltage.

1. Amplifier with hard limit to simulate, say, mechanical limit stops [Fig. 2-9 (A)].
2. Soft limiting (a saturating amplifier or motor) [Fig. 2-9 (B)].
3. "Bang-bang" circuit (a two-state switch) [Fig. 2-9 (C)].
4. Absolute value circuit: $e_o = |e_{in}|$ [Fig. 2-9 (D)].

   The numbers at the inputs of amplifiers are gain factors for the input connection. Thus when e_{in} is positive, diode B conducts and diode A does not, and the feedback connection is such that the output of pot No. 1 *must* be equal to minus the input voltage, e_{in}, (so that the input to amplifier No. 1, a high-gain amplifier, is essentially zero). The sum of the inputs to amplifier No. 2 is $2(-e_{in}) + (e_{in}) = -e_{in}$; hence $e_o = e_{in}$ when e_{in} is positive. When e_{in} is negative, diode A conducts and shorts out the high-gain amplifier, amplifier No. 1. Only the lower input to amplifier No. 2 is active; thus $e_o = -e_{in}$ when e_{in} is negative. In other words, e_o equals the *magnitude* of e_{in}.

$$e_o = |e_{in}|$$

5. "Dead-space" circuit (devices requiring for operation an excitation above some minimum threshold) [Fig. 2-9 (E)].
6. Sawtooth generator [Fig. 2-9 (F)].

Implicit Differentiation

The operation of *squaring* can of course be accomplished with any multiplier by supplying both inputs with the same variable. Also, the x^2dfg has been mentioned as a fixed diode-function generator for squaring. It has been shown that any of the squaring devices can be used to take the square root of a variable, and that any multiplier can be used in a division circuit. These two circuits are reviewed here briefly.

If squaring takes place in the feedback path of a high-gain amplifier, and the squared voltage is subtracted from the input variable $x(t)$, then since the sum of the inputs must be essentially zero, the output voltage varies

Almost ANY NONLINEAR CHARACTERISTIC can be APPROXIMATED by USING DIODES and/or RELAYS

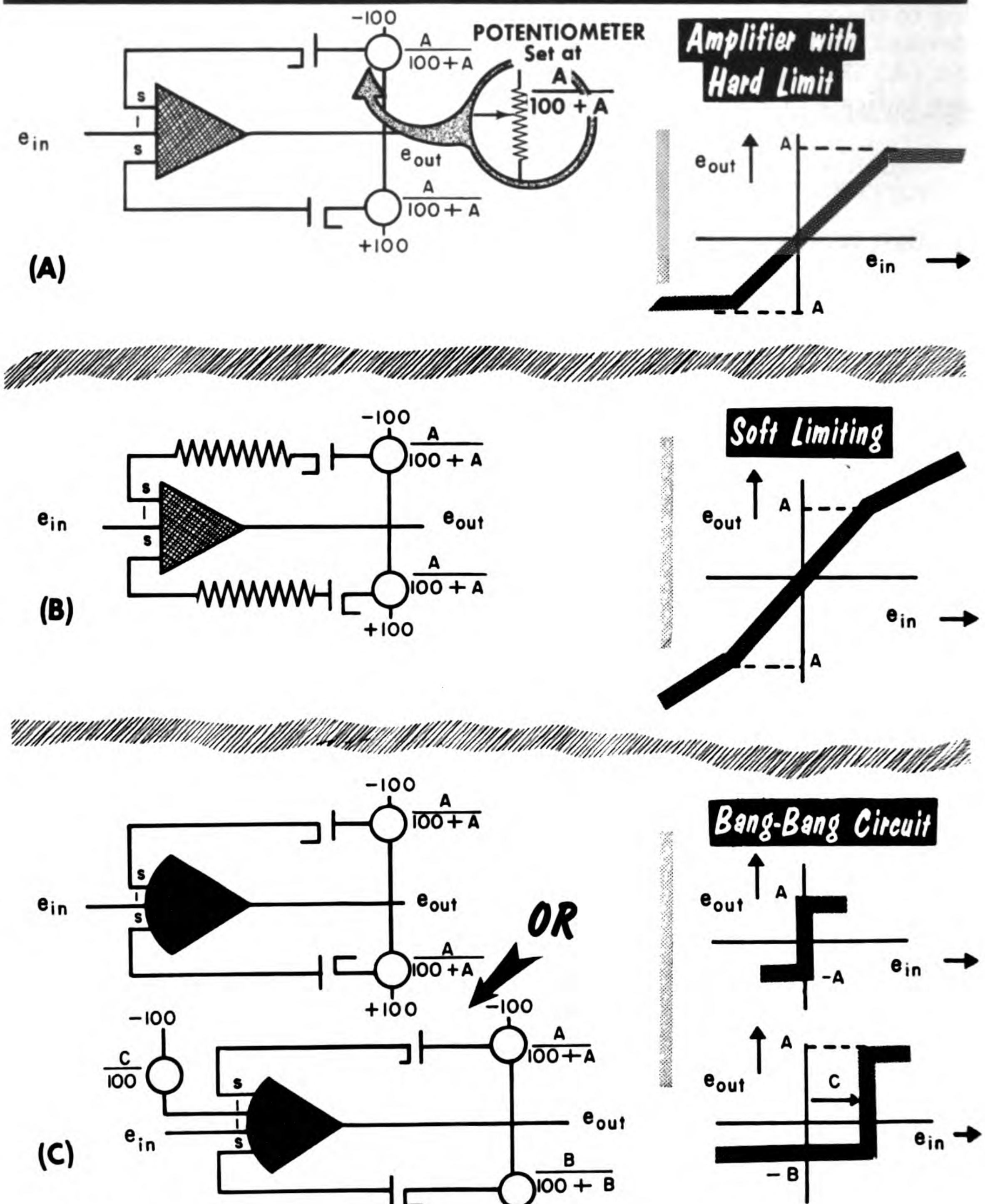


Fig. 2-9

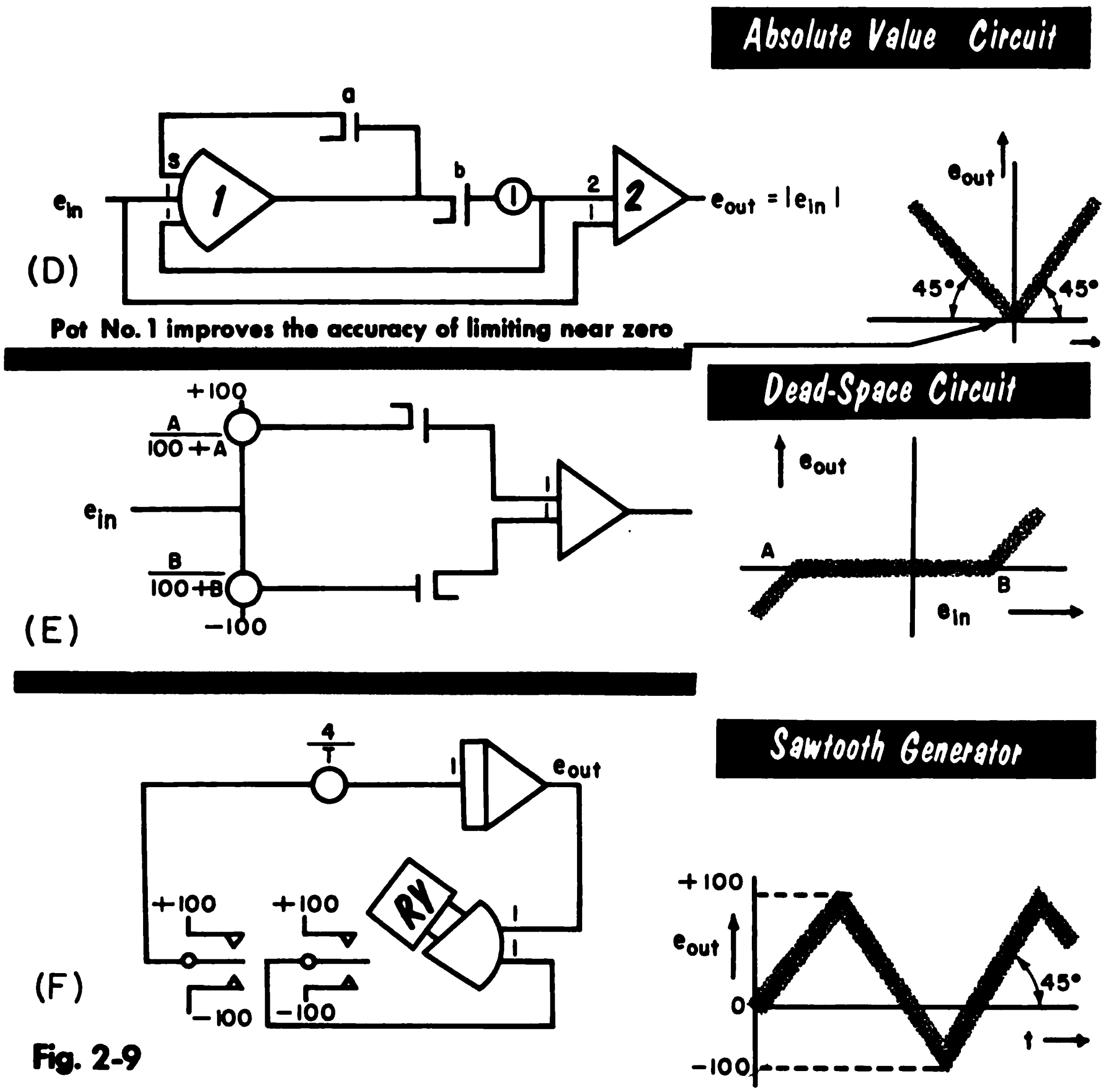


Fig. 2-9

as the square root of $x(t)$. Denote the gain of the amplifier as $-K$, and write an equation for e_o [Fig. 2-10 (A)]:

$$e_o = -K(-x + e_o^2)$$

Dividing through by K, and rearranging the terms:

$$x = e_o^2 + \frac{e_o}{K}$$

But since K is exceedingly large the last term can be neglected, and thus

$$e_o \cong \sqrt{x}$$

If the input had been $+x(t)$, then $e_o \cong -\sqrt{x}$. The computer will never find the root of a negative number.

Implicit division is performed by multiplying in the feedback path. That is,

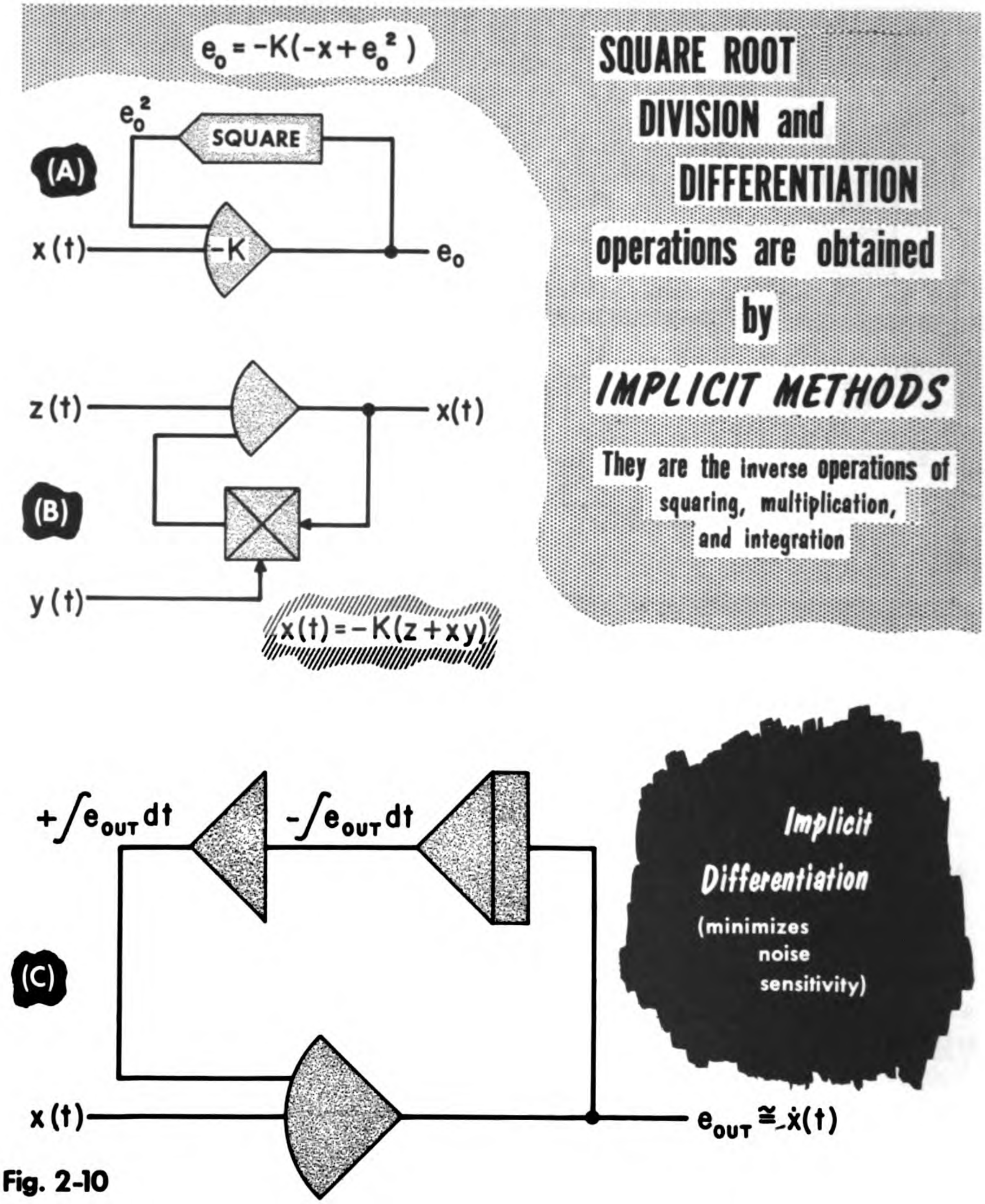


Fig. 2-10

the quotient (or the output) is multiplied by the divisor, *y(t)*, and subtracted from the dividend *z(t)*, at the input of a high-gain amplifier. A small error between z and xy causes the output to assume the correct value of x [Fig. 2-10 (B)].

In referring to Fig. 2-10 (B) note that

$$x = -K(z + xy)$$

or

$$\frac{x}{-K} = z + xy$$

but K is large:

$$z \cong -xy \qquad or \qquad x \cong -\frac{z}{y}$$

Another mathematical operation usually performed by *implicit* means is *differentiation*. We have seen earlier that usually it is not necessary or desired to differentiate, but when forced to do so the circuit shown in Fig. 2-10 (C) is useful. Here differentiation is performed by *integrating* in the feedback path of a high-gain amplifier. (Note that squaring/square rooting, multiplication/division, integrating/differentiating, are pairs of inverse operations, and that one operation can always be accomplished by performing the other in the feedback path of a high-gain amplifier.)

Amplifier No. 3 is required to avoid positive feedback.

$$e_o = -K\,[x + \int e_o dt]$$

or

$$x = -\int e_o dt - \frac{e_o}{K}$$

but the last term is negligible, hence

$$e_o \cong -\dot{x}$$

Special Circuits

Sometimes a desired function can be expressed as a polynomial. For example,

$$f\,[x(t)] = a_4\,[x(t)]^4 + a_3\,[x(t)]^3 + a_2\,[x(t)]^2 + a_1\,x(t) + a_o$$

This function is easily generated with multipliers in several ways as illustrated in Figs. 2-11A and 2-11B.

The function $\dot{f} = (4a_4x^3 + 3a_3x^2 + 2a_2x + a_1)\dot{x}$ is shown in Fig. 2-11C.

If it is desired to generate a polynomial function of time, $f(t) = b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_o$, only integrators are needed (Fig. 2-11D).

We have seen how to generate sin$t$ and cos$t$ *or* sin ωt and cos ωt, but suppose we now wish sinθ and cosθ where $\theta(t)$ is some voltage representing an angle which changes as a function of time. The simplest way is to use a resolver (see p. 2–44) which produces the desired result with one building block. When using a resolver, however, $\theta(t)$ must not rotate through more than 360°. If we are simulating a missile that is continually rolling, we must use the circuit shown in Fig. 2-11E, which assumes only that we know what $\dot{\theta}(t)$ is.

Polynomial and Trigonometric Functions are Obtained in Several Ways

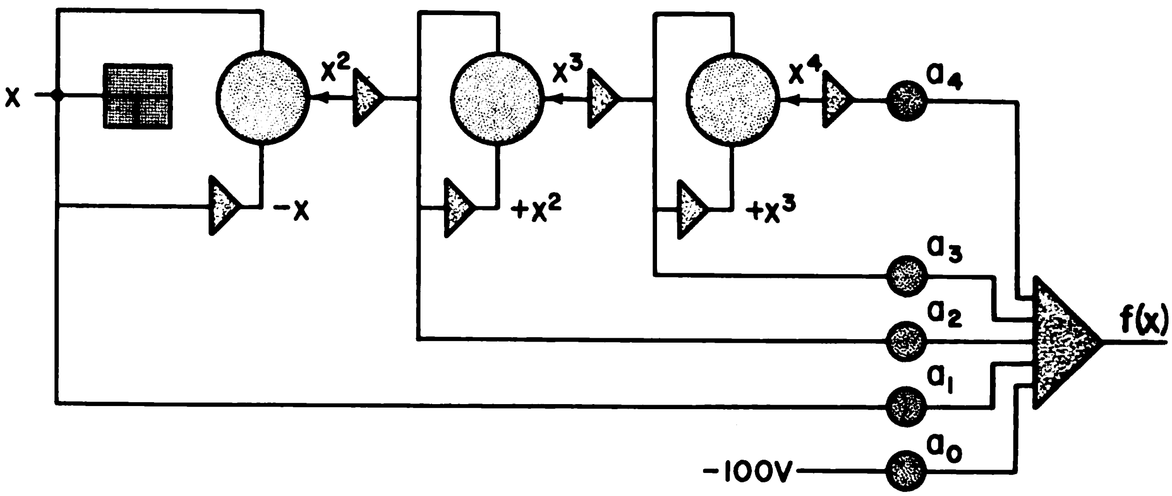


Fig. 2-11A $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

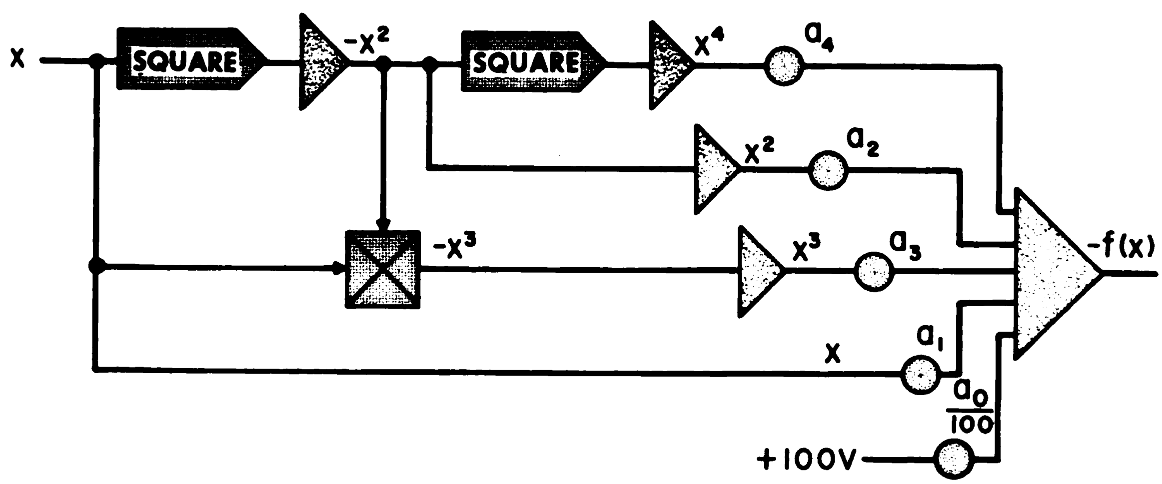


Fig. 2-11B $f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$

This circuit is subject to errors, and may be improved by the addition of a correcting circuit (Fig. 2-11F) which takes note that for all θ, $\cos^2\theta + \sin^2\theta$ must equal one.

Note that since it was necessary to use servomultipliers for $\sin\theta$ and $\cos\theta$

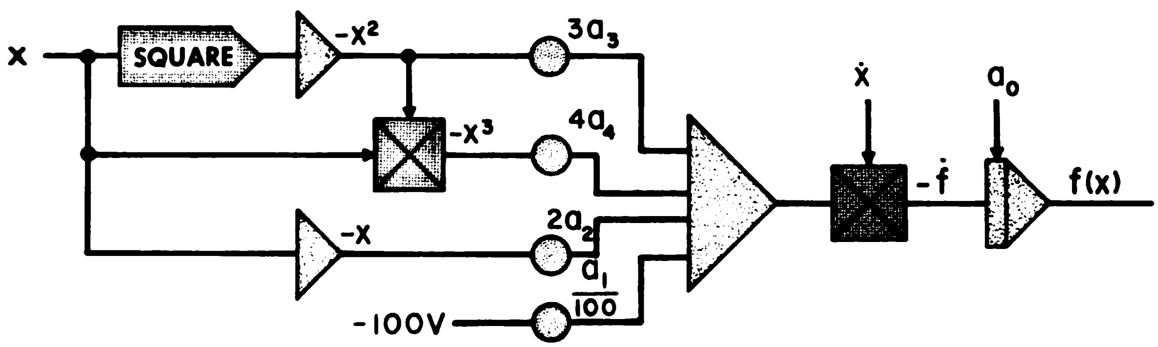


Fig. 2-11C $\dot{f}(x) = [4a_4x^3 + 3a_3x^2 + 2a_2x + a_1]\dot{x}$

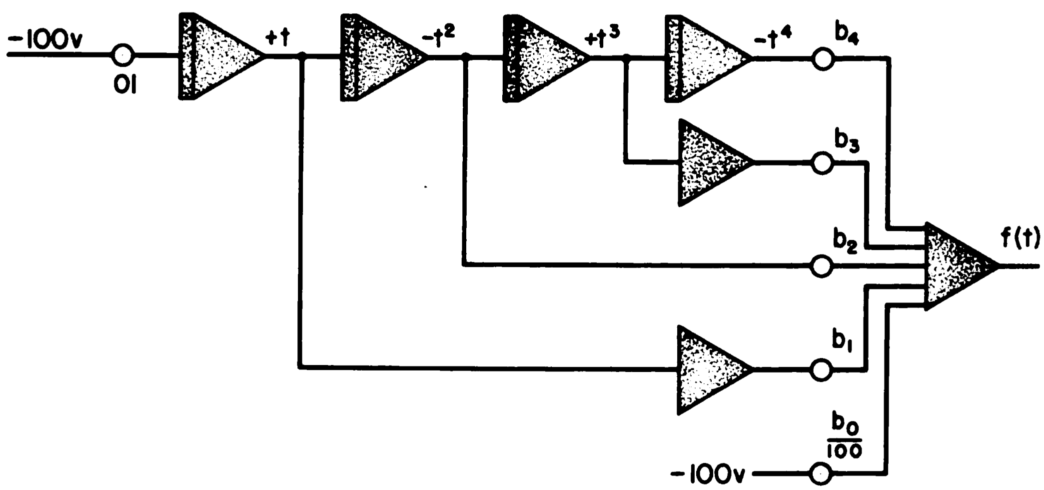


Fig. 2-11D $f(t) = b_4t^4 + b_3t^3 + b_2t^2 + b_1t + b_0$

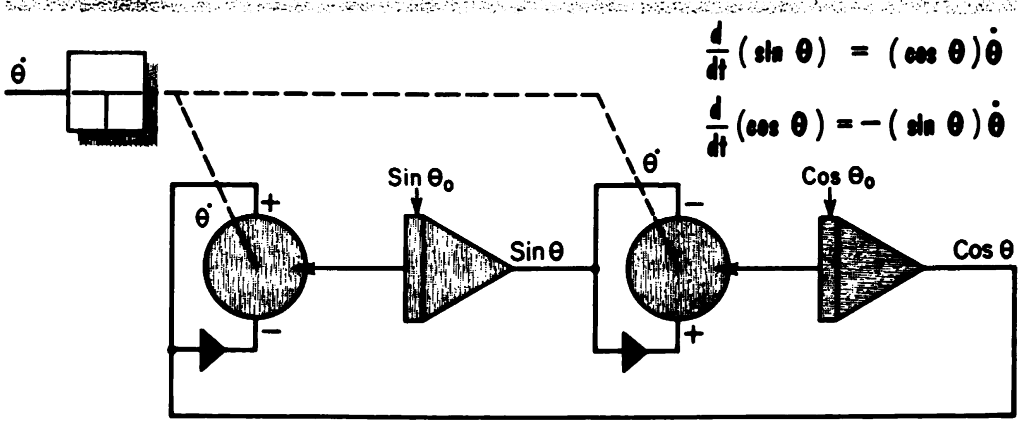


Fig. 2-11E

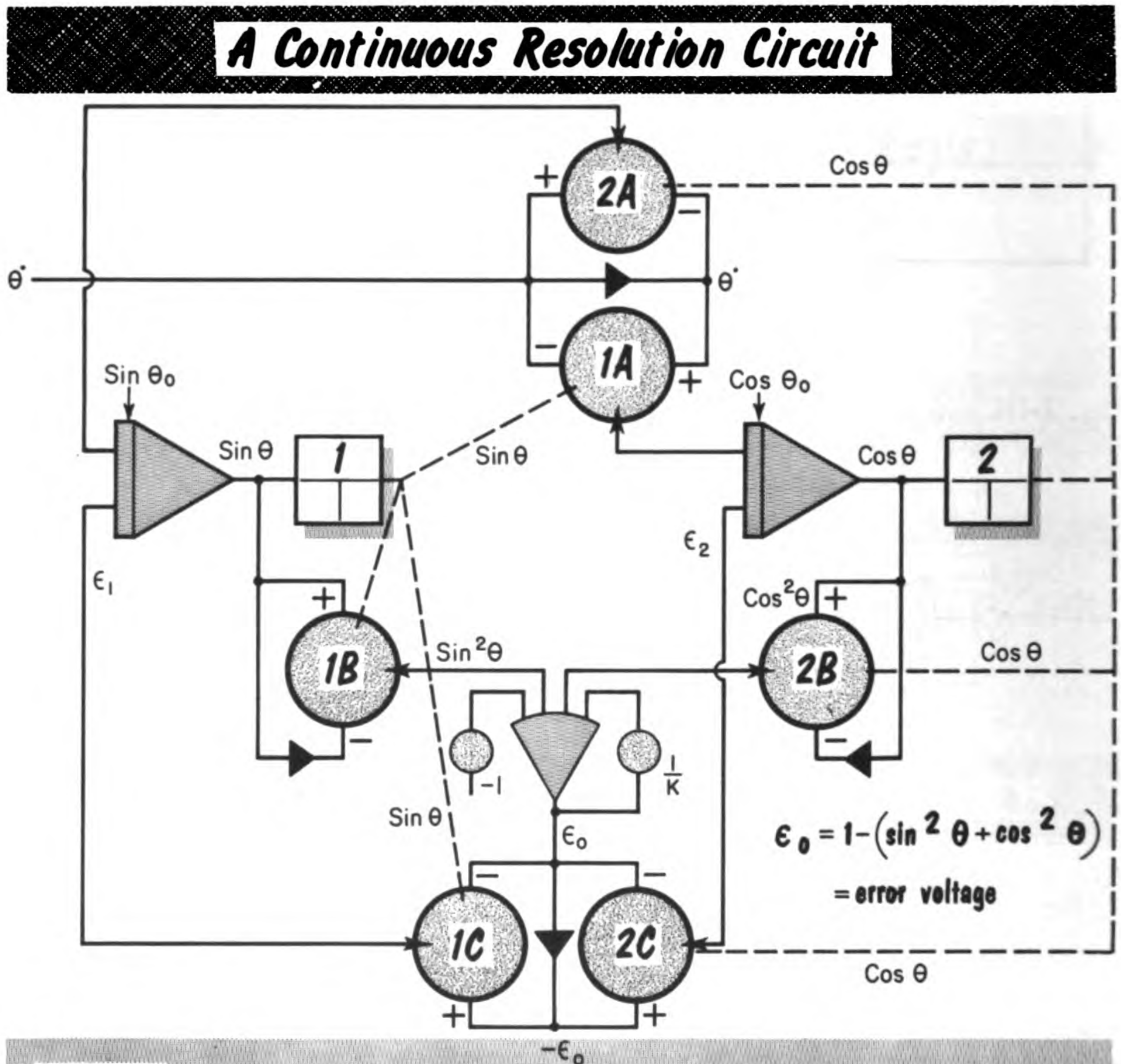


Fig. 2-11F

for the error correcting-circuit, the multiplications of $\dot{\theta}\sin\theta$ and $\dot{\theta}\cos\theta$ are performed with $\sin\theta$ and $\cos\theta$ multipliers rather than using a servomultiplier for $\dot{\theta}$ also, as was first done.

The correcting voltages are:

$$e_o = 1 - (\sin^2\theta + \cos^2\theta)$$
$$\epsilon_1 = Ke_o\sin\theta$$
$$\epsilon_2 = Ke_o\cos\theta$$

AUTOMATIC PROGRAMMING

Patching

With the increasing size of computers and the increasing complexity of problems solved on analog computers, the demands placed upon the com-

puter programmer are growing, too. Because of this, more and more labor saving features are being added to analog computers to a degree that they may soon vie with the modern kitchen in the field of automatic gadgetry!

Early computers used phone cords or wires with plugs to interconnect the

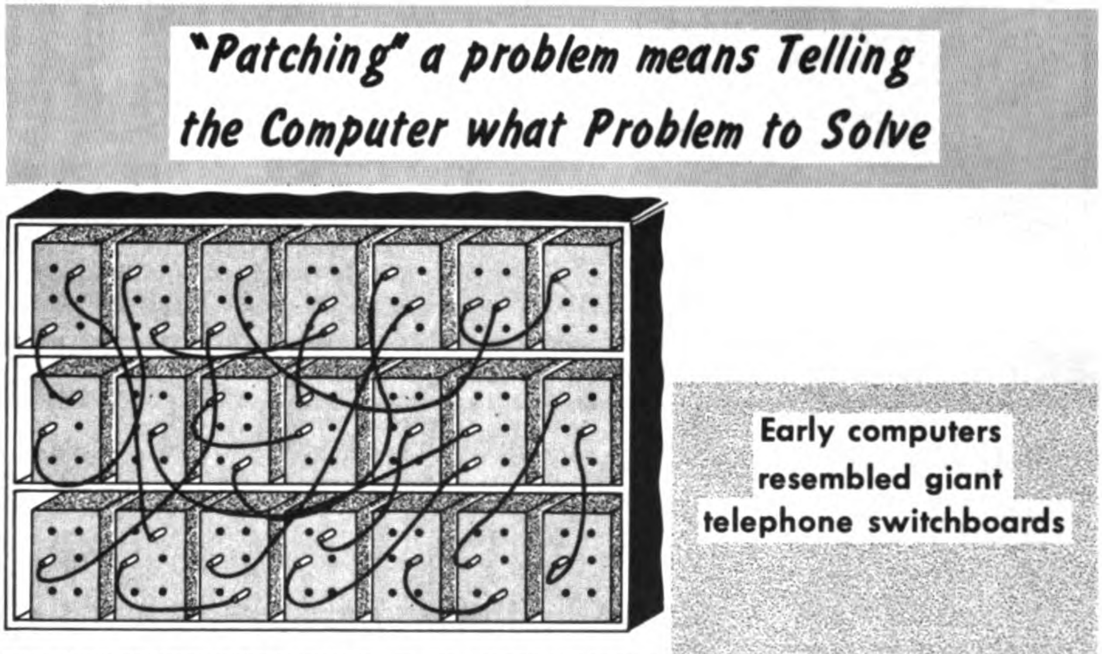


Fig. 2-12

various building blocks directly, so that a computer looked like a giant telephone switchboard (Fig. 2-12).

To provide a means for making computer building blocks available for maintenance checks without disturbing the program wiring, also for quickly changing problems on a computer, and to avoid holding up the use of the computer while the program wiring is being prepared, modern computers use a removable *patch panel*. All program wiring is done on the patch panel. The plugs on the ends of the *patch cords* go through the panel to make contact with small flexible leaves as the patch panel is inserted in the *patch bay* on the console of the computer. The leaf contacts are connected by cables to the inputs and outputs of all building blocks in the computer [Fig. 2-13, (A), (B) and (C)].

The patch panel is prepared directly from the schematic diagram. All the lines on the diagram are interpreted as connections to be made by patch cords between pairs of holes in the patch panel. For a complicated problem, preparation and double checking of a patch panel may require a full day of work; however, the computer itself may be utilized for other purposes during that period. The patch panel also permits storage of a program (in the form of a wired-up patch panel) for problems that may be repeated.

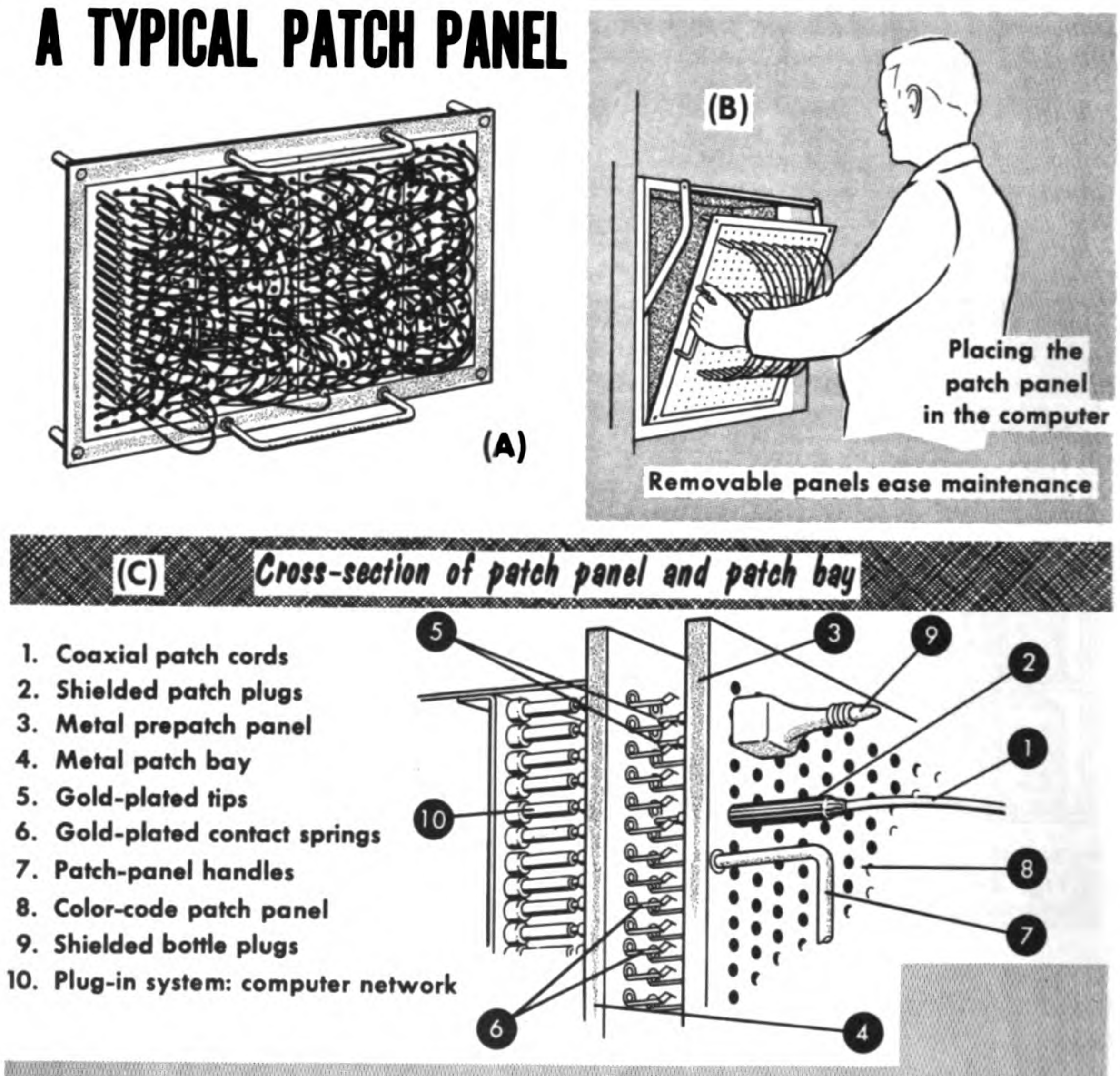


Fig. 2-13

The patch panel, and the automatic features described in the next several pages, increase the flexibility and usefulness of computers by permitting, for example, rapid removal of one problem and insertion of another. Furthermore, these devices reduce the probability of human error by minimizing repetitions and tedious tasks; they have the additional advantage of shortening the total computer time required for a particular study. While *patching* is still a manual operation, there may come a day when automation will take over with crossbar switching systems, as used by the telephone industry, driven by punched-card or punched-tape readers.

Pot-Setting

Throughout the course of many analog computer studies several hundred computer runs are made, each lasting 30 seconds to 2 minutes. Such prob-

lems may require two or more computers to be connected as one. Furthermore, they frequently continue for several days or several weeks. A voluminous amount of graph paper is turned out which requires weeks of analysis by many people.

Large computer studies such as these are expensive in man-hours and computer time, and must be carefully planned and scheduled. Fortunately, the

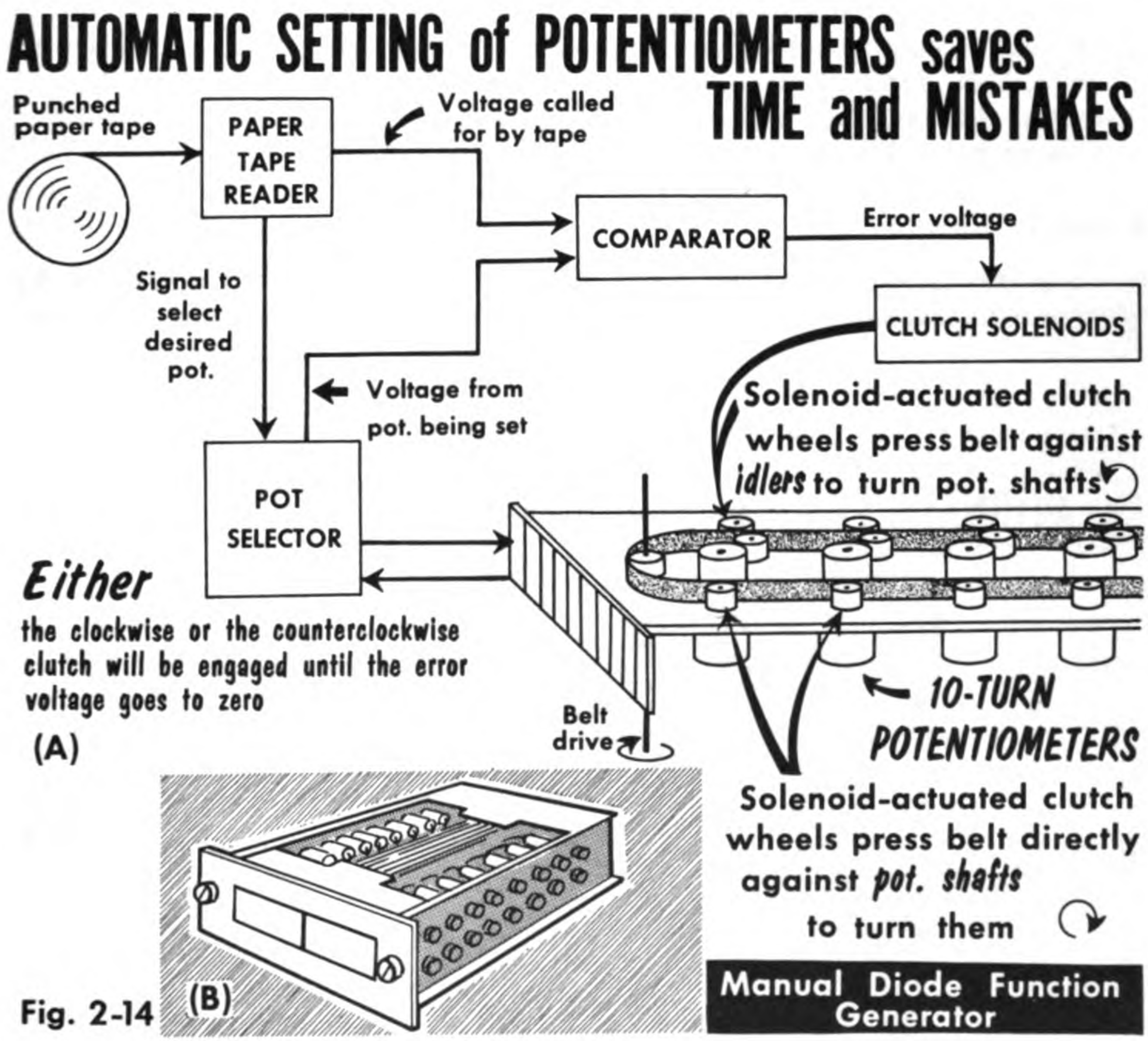


Fig. 2-14

time-consuming job of analysis of computer results can usually be done later, without involving additional computer rental time. On the other hand, there is one operation that takes place during the computer study that is very time consuming, and consequently slows down the entire investigation. This operation is the introduction of constant data into the computer, i.e., the manual setting of potentiometers. Since computer pots are 10-turn units, an operator needs an average time of 30 seconds to set a pot to 4-decimal place accuracy. It is not uncommon for a single problem to require a thousand or more settings and resettings of pots. And that means one or more full days of pot setting.

To remedy this situation computers are now being equipped with automatic pot-setting devices. With this equipment the operator merely feeds in a punched paper tape that has been prepared in advance with pot numbers and settings. The tape-reading unit energizes a selection system, belt drive, and voltage comparator, which set all the pots without further command. Each setting takes about 2 seconds [Fig. 2-14(A)].

One glance at a diode-function generator will reveal that the use of each *dfg* entails the setting of *40* pots. The pots are generally smaller, but it takes just as long to set them. Thus it is obviously no mean task to set up a dozen or so *dfg's* as might be required for a large problem. Fortunately, automatic pot setting is available for *dfg* pots as well as for ordinary coefficient pots [Fig. 2-14 (B)].

Automatic Readout

An important step in any computer study is checking the program for inadvertent mistakes. Double checking is performed at each stage: the com-

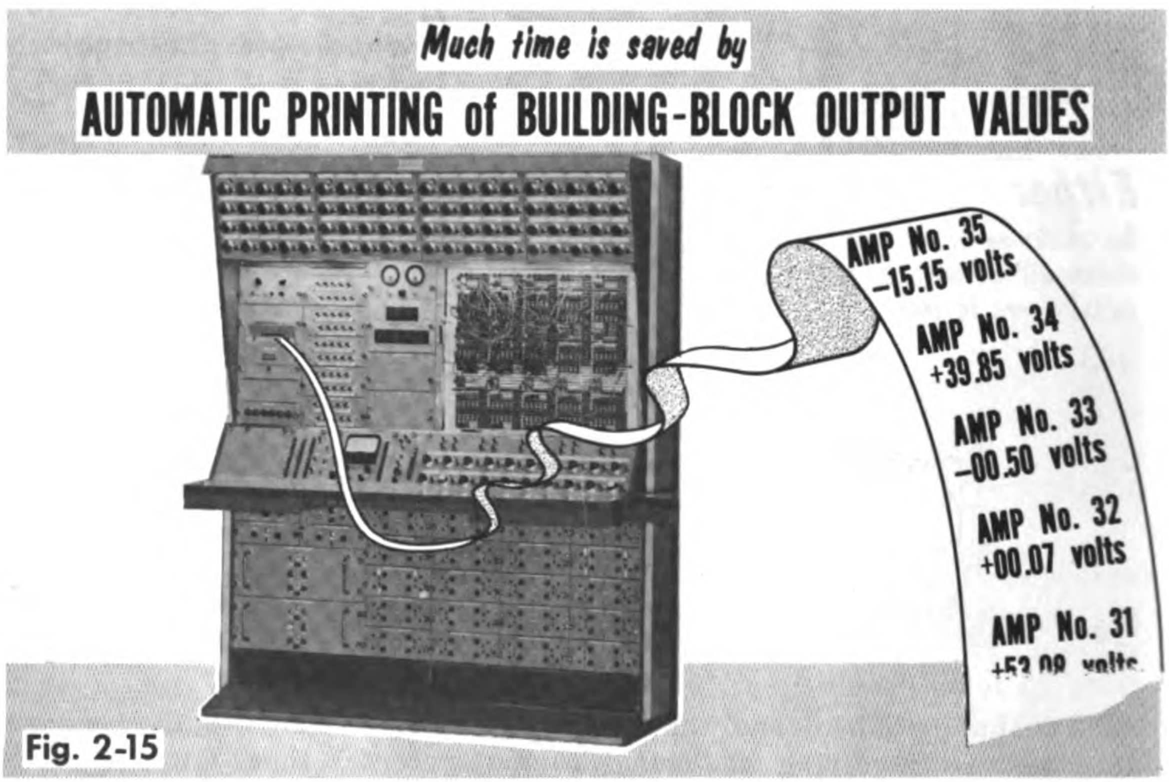


Fig. 2-15

puter equations are checked against the original physical-system equations; the schematic diagram is checked against the equations; the patch-panel wiring is checked against the schematic, and pot settings are checked against original data. Finally, when the patch panel is inserted in the computer the response of every component must be checked. One of the best checks is to precalculate by hand, *from the original equations*, what the

voltage should be at the output of every component, subject to a selected set of initial conditions. The initial conditions are inserted on the integrators and the other components assume appropriate values. The computer remains in the RESET, or INITIAL CONDITION mode. For that reason this check is called a *static check*. Now, the precalculated values are compared with the actual computer-component output voltages. This necessitates selecting several hundred output connections, and measuring and recording several hundred voltages.

To minimize the expense of performing a computer study by reducing the number of time-consuming, repetitive operations on the computer, an automatic voltage-reading and recording system is available for the robot-minded programmer. Typical automatic *readout* systems (Fig. 2-15) measure and record the output voltages of any selected group, or all components, upon the touch of a single button.

Readout of all output voltages is also useful for comparing the state of the computer first thing in the morning with its state the previous evening. Further, one might wish to reproduce the results of a previous computer study when the same problem is being rerun. A quick check of all voltages is very helpful.

QUESTIONS

1. What is a "first-order" system? What can you say about the transient response of a first-order system?
2. What is a "second-order" system? Can you characterize the transient response of a second-order system?
3. Discuss the physical meaning of a "second-order" system, and of the responses referred to in Question 2.
4. Explain the meaning of, and implications of "algebraic loops."
5. What does "implicit" mean? What is an implicit operation?
6. Computer schematic diagrams are often termed "program maps" and "flow diagrams." In these diagrams what quantities are indicated as "flowing," — current? — voltage? — mathematical symbols?
7. Why is a computer "flow diagram" used for programming in preference to a wiring diagram?
8. Explain each of the following: *coupled*, *linear*, *second order*, *ordinary differential equations*. What is the physical implication of each word, i.e., what can you say about the physical system described by such equations?
9. What is positive feedback? — negative feedback?
10. Describe and name several uses of the "automatic readout" system of a computer.

Chapter 3

APPLICATIONS

ENGINEERING DESIGN

Missile Simulation

By far the most important application of analog computers has been in the design of aircraft and missiles. All aspects of the most complicated aerodynamic and aeronautical problems have been investigated with their help. For example:

1. Control-stick and control-surface dynamic response; stiffness; stability
2. Autopilot and automatic-control system response to commands; stability
3. Target-guidance systems
4. Navigational systems

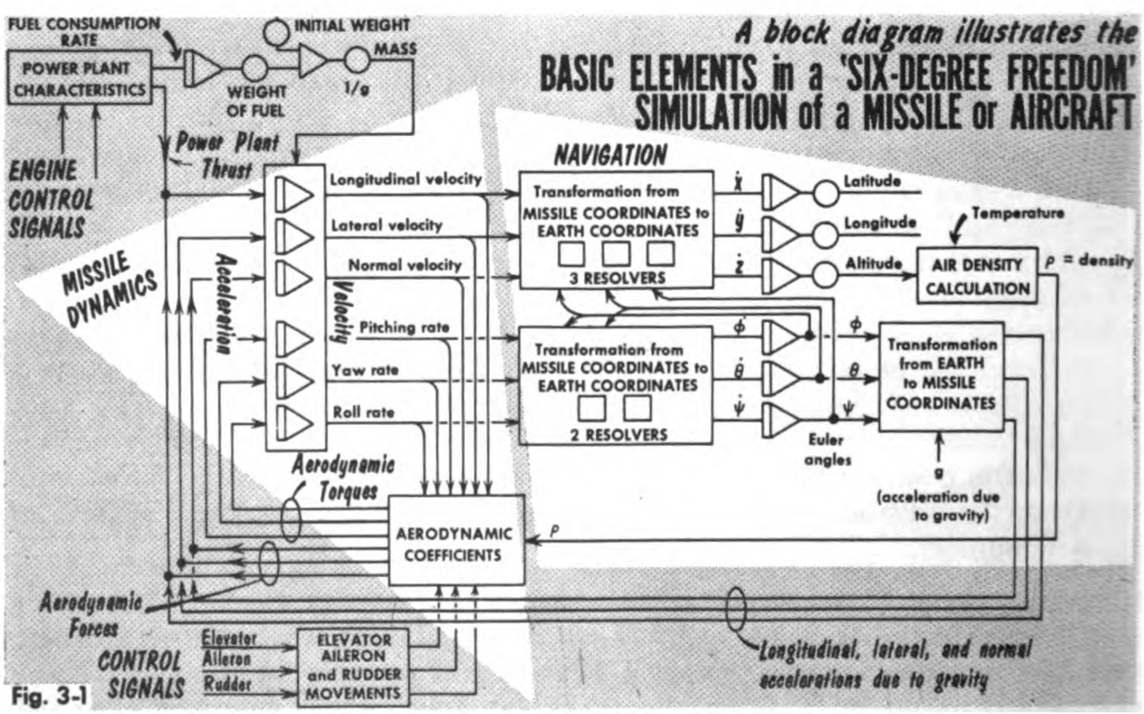


Fig. 3-1

5. Flutter and stresses in wing sections
6. Bending and vibrations of beams and plates
7. Nose-cone heating at high velocities, and turbulence
8. Radar scanning, tracking, homing; gun-laying control systems.
9. Aircraft instrument, electric power, and hydraulic systems.

The block design shown in Fig. 3-1 illustrates the interconnections of groups of computer components in a "six-degree of freedom" simulation of a missile or aircraft. "Six degrees of freedom" means that true flight conditions are simulated where the subject may have three independent translational motions plus three independent rotational motions. In this simulation, control signals, missile dynamics, and missile navigation are represented. To account for gravity and air density, and to record the distance traveled, it is necessary to transform, by means of resolvers, the six missile velocities into a related set of velocities describing the missile motion in terms of earth coordinates, x, y, z, and the Euler angles ϕ, θ, Ψ, which describe the missile orientation about its center of mass. To be useful in determining the total forces along the missile axes, the gravitational acceleration must be transformed back into missile coordinates.

A Chemical Reactor

Suppose that it is desired to investigate the dynamic performance of a chemical reactor in the production of some component C (Fig. 3-2). The reactor is a tank which is heated, and has provision for stirring its contents. Component A is placed in the tank; heat causes a reaction to take place producing component B which in turn reacts to produce component C. The chemist would describe this by the equation:

$$A \xrightarrow{r_1} B \xrightarrow{r_2} C$$

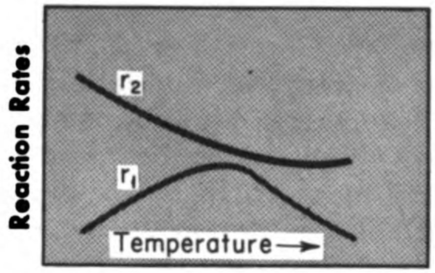
The dynamics of these reactions are described by three first-order, ordinary differential equations, which simply say that the time rate of change of the concentration (n_A) of the component A is equal to minus the reaction rate, r_1, times the concentration itself, and that the concentration (n_B) of B increases as the product $r_1 n_A$ less the rate of conversion into C, $r_2 n_B$:

$$\dot{n}_A = -r_1 n_A$$
$$\dot{n}_B = r_1 n_A - r_2 n_B$$
$$\dot{n}_C = r_2 n_B$$

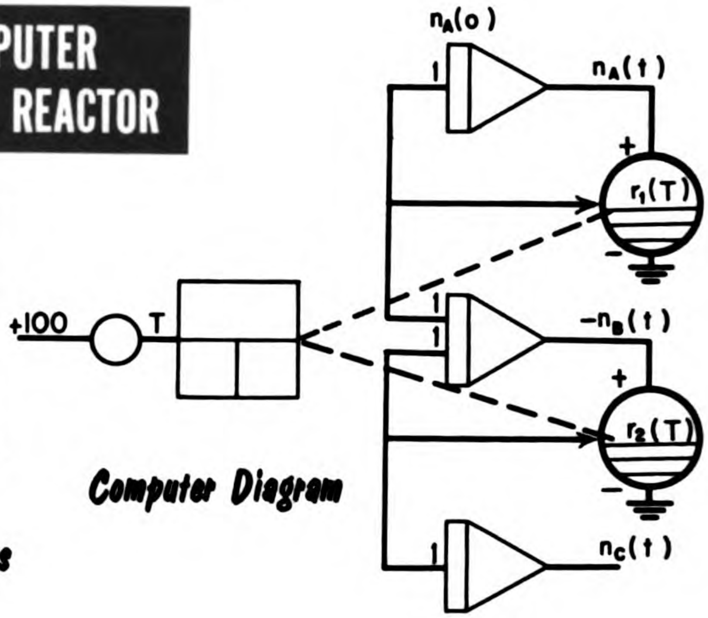
Now let us assume that the stirring or agitation is held constant, that the reaction rates are nonlinearly dependent upon temperature, and that this dependence has been determined by experiment.

In an analog-computer simulation of this reactor, a function generator may be used to reproduce the functional dependence of r_1 and r_2 upon temperature, and the temperature may be set manually with a potentiometer. In a more elaborate simulation the temperature would more than likely depend

An ANALOG COMPUTER SIMULATES a CHEMICAL REACTOR

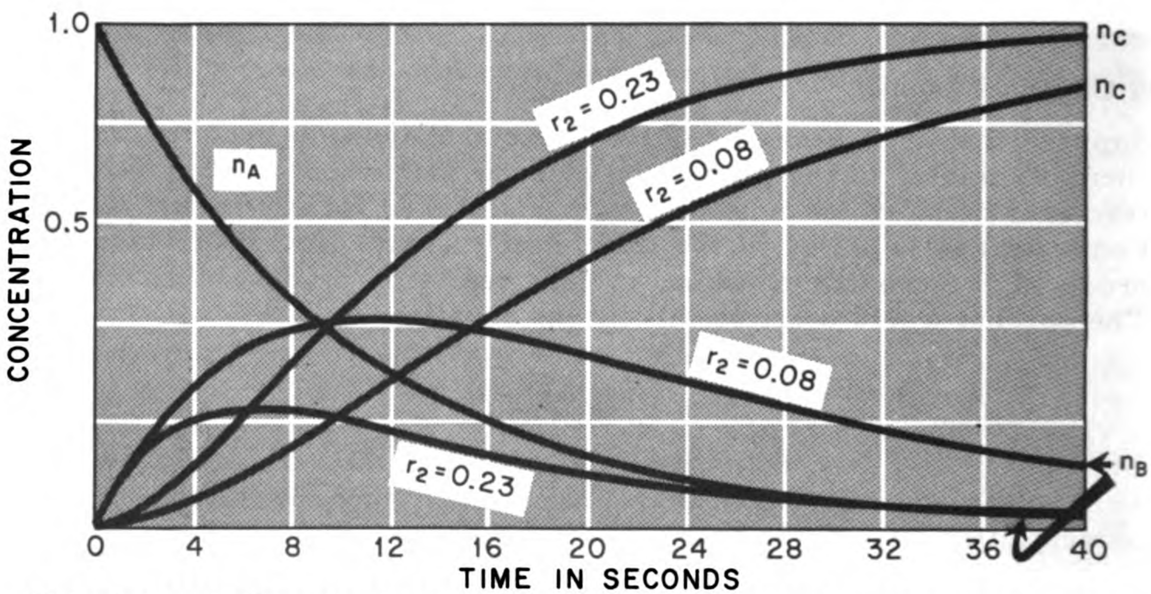


Nonlinear Temperature Dependence of Reaction Rates



Computer Diagram

Plot of Results Obtained



Plot of concentrations as a function of reaction time for different specific reaction rates

Fig. 3-2

upon the heat absorbed or given off by the reaction, as well as the heat supplied by the heater, and thus would not be simply a potentiometer setting.

A Servocontroller

A typical application for analog computers is in the design of servo-control systems. For example, a ship's rudder-control system is shown in Fig. 3-3. The voltage difference, $e_1 - e_2$, is proportional to the error in positioning the rudder. This voltage is amplified and applied to the armature of a d-c motor. The motor repositions the rudder until the error is zero. A voltage

proportional to $\dot{\theta}$ (angular velocity of rudder) is subtracted from the amplifier output to help stabilize the system and prevent the rudder from

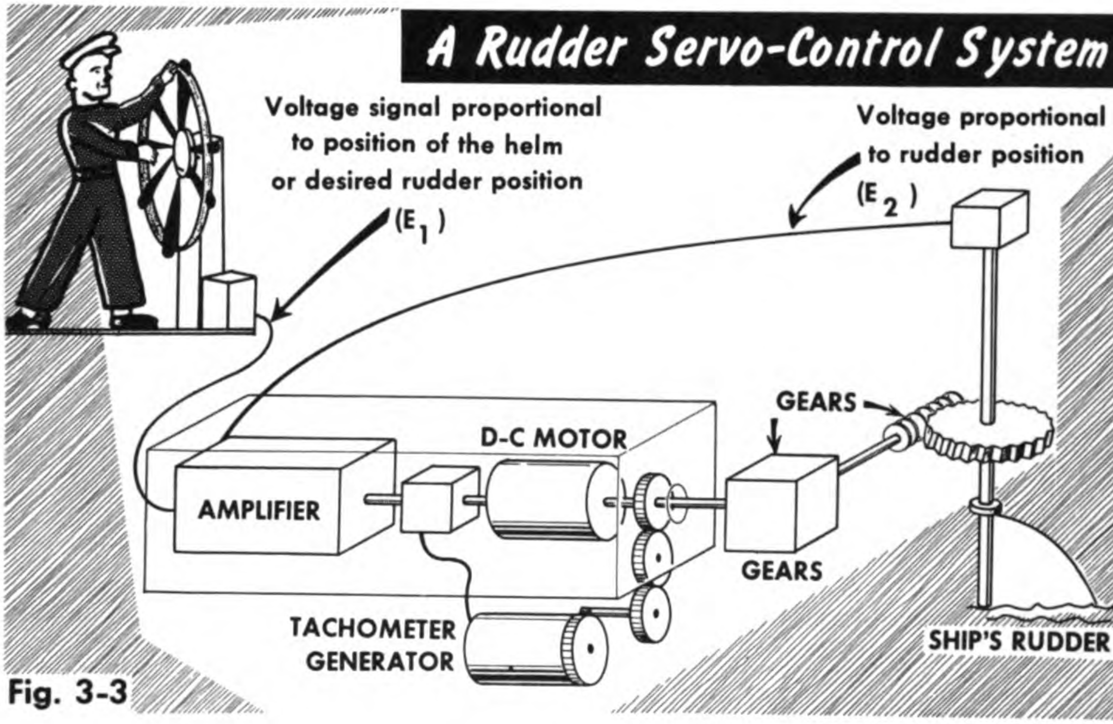


Fig. 3-3

overshooting and oscillating. The equations describing the dynamic behavior of the system are:

Armature voltage: $V_m = A(e_1 - e_2) - k_t\dot{\theta}$

Motor torque: $T = K_T(V_m - K_e\dot{\theta})$

Torque load: $(J_m + \frac{J_L}{N^2})\ddot{\theta} + (f_m + \frac{f_L}{N^2})\dot{\theta} = T$

Combining these equations we have one equation describing the angular motion of the rudder, subject to an input signal e_1.

$$(J_m + \frac{J_L}{N^2})\ddot{\theta} + (f_m + \frac{f_L}{N^2} + k_tK_T + K_eK_T)\dot{\theta} + K_TAb\theta = K_TAe_1$$

$e_2 = b\theta$, b = constant
θ = Angular position of rudder
K_T = Motor-torque constant
A = Amplifier gain
K_e = Motor back - emf constant
J_m = Motor inertia
J_L = Load inertia
f_m = Motor-shaft friction
f_L = Load friction
N = Gear ratio
k_t = Tachometer-generator gain

In most cases, the amplifier gain, A, is made frequency-dependent in a

further attempt to stabilize and improve the dynamic response of the system (Fig. 3-4). Also, for large error signals the amplifier saturates, the gears have backlash, and there is static friction (stiction) present at the rudder. All these effects make the system highly nonlinear, with the result that

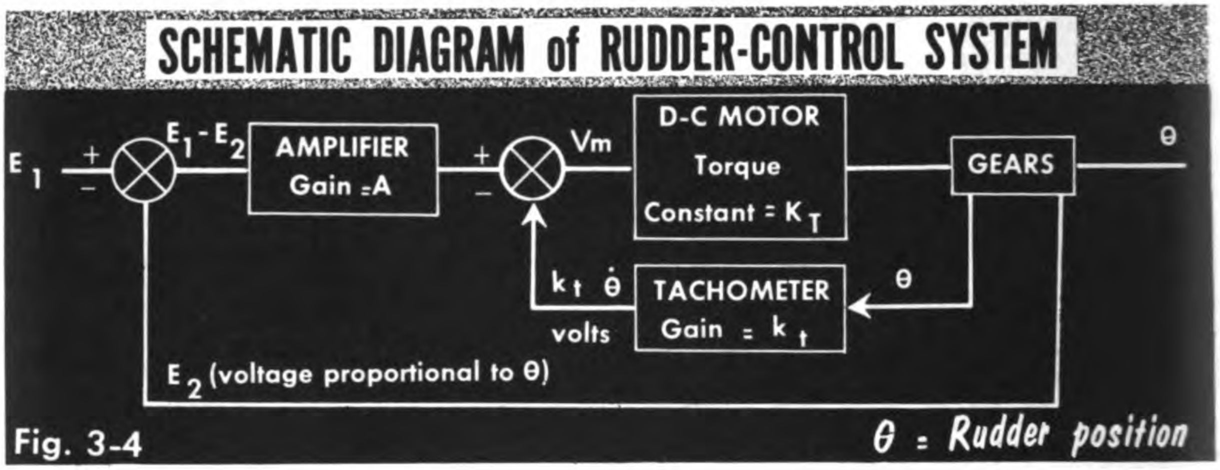


Fig. 3-4

the equations are virtually impossible to solve by any means but dynamic, analog simulation (Fig. 3-5).

Partial Differential Equations

In the mathematical discussions in this book the primary emphasis is upon *ordinary* differential equations. In this context, the opposite of *ordinary* is *partial:* that is, *partial differential equations* (Fig. 3-6). Partial differential equations describe the differential behavior of physical variables which are functions of more than one independent variable, as for example, temperatures, pressures, fluid-flow rates, and stresses and strains which vary with one or more space variables as well as with time. The most common partial differential equations involving time are the:

- Diffusion equation: $\dfrac{\partial^2\theta}{\partial x^2} + \dfrac{\partial^2\theta}{\partial y^2} = K\,\dot{\theta}$

This equation describes the diffusion of heat through solids, as well as the flow of fluids through porous media.

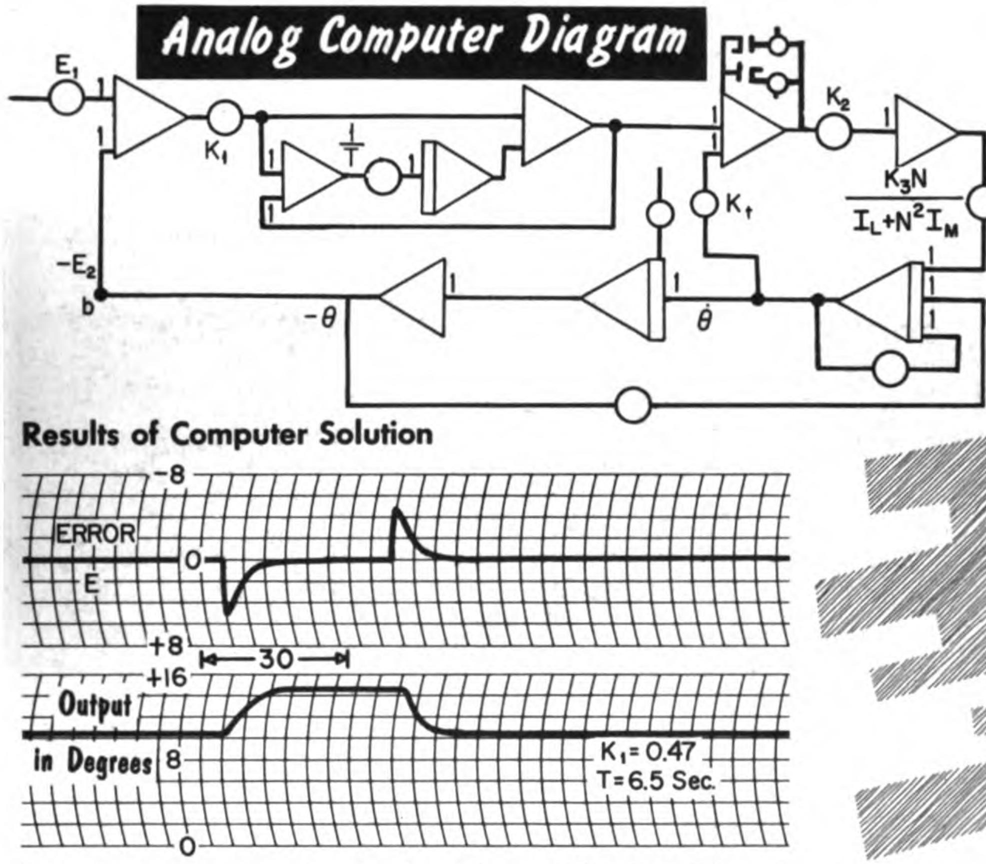
- Wave equation: $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = K\ddot{u}$

This equation describes the propagation of energy by wave phenomena — sound, water, and electromagnetic waves.

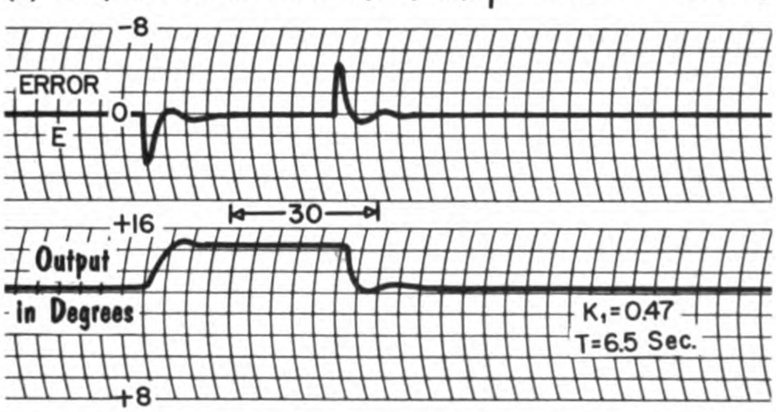
- Beam equation: $\dfrac{\partial^4 R}{\partial x^4} = K\ddot{R}$

This equation describes the bending and vibrations of a long solid beam.

At first it would appear that the analog computer could not solve one



(a) Response of servo for step input (k_1 = 0.47, T = 6.5 sec.)



(b) Response of servo for step input (k_1 = 0.87, T = 6.5 sec.)

Fig. 3-5

of the equations, since the computer has only one independent variable — time. However, if we recall the mathematical definition of a derivative as the following limit as the quantity, Δx, approaches zero:

$$\frac{df(x)}{dx} = \text{limit of } \left\{ \frac{f(x+\Delta x) - f(x)}{\Delta x} \right\}$$

$$\text{as } \Delta x \longrightarrow o$$

we will note that we might approximate the derivative in the following

manner. Suppose Δx is a small division of the total length of x we are interested in, and suppose we do not let Δx approach zero, but keep it fixed. Then we will take as the approximate value of the derivative:

$$\frac{\Delta f}{\Delta x} = \left\{ \frac{f(x + \Delta x) - f(x)}{\Delta x} \right\}$$

Thus a derivative may be approximated by taking the ratio of the difference

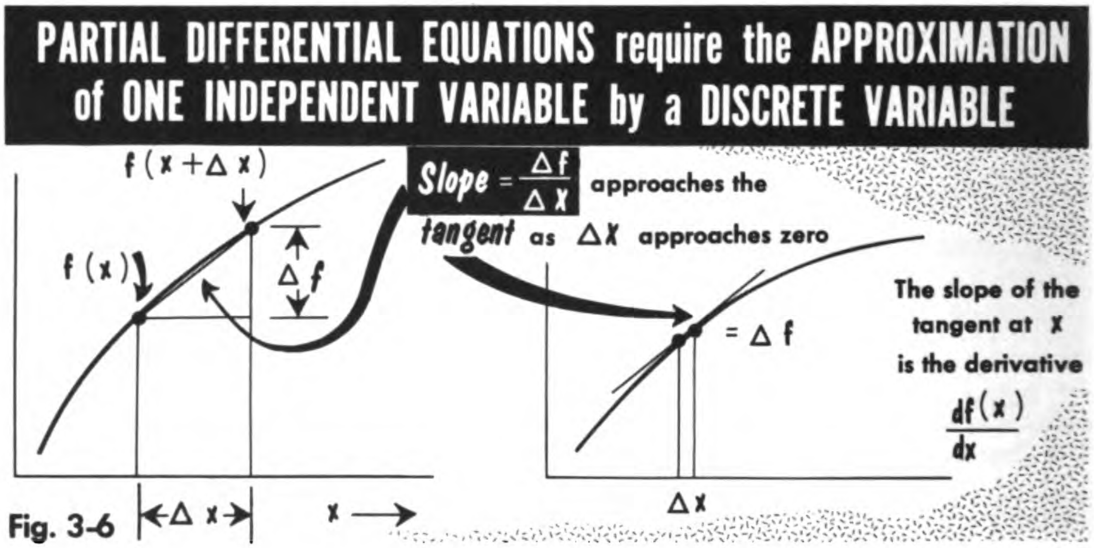


Fig. 3-6

of two neighboring values of $f(x)$ to the distance between them, Δx, as shown in the figure.

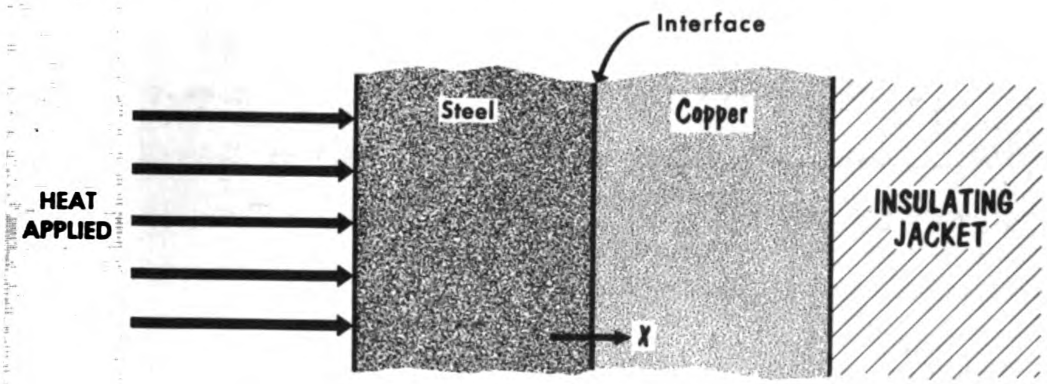
Heat Diffusion

By approximating derivatives as just discussed, all partial differential equations can have their space derivatives replaced by *differences*, which reduces them to sets of ordinary differential equations in time — one equation for each of a set of points in the space dimensions. There is an obvious difficulty with this method: as the number of points is increased to improve the degree of approximation, the number of equations to be solved is correspondingly increased, and hence the amount of equipment available in a particular computer will restrict the number of points that can be taken.

As crude as this technique may seem, it works exceedingly well. The application of analog computers to partial differential equations has been very successful, and its use in this field is constantly growing. Complete books have been written about partial differential equations and their computer solutions, so the amount of space devoted to the subject in this book must not be taken as indicative of the importance of this application.

Differencing of a partial differential equation is illustrated by an example of heat diffusion in the x direction through a plate (with transverse dimensions very large with respect to the thickness) (Fig. 3-7A).

Heat Diffusion through a Metal Slab can be SIMULATED by the solution of several first-order ordinary differential equations describing the temperature variations in several fictitious segments of the slab



One-dimensional heat-diffusion equation: $\dfrac{\partial^2 T}{\partial x^2} = K\dot{T}$

T = temperature
$\dot{T}$ = rate of change of temperature
K = coefficient of diffusion; depends upon material

Fig. 3-7A

EXCHANGE of HEAT between SEGMENTS, and RESULTING TEMPERATURES

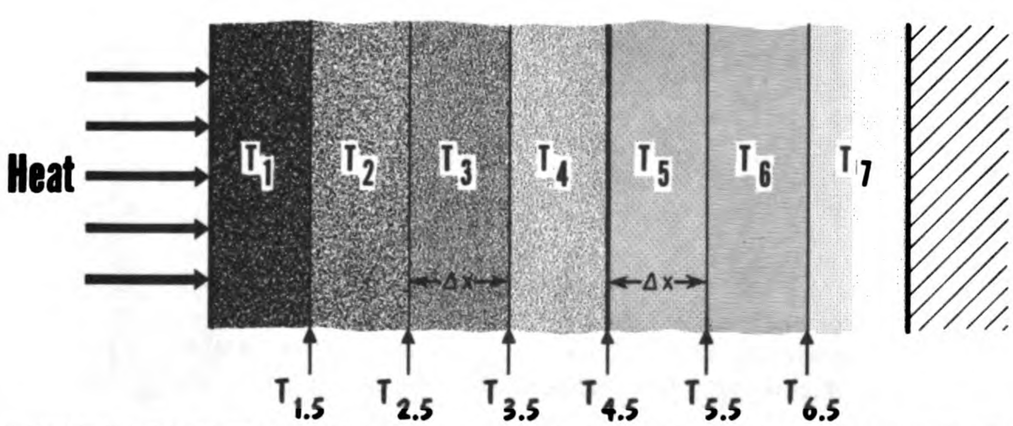


Fig. 3-7B *Temperatures in between sections*

Approximate second derivative

$$\frac{d^2T_{1.5}}{dx^2} \cong \frac{T_1 - 2T_{1.5} + T_2}{\frac{1}{4}(\Delta x)^2}$$

Temperature of each slab is the average of the temperature at its sides

$$T_2 = \frac{T_{1.5} + T_{2.5}}{2}$$

Thus the partial differential *diffusion equation* is replaced by seven ordinary differential equations that can be solved on the analog computer

$$\dot{T}_{1.5} = \frac{4}{K(\Delta x)^2}\left[T_1 - 2T_{1.5} + T_2\right]$$

$$\dot{T}_{2.5} = \frac{4}{K(\Delta x)^2}\left[T_2 - 2T_{2.5} + T_3\right]$$

$$\dot{T}_{3.5} = \frac{4}{K(\Delta x)^2}\left[T_3 - 2T_{3.5} + T_4\right]$$

$$\dot{T}_{4.5} = \frac{4}{K(\Delta x)^2}\left[T_4 - 2T_{4.5} + T_5\right]$$

$$\dot{T}_{5.5} = \frac{4}{K(\Delta x)^2}\left[T_5 - 2T_{5.5} + T_6\right]$$

$$\dot{T}_{6.5} = \frac{4}{K(\Delta x)^2}\left[T_6 - 2T_{6.5} + T_7\right]$$

Boundary Conditions:

$$\begin{cases} \dot{T}_{0.5} = \dfrac{1}{K\Delta x} M(t) & M(t) = \text{heat input function} \\ \dot{T}_{7.5} = \dfrac{1}{K\Delta x}\left[T_7 - T_{6.5}\right] & \left\{\begin{array}{l}\text{Due to insulation: no} \\ \text{heat flow at boundary}\end{array}\right. \end{cases}$$

Fig. 3-7C

The effect of *differencing* the diffusion equation is to replace the above problem with a fictitious problem which has approximately the same solution. The fictitious problem is one where the plate is divided into several small sections each with a particular heat capacity, and each artificial interface having certain heat transfer properties (Fig. 3-7B). The exchange of

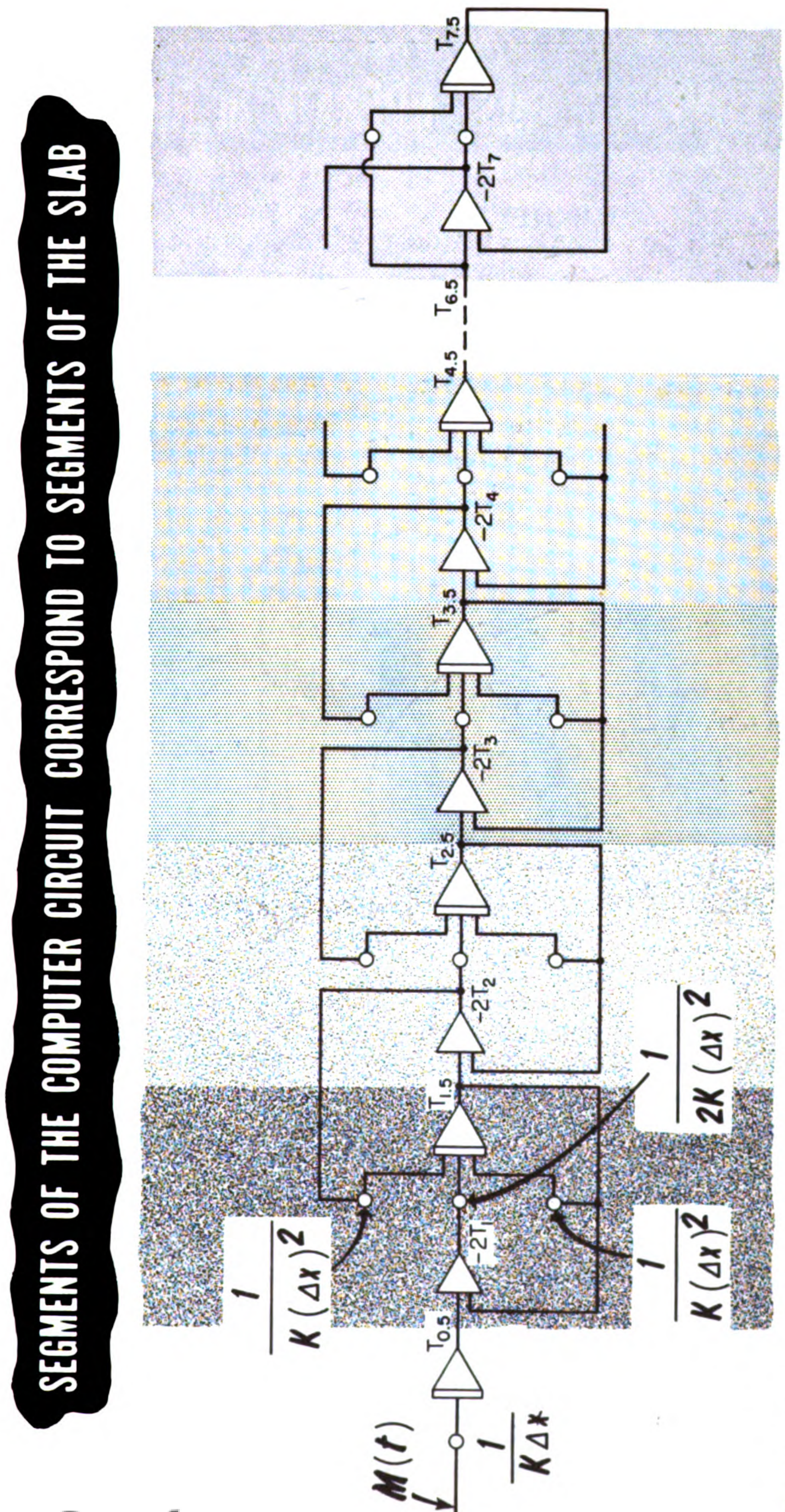

SEGMENTS OF THE COMPUTER CIRCUIT CORRESPOND TO SEGMENTS OF THE SLAB
M(t)
$\frac{1}{K\Delta x}$
$T_{0.5}$
$\frac{1}{K(\Delta x)^2}$
$-2T_1$
$\frac{1}{K(\Delta x)^2}$
$\frac{1}{2K(\Delta x)^2}$
$T_{1.5}$
$-2T_2$
$T_{2.5}$
$-2T_3$
$T_{3.5}$
$-2T_4$
$T_{4.5}$
$T_{6.5}$
$-2T_7$
$T_{7.5}$

Fig. 3-7D

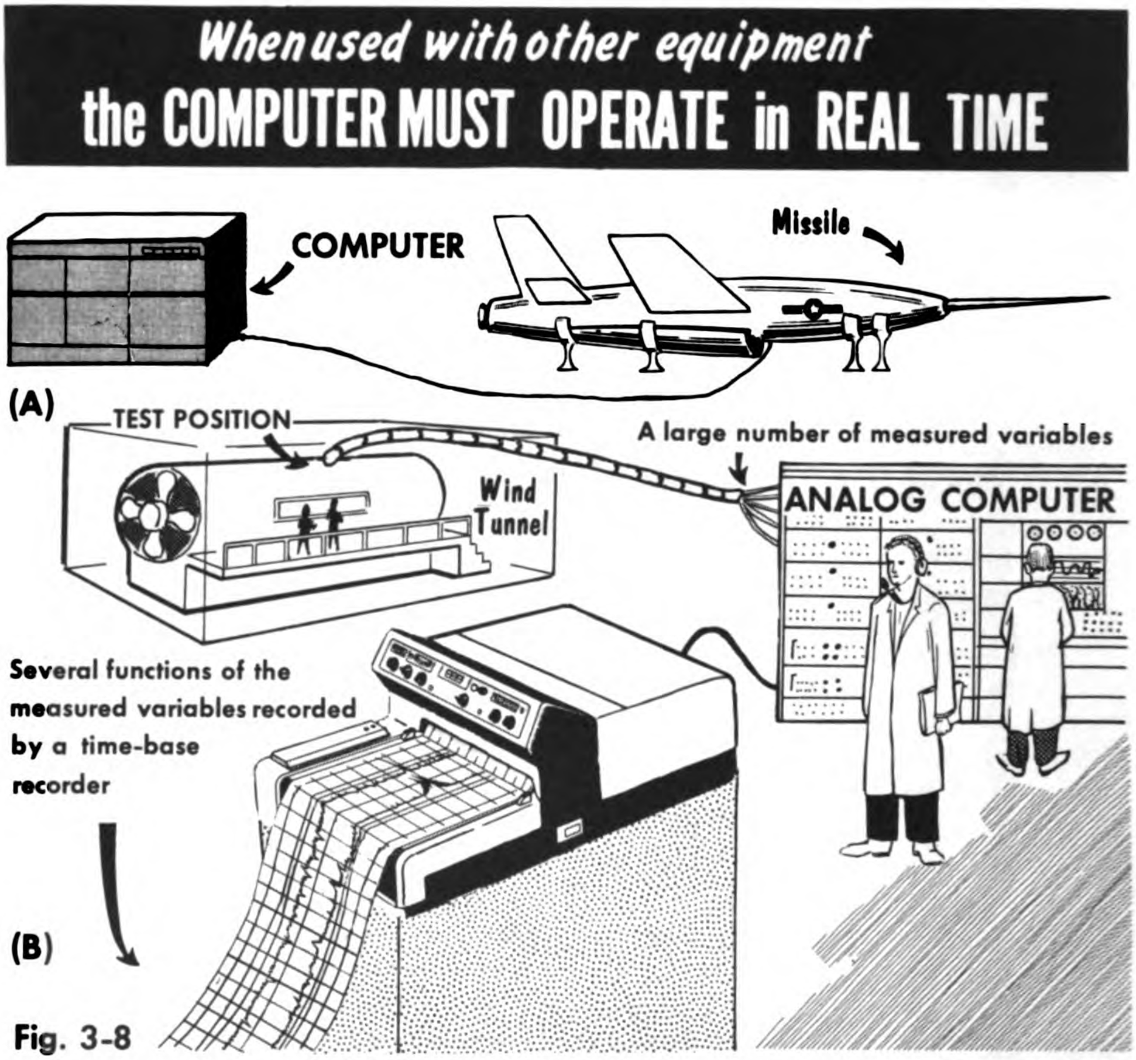


Fig. 3-8

heat between segments, and the resultant temperature for each segment, is determined by solving a set of ordinary differential equations of the form shown here — one for each segment (Fig. 3-7C). The resulting computer flow diagram is shown in Fig. 3-7D; note that the circuits for each segment are identical.

REAL TIME SIMULATION

Computers Used in Conjunction with Other Equipment

When it is said that a computer operates in *real time* it is meant that the analog variables in the computer are changing at the same speed as would their counterparts in the primary physical system. Although real time is the normal mode of operation for an analog computer, a primary system which is very slow or very fast can, when being programmed for computer simulation, easily be speeded up or slowed down by *time scaling* techniques, so that the computer is said to operate in *fast time* or *slow time*.

On the other hand, there are many useful applications of computers wherein it is *mandatory* that the computation be accomplished in real time. Reference is made here to computer studies in which a piece of noncomputer equipment is connected directly to the computer. Instead of just a piece of equipment, an entire system, a wind tunnel, part of a missile, etc., may furnish and accept voltage signals to and from the computer. Such an operation might be termed *on line* operation, in contrast to an isolated laboratory computer operation. It should be clear that the computer cannot be *time-scaled* when connected to active, time-dependent equipment that has no means of time scaling. Therefore, "real time" is required.

Examples of *on-line* computer operations are:

- *Missile Testing* [Fig. 3-8 (A)]: An analog computer may be connected to a missile in a test rig. The missile power plant is not operating, only the control and guidance systems. The computer simulates the environment and other functions necessary to "fly" the missile through a test flight.

- *Wind-Tunnel Data Processing:* In the daily operation of certain test facilities, such as pilot plants and wind tunnels, a large amount of data is produced from several sources, usually in the form of continuous voltage signals that drive time-base recorders. Ordinarily, this raw test data must be subjected to extensive mathematical analysis before useful conclusions can be drawn from it. This data-reduction task is so large and so tedious that any means of simplification is welcomed. It is often possible to introduce the voltage signals directly into an analog computer, and to perform mathematical operations of addition, multiplication, integration, and even statistical correlations, upon the data. Thus, instead of recording for analysis 50 variables, only the 10 or so computer results need be recorded. In such a capacity the computer acts simply as a data processor [Fig. 3-8 (B)].

The Human Centrifuge

An aircraft, missile, or any object free to move around in space is said to have "six degrees of freedom." These degrees correspond to the kinds of motion the object is free to perform; in particular, three translational motions and three rotations [Fig. 3-9 (A)].

Analog computers have seen extensive use in the study of performance and control of six-degree of freedom vehicles (for air, space, and submarine travel). The study of the performance of human beings in such vehicles is another matter, however, for we have yet to discover a set of differential equations that will completely describe human behavior. Consequently, flying laboratories have been used to observe pilot reactions to some conditions of high-speed, high-altitude flight. Flying laboratories, though, have their limitations, and so large centrifuges have been employed to simulate the important characteristics of space flight — in particular the unusual combinations of accelerations and torques felt by the pilot in maneuvering in space.

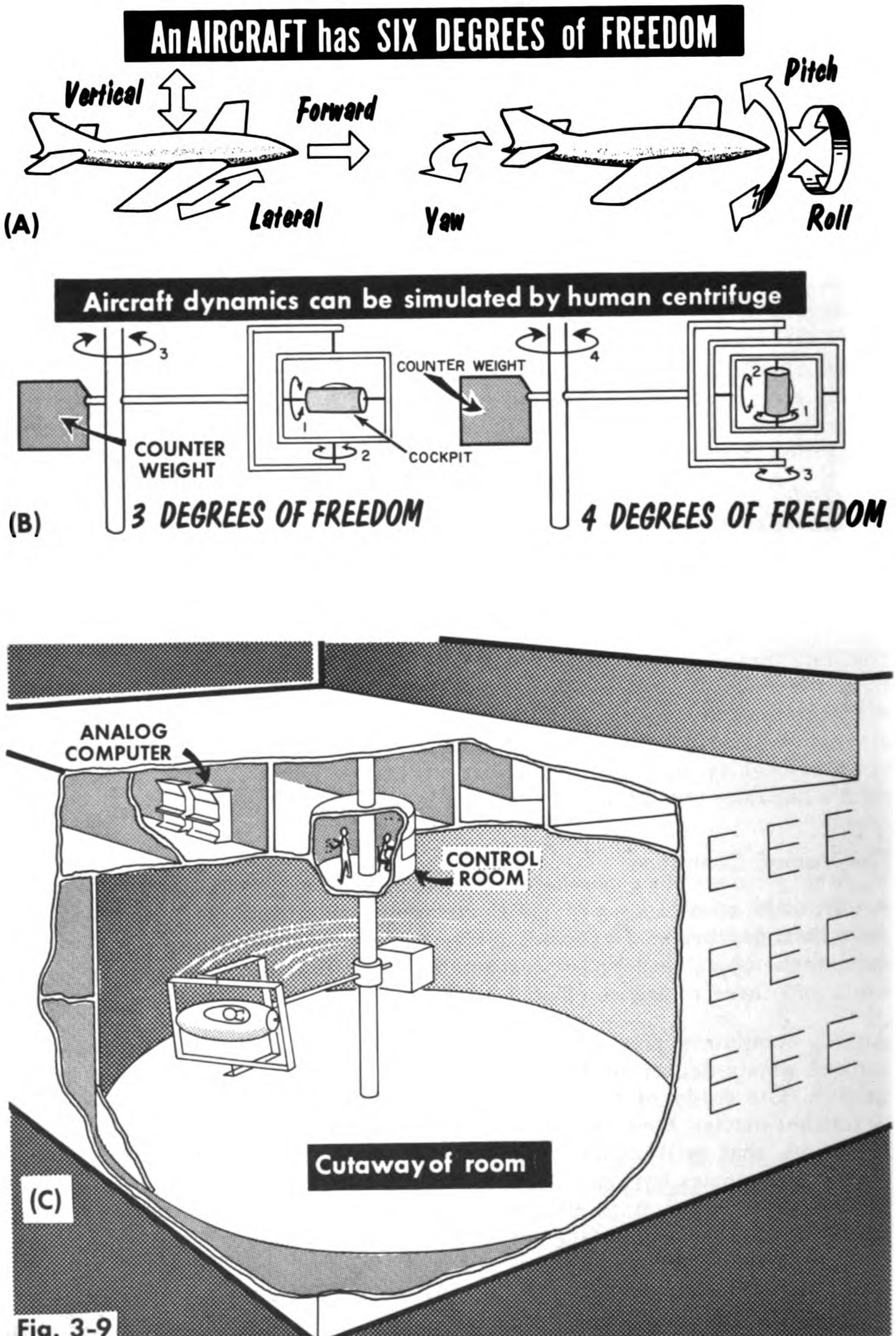

An AIRCRAFT has SIX DEGREES of FREEDOM
Vertical
Forward
Lateral
Yaw
Pitch
Roll
(A)
Aircraft dynamics can be simulated by human centrifuge
COUNTER WEIGHT
COCKPIT
COUNTER WEIGHT
(B)
3 DEGREES OF FREEDOM
4 DEGREES OF FREEDOM
ANALOG COMPUTER
CONTROL ROOM
Cutaway of room
(C)

Fig. 3-9

Such a centrifuge consists of a capsule containing a reconstructed cockpit (and a pilot) supported by gimbal rings at the end of a large boom. The boom revolves, and the capsule is turned in its gimbals much like a carnival "rocket ride." The capsule does not have six degrees of freedom — unless it breaks loose from the boom. It has only three or four degrees of freedom depending upon the number of axes in the system [Fig. 3-9, (B) and (C)].

Now the man in the capsule cannot see what motions the capsule is actually going through. He can only feel accelerations created by the motions. It is possible to arrange the actual motions of the capsule so that the pilot feels he is in a six-degree of freedom vehicle. In fact, he may use the cockpit controls to "fly" a given mission as if in a real space ship. To place the correct forces on the pilot the capsule must go through a weird set of gyrations. It is the responsibility of the analog computer, connected to the centrifuge, to accept the control stick and throttle signals from the pilot, and to compute the correct control signals for the centrifuge so the pilot receives the appropriate accelerations. There is a set of differential equations relating the six-degrees of freedom motion of the fictitious craft, to the three- or four-degrees of freedom motion of the capsule. The computer is programmed to solve these equations, in *real time*, subject to command signals directly from the capsule, the results being used to control the centrifuge motions.

Aircraft Control Systems

We have observed that many special-purpose analog computers have been built as training simulators of specific aircraft for the training of pilots and

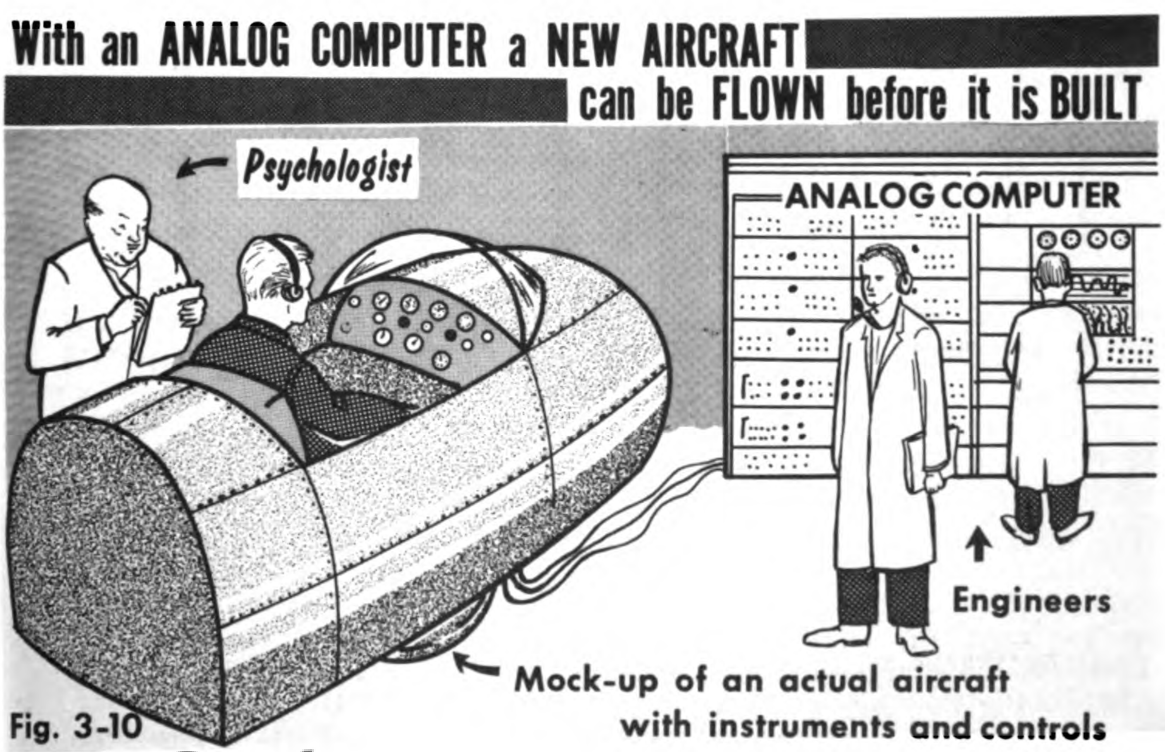


Fig. 3-10

flight crews. General-purpose computers are often programmed to solve the same equations of flight as above, but with the purpose of introducing into the computer setup, a pilot and/or a piece of aircraft equipment to study the effect of a particular design upon pilot reaction and overall system performance. Helicopter control systems are also studied in this way. Entire cockpits may be instrumented and connected to an analog computer which simulates the remainder of the aircraft (Fig. 3-10).

A similar application is where an autopilot system is connected to a computer. The computer "plays" airplane or missile, and the autopilot "flies" it around the laboratory.

Nuclear Reactor Problems

In the operation of a nuclear reactor, energy in the form of heat is generated by the fission of uranium, splitting uranium atoms into two or more

The amount of Xenon present in a Reactor
can be computed from a record of the power level

$$\frac{dI}{dt} + \lambda_1 I = K_1 P$$

$$\frac{dx}{dt} + \lambda_2 x = \lambda_1 I + K_2 P(1 - \alpha x)$$

$$R = K_3 x$$

I = Concentration of iodine, initially zero
X = Concentration of xenon, initially zero
R = Reactivity required by xenon
P = Reactor power level, a known function of time

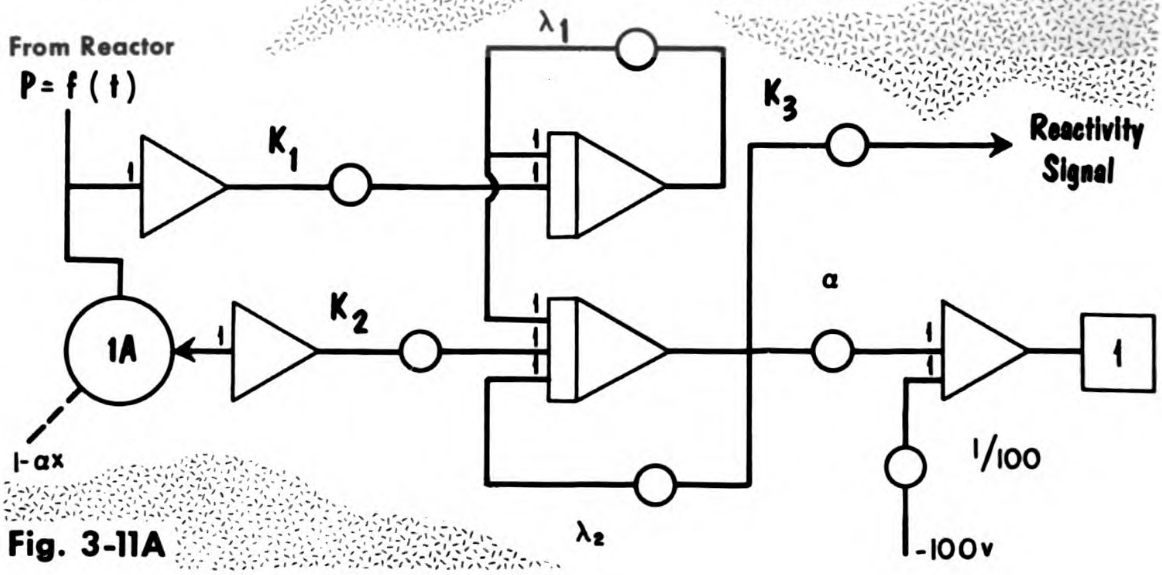


Fig. 3-11A

particles. These particles are other elements such as iodine, chlorine, xenon, samarium, barium, tellurium, etc. They are individually much lighter than uranium, and in fact the total mass of the fission products is less than that

of the uranium destroyed. The difference in mass is equivalent to the energy generated according to Einstein's law:

$$\text{Energy released} = (\text{mass change}) \times (\text{speed of light})^2$$

The fission products can be stable, taking no further part in the reactor processes, or they can be radioactive, and then they will behave in some

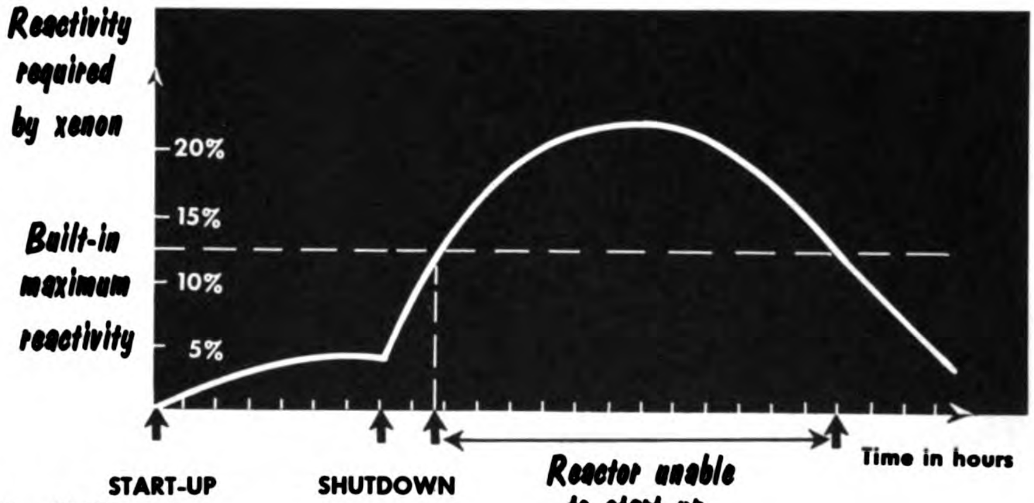


Fig. 3-11B

way or other which will influence the behavior of the reactor. Many of the radioactive substances produced have so little effect on the behavior of the reactor that they need no consideration. A few, however, exercise such a severe influence that their presence must be carefully considered in any design, and monitored in the operation of the reactor. The most important fission product from this consideration is a form of xenon. Its presence in the reactor can only be physically determined by chemical analysis of the reactor fuel rods. The amount of xenon present in the reactor, however, can be computed from a record of the power level, the computation requiring the solution of two time-varying differential equations (Fig. 3-11A).

Xenon prevents the fission of uranium by using the neutrons required for fission to change itself into another form — another isotope. The chances of a neutron producing fission in the presence of an atom of uranium and an atom of xenon is so small, that one can say that no fission will occur. Thus sufficient neutrons must be made available for the xenon, and for maintaining the fission process; the reactivity of the reactor must be large enough to maintain both processes. Under steady operating conditions, the xenon in a power reactor requires a reactivity of approximately 5%, and therefore it is normal to design a reactor to have a possible reactivity greater than this minimum. However, due to the mechanism by which the xenon is produced and removed should the reactor be closed down for any reason, the level of xenon rises, and will within a short time demand a reactivity much greater than 5%, and frequently greater than the value available in most reactors (Fig. 3-11B). When this condition occurs no

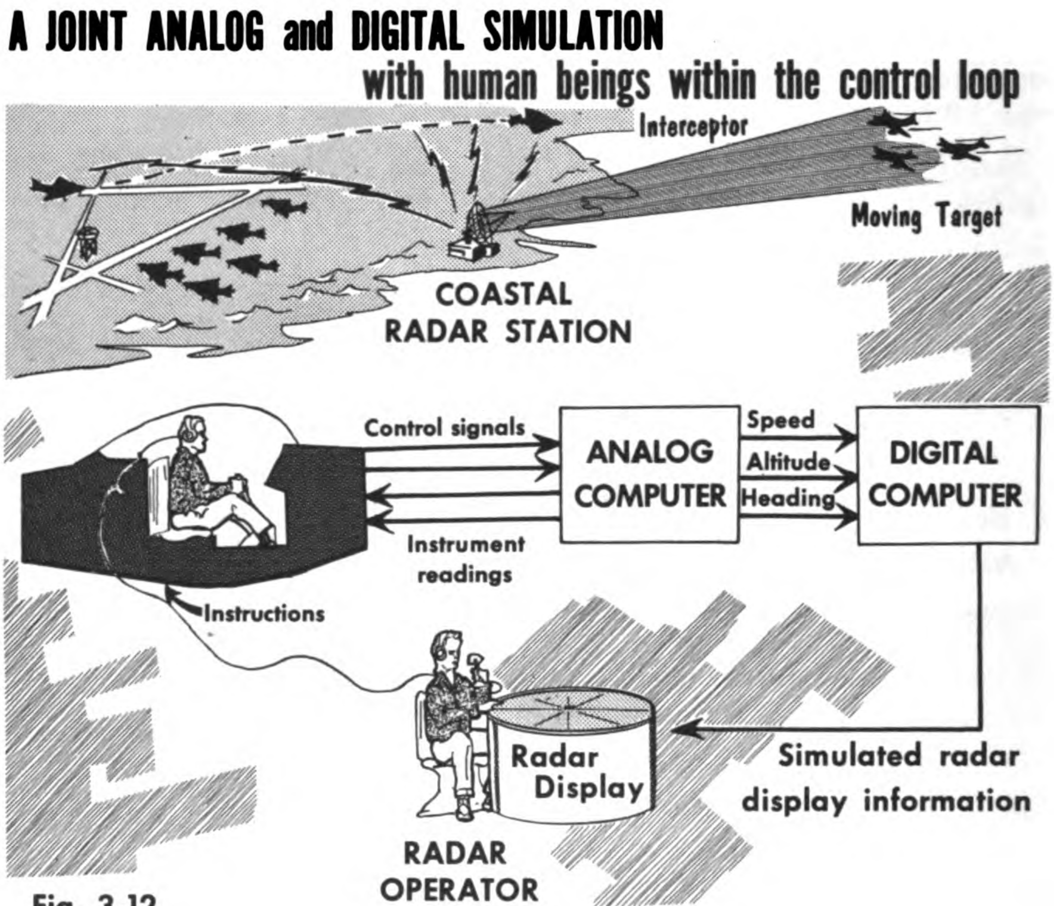


Fig. 3-12

neutrons will be available for fission, and the reactor will be unable to start up again until the level of xenon has been diminished by natural decay to cesium, a time-consuming process. A reactor could remain in this useless condition for a couple of days. Thus, if the reactor has to be shut down, it is essential to know what time is available before start-up is necessary, to prevent the level of xenon becoming critical. A simple analog-computer circuit is used to solve the computing problem and to present a continual record of the xenon level. The circuit receives a signal representing power level from the reactor. It can be operated in real time, or, by storing the power-level signal and speeding up the computer solution, the xenon level can be determined at regular intervals—every couple of hours.

REAL TIME JOINT ANALOG-DIGITAL SIMULATION

GCI Simulation

The GCI (Ground-Controlled Intercept) system is a network of long-range radar installations and jet-interceptor aircraft squadrons, on twenty-four hour duty and constant alert. Detection of unscheduled aircraft approaching the U.S. results in the take-off, *within minutes*, of the latest fighter

aircraft. Pilots are directed to the target by radio from the ground radar station. Since the radar operator can observe both the target and the interceptor aircraft, the whole system acts as a closed-loop feedback-control system. The radar operator and pilot are part of the loop.

In an effort to determine, among other things, the effect of human beings upon the performance of the system, investigators at the National Bureau of Standards have conducted a very interesting computer study to simulate the system. This study was one of the first to use both digital and analog computers together in a single simulation. Human operators were used for pilot and radar observer, the analog computer simulated the interceptor aircraft dynamics, while the digital computer substituted for the radar. The digital computer calculated the *position* of the simulated interceptor, receiving the speed, altitude, and heading information from the analog computer. A program in the digital computer furnished the target data, and the computer fed signals to a radar-type display unit for interpretation by the radar observer (Fig. 3-12). The pilot would then "fly" the analog computer, following instruction from the radar observer.

ICBM Simulation

As has been suggested earlier, the analog computer for many years has been the ideal tool for aircraft and missile designers. The flying of models was expensive and time consuming. The computer provided a means of

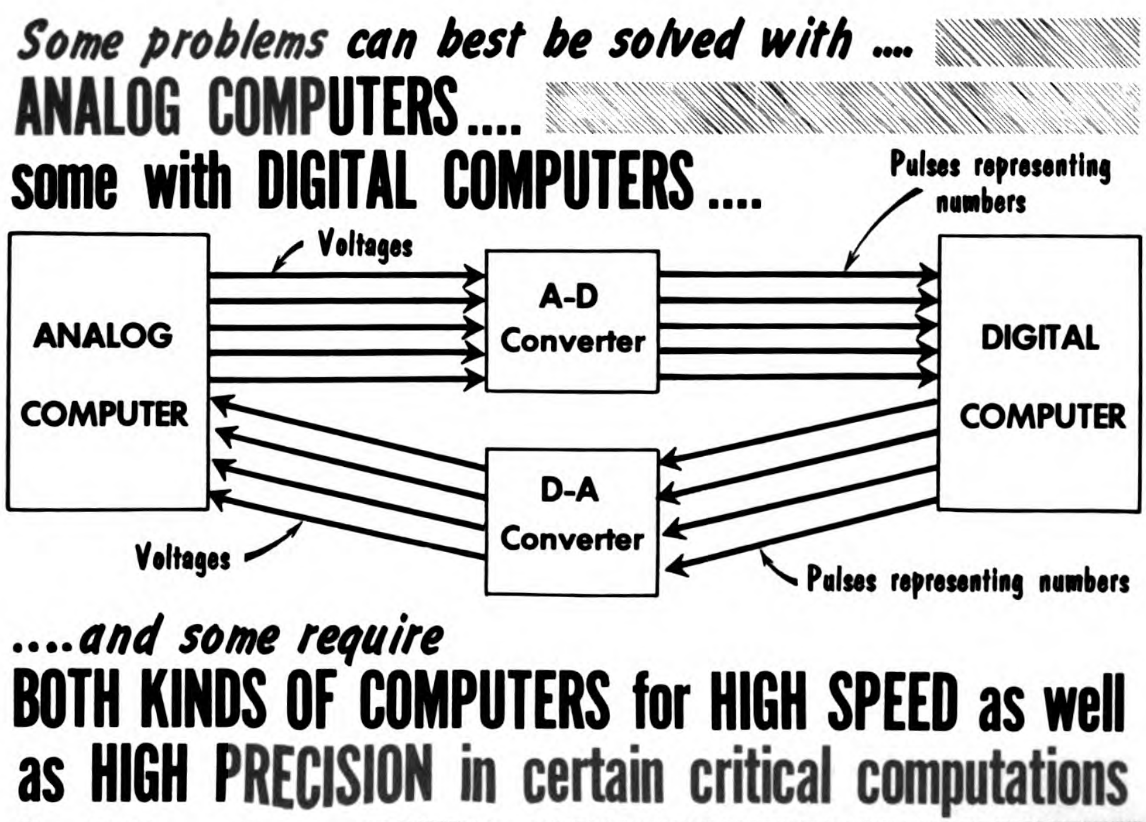


Fig. 3-13

"flying" a model right in the designers office, and the model could be changed with the turn of a knob, stopped "in mid-air," or completely dismantled in a few minutes. As missile-control systems grew more complex, the analog computer met the challenge and gave the designers an intimate feel for the very complicated systems they were designing. As design tolerances became more crucial, computer component precision was improved to the extent that adequate accuracy was maintained for the engineering job at hand. Missiles and analog computers grew together in the 1950's. Neither could have advanced as rapidly alone.

The development of an intercontinental ballistic missile posed a new kind of problem. An ICBM must be able to navigate over 5000 to 10,000 miles and hit a predetermined fixed target. To evaluate a particular design for the missile navigation and guidance system, the missile is simulated over its entire flight. However, a "miss" of 1 or 2 miles is such a small part of the total distance traveled, that it is difficult for an analog computer to accurately represent both the total distance and the "miss distance" at the same scale. Drift in the integrators during the relatively long computer run, and small noise voltages, contribute to computer errors of the same order of magnitude as the "miss distance" itself. Note that the ratio of "miss distance" to total distance is about the same as the minimum resolution of a good analog computer.

The answer is obtained by the combined use of analog and digital computers (Fig. 3-13) for ICBM simulation. The analog computer handles the many rapidly changing variables in the simulation of the missile itself, its guidance, and stabilization systems. The digital computer accepts speed, direction, and altitude information through an analog-to-digital converter, integrates speed to obtain distance traveled, and with its high resolution and drift-free operation keeps track of the exact position of the missile. In effect, the digital computer simulates the world, space, and the target, and acts as navigator for the simulated missile. Several missile manufacturers have engaged in large-scale studies of this kind, employing 300–400 analog-computer amplifiers and IBM-704 and UNIVAC-1103 digital computers — the largest and fastest available at the time. It should be noted that even these large, fast digital computers could not handle the entire ICBM simulation because they are unable to handle the complicated equations of missile dynamics with adequate speed.

QUESTIONS

1. What are partial differential equations? What kind of physical systems do they describe? Can they be solved successfully with the analog computer?
2. Discuss "real-time simulation." What is "real time"?
3. Why does the solution of partial differential equations on a computer appear to require more computing equipment?

4. Can you characterize with one or more general classifications the mathematical problems solved on analog computers?
5. Give an explanation of "three degrees of freedom", — "a six-degree of freedom simulation". Give examples.
6. Where are the "degrees of freedom" in the computer? Explain.
7. What problems arise when analog and digital computers are operated in tandem?
8. Are analog and digital computers compatible? What interconnecting equipment is required?
9. Why would one want to use analog and digital computers together?
10. Explain the terms, "computer model", "computer simulation".

GLOSSARY

AMPLIFIER, HIGH-GAIN: An operational amplifier having no feedback component. Extremely large changes in the output voltage occur for small changes in the input voltages.

AMPLIFIER, INTEGRATING: An operational amplifier having input resistors and a feedback capacitor arranged to cause the output voltage to be a precise time integral of the input voltages, each multiplied by a coefficient determined by the value of its input resistor:

$$E_{OUT} = -\int_{o}^{t} [a_1E_1 + a_2E_2 + \ldots + a_nE_n]\,dt$$

AMPLIFIER, INVERTING: An operational amplifier with equal input and feedback resistors able to change the sign of a varying voltage precisely.

AMPLIFIER, OPERATIONAL: The direct-voltage amplifier specially designed for use in the electronic differential analyzer to perform by simple feedback arrangements the mathematical operations of inversion, constant multiplication, algebraic summation, and/or integration with respect to time.

AMPLIFIER, SUMMING: An operational amplifier having input and feedback resistors arranged to cause the output voltage to be a precise algebraic sum of the input voltages, each multiplied by a coefficient determined by the value of its input resistor:

$$E_{OUT} = -[a_1E_1 + a_2E_2 + \ldots + a_nE_n]$$

AMPLITUDE SCALE FACTOR: A constant relating a computer voltage to the physical system characteristic it represents.

AUTOMATA (PLURAL OF AUTOMATION): Mechanical, electromechanical, or electronic devices which simulate human behavior to some useful degree.

BANDWIDTH: The bandwidth of a device is the frequency interval over which the frequency response is approximately constant, less than 30% change in amplitude response.

BINARY NUMBER: A number using base 2 and therefore needing only two symbols, 0 and 1.

BLOCK DIAGRAM: A pictorial representation of the mathematical description of a physical system.

CALCULUS: That branch of mathematics encompassing the methods of integration and differentiation.

CATHODE FOLLOWER: An arrangement of electron-tube circuitry in which the output voltage, taken from the tube cathode, follows closely the value of the input voltage applied to the tube control grid.

COMPUTER, ACTIVE-ELEMENT: A computer containing power-amplifying devices, which are used within individual computing components (integrators, multipliers, etc.,) to improve their per-

formance. It is capable of simulating the dynamic behavior of the majority of interesting physical systems.

Computer, Analog: A computer which permits the building of a model of a primary system whose behavior is to be investigated. The form of the computer provides flexibility in the model, simplicity in measurement of the values which describe the state of the system, and economy in the time, space and cost of the experiments to be performed.

Computer, Digital: A computer able to calculate rapidly in terms of the basic arithmetical operations of addition and subtraction.

Computer, Passive-Element: A computer containing no power-amplifying devices, which allows the building of a potential or displacement model of a primary system.

Computer Time: An artificial time variable employed in an analog computer simulation of a physical system. By a procedure called *time scaling*, events occurring in one second of time in the primary system may be investigated over one minute, say, in the simulation, or those occurring over a period of one hour can be considered in the computer in five seconds.

Computer Variable: A measurable variable in a computer. In the d-c analog computer, the computer variables are the voltages at outputs of computer building blocks which may vary continuously up to ± 100 volts. By suitable circuit arrangements, computer variables are forced to behave in a fashion analogous to the physical variables which describe the condition of the primary system.

Cosine: A trigonometric function. The cosine of an angle in a right triangle is equal to the ratio of a length of the side adjacent the angle to the length of the hypotenuse.

Coupled Equations: *See Simultaneous Equations.*

Damped Sinusoidal Function: A sinusoidal function with a constant periodicity and ever-decreasing positive and negative peak values. A very common function used to describe the dynamic behavior of oscillatory physical systems.

Decimal Number: A number using base 10 and therefore needing 10 symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Dependent Variable: *See Function.*

Derivative: The result of a differentiation process.

Differential Analyzer: A device for providing solutions to differential equations. The solutions are produced as time variations of easily measurable quantities (voltages, displacements).

Differential Equation: A mathematical statement (equations) including terms which are derivatives of variables — solved by methods of integration.

Differentiation: A mathematical process of determining the rate of change of a dependent variable with respect to changes in an independent variable.

Digital Differential Analyzer: A differential analyzer which employs the techniques and circuitry of a digital computer, thereby producing incremental rather than continuous changes in the values of the variables whose time behavior is desired.

Digital Voltmeter: A precision voltmeter which employs counting and logic circuits to measure and display numerically, the value of a voltage.

Diode-Function Generator: A function generator which contains diode bridges operating in parallel and biased to different voltage levels, to change the (output voltage) / (input voltage) relationship along a series of straight lines.

Drift: A relatively long-term fluctuation in value; of particular concern in precisely-balanced electronic computing circuits where long-term changes in component characteristics cause a change in circuit balance.

Dynamic Behavior: The time variations of the characteristics (e.g., position, velocity, voltage, current, flux density) of a physical system which is losing stored energy to its environment, or gaining energy due to external force (electrical, mechanical, or chemical).

Equation: A mathematical statement that a combination of variables and numbers is always equal to some other combination of variables and numbers, for all values of the independent variables included in the statement.

First-Order System: A physical system having one energy store and described by a first-order differential equation.

Frequency Response: The performance of a device in terms of the amplitude and phase of its output signal for a unit amplitude, zero phase, variable frequency input signal.

Function: A mathematical term for a variable which can assume changing values determined by the value of some other variable. The first is said to be a *function* of the latter variable. A function is a *dependent variable,* the value of which depends upon an independent variable.

Function Generator: A device which is able to produce an output variable related to the input variable driving the generator by a given relationship or known graphical correspondence.

Independent Variable: A variable whose values are independent of all other variables, a function of no other variable. To describe the behavior of any physical system, its characteristics are stated in terms of the independent variables of space and time (the position and time at which they are measured).

Initial Value: Initial Condition: The value of an integral at the beginning of the interval of integration. Initial conditions used refer to the initial values of variables in the solution of differential equations.

Integral: The result of an integration process.

Integrand: A variable which is to be integrated.

Integration: A mathematical process of continuing summation of the values of a dependent variable over some range of values of an independent variable.

Inverse Logarithm: or Antilogarithm ($\log_{10}^{-1}y$, $\log_e^{-1}y$): The inverse "log" of y is the number x which has a logarithm equal to y.

Inverse Tangent: or Arc Tangent: An inverse trigonometric function. The arc tangent of the number x is that angle which has a tangent equal to x.

Loading Error: The voltage error caused at the output of a computing component by a flow of current from that component to the succeeding ones to which it is connected. This error is serious only when the component from which the output voltage is taken has a high output impedance.

Logarithm, Base e — ($\log_e x$): A mathematical function of a variable x with a value equal to the power to which the number e must be raised to give a value of x. The number e, is approximately equal to 2.17183. $\log_e x$ is used extensively in the mathematics of physical systems.

Logarithm, Base 10 — ($\log_{10}x$): A mathematical function of a variable x with a value equal to the power of 10 which is equal to the number x.

Mathematical Variable: A mathematical entity, usually identified by a letter, such as x, y, and Θ, which can assume various numerical values. Mathematical variables are employed to describe the behavior of physical variables.

Multiplier, Constant: A computing component able to multiply a variable quantity by a constant coefficient value. The coefficient value is adjustable between computations, but not variable during computation.

Multiplier, Electronic: An all-electronic computing component able to produce a voltage proportional to the product of two time-varying voltages.

Multiplier, Quarter-Square: An electronic multiplier which employs the mathematical relationship,

$$xy = \frac{1}{4}\left[(x+y)^2 - (x-y)^2\right]$$

Multiplier, Servo: *See Servomultiplier.*

Negative Feedback: The transmission of energy from the output to the input of a device so that the output signal opposes the input signal. Negative feedback reduces the overall gain but improves the performance of the device from other considerations.

NETWORK ANALYZER: A convenient collection of passive electrical components which can be arranged as desired to form, under the influence of suitable current sources, a voltage-current distribution similar to the space distribution of an interesting quantity in a primary system.

PHYSICAL VARIABLE: A characteristic of a physical system which can have different values, as time changes, or as some other variables change.

"POT": Abbreviation for potentiometer.

POT-PADDER: *See Tapped-Potentiometer Function Generator.*

PRIMARY SYSTEM: A physical system which is to be described, analyzed, and studied by scale, mathematical, or computer models.

REAL TIME: The elapsed time indicated by a common clock, which is the common independent variable for the mathematics of primary systems, and which through time scaling may be compressed or expanded in an analog computer simulation. *See Computer Time.*

REPETITIVE OPERATION: A mode of computer control which alternates the computer between an OPERATING and RESET condition.

RESOLVER: A fixed-function generator which produces the trigonometric functions, sine and cosine, and is used to resolve the polar coordinates of a point into the corresponding rectangular coordinates.

SECOND-ORDER SYSTEM: A physical system having two energy stores, described by a second-order differential equation.

SERVO, OR SERVOMECHANISM: Usually an electromechanical device which causes the variation of an output *mechanical* variable to follow or to be the same as the variations of an input *electrical* variable, independent of changes in the load placed upon the output of the device.

SERVOMULTIPLIER: An electromechanical computing component able to produce a voltage proportional to the product of two time-varying voltages.

SIMULTANEOUS EQUATIONS: Algebraic and/or differential equations stating interdependent conditions upon the relationships of several variables, which must be solved simultaneously for any variable to be determined.

SIMULATION: The procedure of describing, analyzing, and studying the dynamic behavior of one physical system through the analogous behavior of a second physical system, a scale model, or in particular, an analog computer model.

SINE: A trigonometric function. The sine of an angle in a right triangle is equal to the ratio of the length of the side opposite the angle to the length of the hypotenuse.

SINUSOIDAL FUNCTION (OF AN ANGLE): A function which, as all the possible values of the angle are considered, varies as the sine or cosine of the angle. As a graph, the function is periodic, every 360° of angle, with equal positive and negative peak values.

STRIP-CHART RECORDER: An output-display device which traces graphically the changing values of voltages by deflecting a pen or stylus bearing on a strip of paper moving with constant velocity. The displacement of the pen is perpendicular to that of the paper, and is proportional to the voltage.

TANGENT: A trigonometric function. The tangent of an angle in a right triangle is equal to the ratio of the lengths of the opposite and adjacent sides of the angle.

TAPPED-POTENTIOMETER FUNCTION GENERATOR: A function generator using a potentiometer having many equally-spaced terminals to which selected constant voltages can be applied. The potentiometer wiper positioned by the input voltage provides an output voltage related as desired to the input voltage.

VARIABLE, A: Something which may take on a sequence of values. *See Physical Variable, Mathematical Variable, Computer Variable, Function.*

X-Y PLOTTER: An electromechanical output-display device which positions a pen at a point on a sheet of graph paper so that its rectangular coordinates referred to suitable axes are proportional to two input voltages.

NOTATION

Electrical Terms

V, E, v, e	voltage; measured in volts, millivolts (mv), microvolts (μv)
I, i	current; measured in amperes, milliamperes (ma), microamperes (μa)
R	resistance; measured in ohms (Ω), megohms (MΩ)
L	inductance; a property of coils, called inductors; measured in henries (hy), millihenries (mh)
C	capacitance; a property of capacitors, or condensers; measured in microfarads (μfd, mfd)
q	electric charge; measured in coulombs
P	power, electric; measured in watts (w)
P(t)	instantaneous power, or energy flow, as a function of time
W	stored electrical or mechanical energy
ϕ	the phase angle of an a-c electrical variable
B+	the positive d-c voltage which supplies current to the plate circuits of vacuum tubes

Mechanical Terms

F	force; measured in pounds
v, $\dot{x}$	velocity; measured in feet per second (fps), miles per hour (mph)
a, $\ddot{x}$	acceleration; measured in feet per second per second (fps^2), or in miles per hour per second (mph/sec)
M	mass; measured in slugs
W	weight; measured in pounds (lb)
K	constant of proportionality; often used as a "spring constant" describing stiffness of a mechanical spring, measured in pounds per foot (lb/ft)
D	damping constant; used to describe effect of air friction, shock absorbers, dash-pots; measured in pounds per foot per second (lb/fps)
P	pressure; measured in pounds per square foot (lb/ft^2)
x	a distance; measured in inches, feet, miles, etc.
Θ, α, β	angles; measured in degrees, minutes, and seconds, or in radians
x, y	rectangular (or cartesian) coordinates of a point in two dimensions; measured in any units of length
x, y, z	cartesian coordinates of a point in three dimensions
R	a radius, or radial distance
R, Θ	polar coordinates of a point in two dimensions

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